Coordination of Multiple Homogeneous or Heterogeneous Mobile Vehicles with Various Constraints

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Control of Autonomous Multi-Agent Systems II

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Introduction

- Objective: Multi-vehicle coordination control with heterogeneous systems
- Previous work focuses mainly on multi-agent systems with strict equality constraints
- Main contribution: Tackle a more general problem that includes inequality constraints or a mix of equality and inequality constraints

Problem Formulation

- Consider a group of n vehicles, whose kinematic equations are described by

$$\dot{p}_i = f_{i,0} + \sum_{j=1}^{l_i} f_{i,j} u_{i,j}$$

- Consider assigning the vehicle group with a coordination task described by inequality constraints
- Two key problems to be addressed
  - Determine if the group can perform the coordination task with various constraints
  - Obtain feasible motions that allow the group to perform the task

Vehicle Kinematics Constraints

- Vehicle's kinematics equivalent constraints

$$\omega_{i,j}(p_i)\dot{p}_i = q_{i,j}, \qquad j = 1, ..., n_i - l_i$$

- Collect all row covectors as

$$\Omega_{K_i} = \begin{bmatrix} \omega_{i,1}^T & \omega_{i,2}^T & \dots & \omega_{i,n_i-l_i}^T \end{bmatrix}^T$$

$$T_{K_i} = \begin{bmatrix} q_{i,1}^T & q_{i,2}^T & \dots & q_{i,n_i-l_i}^T \end{bmatrix}^T$$

$$\Omega_{K_i}\dot{p}_i = T_{K_i}$$

- For all n vehicles

$$\Omega_K(\dot{P}) = T_K$$

Coordination Task Equality Constraints (1/2)

- Assume a networked multi-vehicle control system modelled by an undirected graph $\mathcal{G}$ s.t. $\mathcal{V}$ denotes vertex set and $\mathcal{E}$ denotes the edge set
- A family of equality constraints Φ is indexed by the edge set
- The constraint for edge (i, j) is enforced if

$$\Phi_{ij}(p_i, p_j) = 0$$

Coordination Task Equality Constraints (2/2)

- To satisfy the equality constraint

$$\frac{d}{dt}\Phi_{ij}(p_i, p_j, t) = \frac{\partial \Phi_{ij}}{\partial p_i}\dot{p}_i + \frac{\partial \Phi_{ij}}{\partial p_j}\dot{p}_j + \frac{\partial \Phi_{ij}}{\partial t} = 0$$

$$\frac{d}{dt}\Phi(P, t) = \frac{\partial \Phi}{\partial P}\dot{P} + \frac{\partial \Phi}{\partial t} = 0$$

- Group all the constraints for all the edges

$$\Omega_E = \frac{\partial \Phi}{\partial P}$$

$$T_E = -\begin{bmatrix} \dots & (\frac{\partial \Phi_{ij}}{\partial t})^T & \dots \end{bmatrix}^T$$

$$\Omega_E \dot{P} = T_E$$

Coordination Task Inequality Constraints (1/2)

- A family of inequality coordination constraints $\mathcal{I}_{\mathcal{E}} = \{\mathcal{I}_{ij}\}_{(i,j)}$ is indexed by the edge set $\mathcal{E}$
- The constraints for the edge (i, j) are enforced if

$$\mathcal{I}_{ij}(p_i(t), p_j(t)) \leq 0 \qquad \forall t$$

- Consider subset of active constraints among all edges

$$\mathcal{X}(P) = \{(i,j), i,j = 1, 2, ..., n \mid \mathcal{I}_{ij}(p_i, p_j) = 0\}$$

Coordination Task Inequality Constraints (2/2)

- An inequality constraint for edge (i, j) is maintained if

$$\frac{d}{dt}\mathcal{I}_{ij} = \frac{\partial \mathcal{I}_{ij}}{\partial p_i}\dot{p}_i + \frac{\partial \mathcal{I}_{ij}}{\partial p_j}\dot{p}_j \leq 0, \qquad \forall (i, j) \in \mathcal{X}(P)$$

- Group all the inequality constraints for all active edges

$$\Omega_I = \frac{\partial \mathcal{I}}{\partial P}$$

$$\Omega_I \dot{P}(P, u(t)) \leq 0, \qquad \forall (i, j) \in \mathcal{X}(P)$$

Coordination Feasibility

- The coordination task with inequality constraints has feasible motions if the following mixed equations and inequalities have a solution

$$\Omega_K \dot{P} = T_K$$

$$\Omega_I \dot{P} \leq 0 \qquad \forall (i,j) \in \mathcal{X}(P)$$

- The coordination task with both equality and inequality constraints has feasible motions if the following mixed equations and inequalities have a solution

$$\Omega_K \dot{P} = T_K$$

$$\Omega_E \dot{P} = T_E$$

$$\Omega_I \dot{P} \leq 0 \qquad \forall (i,j) \in \mathcal{X}(P)$$

Generating Feasible Motion Algorithm (1/2)

- Initialize: $\Omega_K, T_K, \Omega_E, T_E, \mathcal{X}(P), \Omega_I$
- While running
  - Solve equality $\begin{bmatrix} \Omega_K \\ \Omega_E \end{bmatrix} \dot{P} = \begin{bmatrix} T_K \\ T_E \end{bmatrix}$
  - If solution exists
    - Calculate special solution $\overline{K}$
    - Calculate basis vectors $K_1, ..., K_{\mathcal{N}}$ of $Null(\begin{bmatrix} \Omega_K \\ \Omega_E \end{bmatrix})$
    - Such that the solution can be written as

$$\dot{P} = \overline{K} + \sum_{l=1}^{\mathcal{K}} K_l \omega_l$$

    - ω_l is a set of virtual inputs that activate the associated vector field K_l

Generating Feasible Motion Algorithm (2/2)

- If $\mathcal{X}(P) = \phi$ (No active inequality constraints)
  - Return set of feasible motions $\dot{P} = \overline{K} + \sum_{l=1}^{\mathcal{K}} K_l \omega_l$
- else
  - Calculate the co-distribution matrix Ω_l for *active inequality constraints*
  - If $\Omega_l(\overline{K} + K_l \omega_l) \leq 0$ for certain ω_l
    - Return feasible motion $\dot{P} = \overline{K} + K_l \omega_l$

Coordinating Heterogeneous Vehicles with Inequality Constraints (1/6)

- Example 1: Unicycle and Car-like vehicle
- Unicycle Dynamics:

$$\dot{x}_1 = v_1 cos(\theta_1)$$

$$\dot{y}_1 = v_1 sin(\theta_1)$$

$$\dot{\theta}_1 = u_1$$

- Kinematic Constraint:

$$\Omega_{K_1} = span\{sin(\theta_1)dx_1 - cos(\theta_1)dy_1\}$$

Coordinating Heterogeneous Vehicles with Inequality Constraints (2/6)

- Car-like Dynamics:

$$\dot{x}_2 = v_2 cos(\theta_2)$$

$$\dot{y}_2 = v_2 sin(\theta_2)$$

$$\dot{\theta}_2 = v_2(1/l) tan(\phi_2)$$

$$\dot{\phi}_2 = u_2$$

- Kinematic Constraint:

$$\Omega_{K_2} = span\{sin(\theta_2+\phi_2)dx_2 - cos(\theta_2+\phi_2)dy_2 - lcos(\phi_2)d\theta_2, sin(\theta_2)dx_2 - cos(\theta_2)dy_2\}$$

Coordinating Heterogeneous Vehicles with Inequality Constraints (3/6)

- Solving $\Omega(\dot{P}) = T = 0$, while $\dot{P} = \sum_{l=1}^{3} \omega_l K_l$
- Thus, $K_1 = [0, 0, 1, 0, 0, 0, 0]^T$, $K_2 = [0, 0, 0, 0, 0, 0, 1]^T$ and $K_3 =$

$$\begin{bmatrix} \cos(\theta_1)(\cos(\theta_2)(x_1 - x_2) + \sin(\theta_2)(y_1 - y_2)) \\ \sin(\theta_1)(\cos(\theta_2)(x_1 - x_2) + \sin(\theta_2)(y_1 - y_2)) \\ 0 \\ \cos(\theta_2)(\cos(\theta_1)(x_1 - x_2) + \sin(\theta_1)(y_1 - y_2)) \\ \sin(\theta_2)(\cos(\theta_1)(x_1 - x_2) + \sin(\theta_1)(y_1 - y_2)) \\ \frac{1}{l_2}\tan(\phi_2)(\cos(\theta_1)(x_1 - x_2) + \sin(\theta_1)(y_1 - y_2)) \\ 0 \end{bmatrix}$$

Coordinating Heterogeneous Vehicles with Inequality Constraints (4/6)

- Consider the distance equality constraint:

$$\Phi_{ij} : \frac{1}{2}(x_i - x_j)^2 + \frac{1}{2}(y_i - y_j)^2 - \frac{1}{2}(d_{ij})^2 = 0$$

- with a co-distribution matrix:

$$\Omega_{E,ij} : (x_i - x_j)(dx_i - dx_j) + (y_i - y_j)(dy_i - dy_j)$$

Coordinating Heterogeneous Vehicles with Inequality Constraints (5/6)

- In practical application, it's more useful to consider distance as an inequality constraint:

$$\mathcal{I}_{ij}^{(1)} : \frac{1}{2}(d_{ij}^{-})^2 \leq \frac{1}{2}(x_i - x_j)^2 + \frac{1}{2}(y_i - y_j)^2 \leq \frac{1}{2}(d_{ij}^{+})^2$$

$$\text{where } d_{ij}^{-}, d_{ij}^{+} > 0$$

- And co-distribution matrix is given by: $\Omega_{I,ij} = \Omega_{E,ij}$ if the right inequality is active, or $\Omega_{I,ij} = -\Omega_{E,ij}$ if the left inequality is active.

Coordinating Heterogeneous Vehicles with Inequality Constraints (6/6)

- Some additional remarks:
- When constraints are satisfied both vehicles move randomly
- Special solution to kinematic constraints is taken as zero (driftless vehicles)
- Algorithm 1 provides a satisfying system velocity $\dot{P} = [\dot{x}_1, \dot{y}_1, \dot{\theta}_1, \dot{x}_2, \dot{y}_2, \dot{\theta}_2, \dot{\phi}_2]$, from which we then compute the controls required to achieve it as:

$$v_1 = \mathrm{sgn}(\mathrm{sgn}(\dot{x}_1)\,\mathrm{sgn}(\cos\theta_1) + \mathrm{sgn}(\dot{y}_1)\,\mathrm{sgn}(\sin\theta_1))\sqrt{\dot{x}_1^2 + \dot{y}_1^2}, u_1 = \dot{\theta}_1$$

$$v_2 = \mathrm{sgn}(\mathrm{sgn}(\dot{x}_2)\,\mathrm{sgn}(\cos\theta_2) + \mathrm{sgn}(\dot{y}_2)\,\mathrm{sgn}(\sin\theta_2))\sqrt{\dot{x}_2^2 + \dot{y}_2^2}, u_2 = \dot{\phi}_2$$

Coordinating Heterogeneous Vehicles with Inequality Constraints / Simulation (1/2)

- Building up the system model:

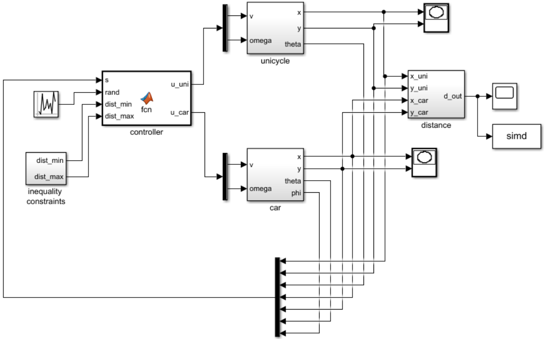

Figure 1: Simulink model of a unicycle and a car-like vehicle simulation

Coordinating Heterogeneous Vehicles with Inequality Constraints / Simulation (2/2)

- Following Algorithm 1, the executed results are shown in figure 2:

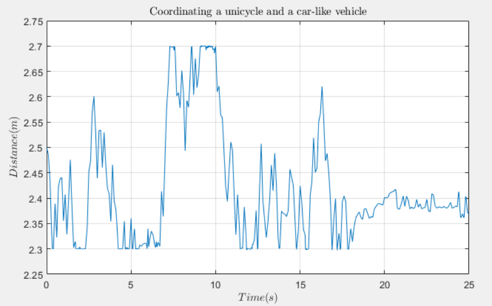

Figure 2: Coordinating a unicycle and a car-like vehicle

Coordinating Homogeneous Vehicles with Equality and Inequality Constraints (1/8)

- Example 2: Three Unicycle vehicles
- Multiple homogeneous vehicles with mixed constraints
- One leader and two followers
- Unicycles: i=1,2,3

$$\dot{x}_i = v_i cos(\theta_i)$$

$$\dot{y}_i = v_i sin(\theta_i)$$

$$\dot{\theta}_i = u_i$$

Coordinating Homogeneous Vehicles with Equality and Inequality Constraints (2/8)

- As before, the kinematics of each vehicle is expressed as

$$\Omega_{K_i} = span\{sin(\theta_i)dx_i - cos(\theta_i)dy_i\}$$

- Resulting in

$$\Omega_K \in \mathbb{R}^{3x9}$$

$$T_K = [0, 0, 0]^T$$

Coordinating Homogeneous Vehicles with Equality and Inequality Constraints (3/8)

- The leader is constrained to follow an arbitrary reference trajectory in terms of two continuous control inputs

$$v_{1,r}(t), u_{1,r}(t)$$

- These time-varying speeds are incorporated as two equality constraints:

$$cos(\theta_1)dx_1 + sin(\theta_1)dy_1 = v_{1,r}(t)$$

$$d\theta_1 = u_{1,r}(t)$$

- Obtaining

$$\Omega_E \in \mathbb{R}^{2x9}, T_E = [v_{1,r}(t), u_{1,r}(t)]^T \in \mathbb{R}^2$$

Coordinating Homogeneous Vehicles with Equality and Inequality Constraints (4/8)

- Solving the equality $\begin{bmatrix} \Omega_K \\ \Omega_E \end{bmatrix} \dot{P} = \begin{bmatrix} T_K \\ T_E \end{bmatrix}$
- We obtained the special solution

$$\overline{K} = \begin{bmatrix} v_{1,r}cos(\theta_1) & v_{1,r}sin(\theta_1) & u_{1,r} & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}^T$$

- The basis vectors $K_1, ..., K_{\mathcal{N}}$ of $Null(\begin{bmatrix} \Omega_K \\ \Omega_E \end{bmatrix})$

$$K_1 = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \end{bmatrix}^T$$
$$K_2 = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}^T$$
$$K_3 = \begin{bmatrix} 0 & 0 & 0 & cos(\theta_2) & sin(\theta_2) & 0 & 0 & 0 & 0 \end{bmatrix}^T$$
$$K_4 = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & cos(\theta_3) & sin(\theta_3) & 0 \end{bmatrix}^T$$

Coordinating Homogeneous Vehicles with Equality and Inequality Constraints (5/8)

- Write distance constraint as two inequalities

$$\mathcal{I}_{ij}^{(1)} : \frac{1}{2}(d_{ij}^{-})^2 \leq \frac{1}{2}(x_i - x_j)^2 + \frac{1}{2}(y_i - y_j)^2 \leq \frac{1}{2}(d_{ij}^{+})^2$$

$$\text{s.t. } d_{ij}^{-}, d_{ij}^{+} > 0$$

- If the right inequality becomes active, co-distribution matrix is given by

$$\Omega_{I,ij}^{(1)} = (x_i - x_j)(dx_i - dx_j) + (y_i - y_j)(dy_i - dy_j)$$

- If the left inequality becomes active, co-distribution matrix is given by

$$\Omega_{I,ij}^{(1)} = -(x_i - x_j)(dx_i - dx_j) - (y_i - y_j)(dy_i - dy_j)$$

Coordinating Homogeneous Vehicles with Equality and Inequality Constraints (6/8)

- We want the followers to maintain visibility of the leader; thus, we pose the visibility inequality constraint

$$\mathcal{I}_{ij}^{(2)} : cos(\alpha_{ij})\langle a_{ij}, a_{ij}\rangle^{1/2} \leq \langle a_{ij}, b_j\rangle$$

- If the inequality constraint becomes active, the associated co-distribution matrix is given by

$$\Omega_{I,ij}^{(2)} = \frac{cos(\alpha_{ij})}{\sqrt{\langle a_{ij}, a_{ij}\rangle}} \left\langle a_{ij}, \begin{bmatrix} dx_i - dx_j \\ dy_i - dy_j \end{bmatrix} \right\rangle - \left\langle \begin{bmatrix} dx_i - dx_j \\ dy_i - dy_j \end{bmatrix}, b_j \right\rangle - \langle a_{ij}, c_j\rangle d\theta_j$$

s.t. $a_{ij} = \begin{bmatrix} x_i - x_j & y_i - y_j \end{bmatrix}, \quad b_j = \begin{bmatrix} cos(\theta_j) & sin(\theta_j) \end{bmatrix}, \quad c_j = \begin{bmatrix} -sin(\theta_j) & cos(\theta_j) \end{bmatrix}$

Coordinating Homogeneous Vehicles with Equality and Inequality Constraints (7/8)

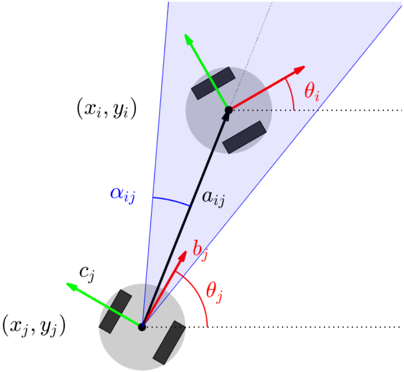

Figure 3: Visibility Inequality Constraint

Coordinating Homogeneous Vehicles with Equality and Inequality Constraints (8/8)

- Constant parameters will be

$$\alpha_{12} = \alpha_{13} = 0.4, \quad d_{12}^- = d_{13}^- = 1, \quad d_{12}^+ = d_{13}^+ = 2$$

- Leader vehicle reference trajectory

$$v_{1,r}(t) = 2sin(t), \quad u_{1,r}(t) = 2cos(2t)$$

- Combining the constraints

$$\Omega_l = \begin{bmatrix} (\Omega_{l,12}^{(1)})^T & (\Omega_{l,13}^{(1)})^T & (\Omega_{l,12}^{(2)})^T & (\Omega_{l,13}^{(2)})^T \end{bmatrix} \in \mathbb{R}^{6\times 9}$$

- At most four constraints may be active at any point in time

Coordinating Homogeneous ... / Simulation (1/5)

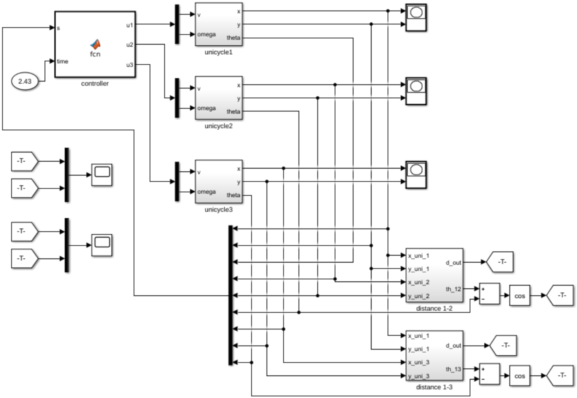

Figure 4: Simulink model of Coordinating 3 unicycle vehicles simulation

Coordinating Homogeneous ... / Simulation (2/5)

Coordinating Homogeneous ... / Simulation (3/5)

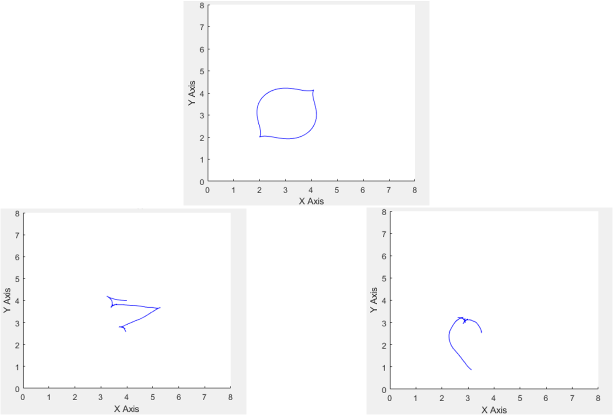

Figure 5: Trajectories for the three unicycle vehicles

Coordinating Homogeneous ... / Simulation (4/5)

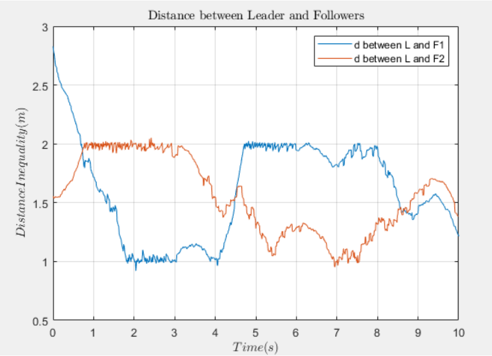

Figure 6: Distance between leader and followers unicycle vehicles

Coordinating Homogeneous ... / Simulation (5/5)

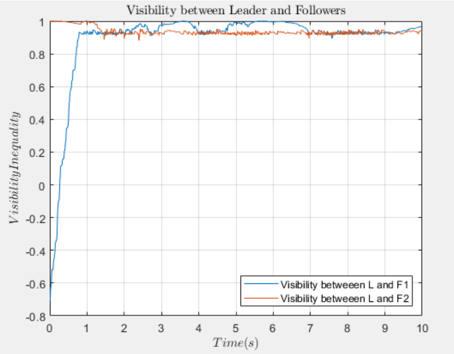

Figure 7: Visibility between leader and followers unicycle vehicles

Conclusions

- This work discusses the coordination control problem for multiple mobile vehicles subject to various constraints (nonholonomic motion constraints, holonomic formation constraints, equality or inequality constraints, among others)
- A heuristic algorithm is proposed to find feasible motions and trajectories
- The algorithm does not generate all feasible motion directions; generalizations of the algorithm and selections of optimal motion directions are issues for future research

Thank you for your attention

References

[1] Sun Z, Greiff M, Robertsson A, Johansson R. Feasible coordination of multiple homogeneous or heterogeneous mobile vehicles with various constraints. In2019 International Conference on Robotics and Automation (ICRA) 2019 May 20 (pp. 1008-1013). IEEE.