1-1 Using *Successive Approximation Method*
Let initial guess $y_0(t) = x$ Then

$$\begin{aligned} y_1(t) &= x - \int_0^x (x-t)t\,dt = x - \frac{x^3}{3!} \\ y_2(t) &= x - \int_0^x (x-t)(t - \frac{t^3}{3!})\,dt = x - \frac{x^3}{3!} + \frac{x^5}{5!} \end{aligned}$$

1-2 Since, $k(t,s) = ts$ separable, we can use *Direct Computational Medthod*

$$y(t) = \frac{7}{8}t + \frac{1}{2}t\int_0^1 tsy^2(s)\,ds$$

Let $C := \int_0^1 sy^2(s)\,ds$

$$\begin{aligned} y(t) &= \frac{7}{8}t + \frac{1}{2}tC \\ C &= \int_0^1 s\left[\frac{7}{8}s + \frac{1}{2}sC\right]^2\,ds \\ C &= \int_0^1 s^3\left[\frac{7}{8} + \frac{1}{2}C\right]^2\,ds = \frac{1}{4}\left[\frac{7}{8} + \frac{1}{2}C\right]^2 \end{aligned}$$

$$16C^2 - 200C + 49 = 0$$

We get two solutions

$$\begin{cases} y_1(t) = 7t\mathrm{At}C_1 = \frac{49}{4} \\ y_2(t) = t\mathrm{At}C_2 = \frac{1}{4} \end{cases}$$

Theorem//

Since $k(t,s)$ is continuous on closed domain $[a,b]\times[a,b]$. so it guarantees the existence of a finite $L > 0$
From *Existence and uniqueness theorem* we are able to prove that the integral equation has a unique continuous solution $y \in C[a,b]$
Since $f, k(t,.) \in C[a,b] \subset L_2[a,b] \quad t \in [a,b]$
Then $M = \int_a^b \int_a^b |k(t,s)|^2\,dsdt \leq \int_a^b \int_a^b L^2\,dsdt = L^2(b-a)^2$

$$\sqrt{M} \leq L(b-a)$$

Thus, we deduce that

$$|\lambda| < \frac{1}{L(b-a)}$$