Alexandria University - Faculty of Science
Mid term examination – 2024

Department: Mathematics and Computer Science
Course Title: Integral equations
Course Code: Integral equations
Lecturer: Dr.Hanna R. Ebead
Total Marks: 30

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1. Solve the integral equations.
   (a) $y(x) = x - \int_0^x (x-t)y(t)\,dt$
   (b) $y(t) = \frac{7}{8}t + \frac{1}{2}\int_0^1 tsy^2(s)\,ds$

2. (a) Let $f \in C[a,b]$. If $k : [a,b]\times[a,b] \to \mathbb{R}$ is continuous on $[a,b]\times[a,b]$ Then prove that

$$y(t) = f(t) + \lambda \int_a^b k(t,s)y(s)\,ds \quad t \in [a,b]$$

   has a unique continuous solution $y \in C[a,b]$
   Provided that

$$|\lambda| < \frac{1}{L(b-a)}, \quad \text{Where} \quad L = \max_{t,s\in[a,b]\times[a,b]} |k(t,s)|$$

   (b) Give an counter example to show that the previous theorem is sufficient but not necessary

3. Convert the following IVP

$$\begin{cases} \frac{\mathrm{d}^2y}{\mathrm{d}x^2} - 2x\frac{\mathrm{d}y}{\mathrm{d}x} - 3y = 0 \\ y(0) = 1, y'(0) = 0 \end{cases}$$

   Into equivalent Volterra integral equation