

Department : Mathematics and Computer Science
Level : Third Level
Course Code : 040101309
Course Title : Integral Equations
Semester : Fall 2023
Time Allowed : 30 minutes
Lecturer : Dr.Hanna R. Ebead
Total Marks : 30 Points



Mid-Term Examination Paper

1. Solve the Integral Equations

(a) $y(x) = x - \int_0^x (x-t)y(t)dt$

(b) $y(t) = \frac{7}{8}t + \frac{1}{2} \int_0^1 tsy^2(s) ds$

2. (a) Let $f \in C[a, b]$. If $k : [a, b] \times [a, b] \rightarrow \mathbb{R}$ is continuous on $[a, b] \times [a, b]$ Then prove that

$$y(t) = f(t) + \lambda \int_a^b k(t, s)y(s) ds, \quad t \in [a, b]$$

has a unique continuous solution $y \in C[a, b]$

Provided that $|\lambda| < \frac{1}{L(b-a)}$, Where $L = \max_{t, s \in [a, b] \times [a, b]} |k(t, s)|$

(b) Give a counter example to show that the previous theorem is sufficient but not necessary

3. Convert the following IVP

$$\begin{cases} \frac{d^2 y(x)}{dx^2} - 2x \frac{dy(x)}{dx} - 3y(x) = 0 \\ y(0) = 1, y'(0) = 0 \end{cases}$$

Into equivalent Volterra Integral Equation
