

Department : Mathematics and Computer Science  
Year : Third Level  
Course Code: 040101309  
Course Title: Integral equations  
Semester : Fall 2023  
Time Allowed : 30 minutes  
Lecturer: Dr.Hanna R. Ebead  
Total Marks: 30



## Mid-Term Examination Paper

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1. Solve the integral equations.

(a)  $y(x) = x - \int_0^x (x-t)y(t)dt$

(b)  $y(t) = \frac{7}{8}t + \frac{1}{2} \int_0^1 tsy^2(s) ds$

2. (a) Let  $f \in C[a, b]$ . If  $k : [a, b] \times [a, b] \rightarrow \mathbb{R}$  is continuous on  $[a, b] \times [a, b]$  Then prove that

$$y(t) = f(t) + \lambda \int_a^b k(t, s)y(s) ds \quad t \in [a, b]$$

has a unique continuous solution  $y \in C[a, b]$

Provided that  $|\lambda| < \frac{1}{L(b-a)}$ , Where  $L = \max_{t,s \in [a,b] \times [a,b]} |k(t, s)|$

- (b) Give an counter example to show that the previous theorem is sufficient but not necessary

3. Convert the following IVP

$$\begin{cases} \frac{d^2y}{dx^2} - 2x \frac{dy}{dx} - 3y = 0 \\ y(0) = 1, y'(0) = 0 \end{cases}$$

Into equivalent Volterra integral equation

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