Voronoi diagrams have linear complexity

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From Handshaking theorem

$$\sum_{v \in Vor(P)} deg(v) = 2|e| \tag{1}$$

$$\forall v \in Vor(P), \qquad deg(v) \geq 3$$

$$2|e| \geq 3(|v| + 1), \tag{2}$$

Euler's formula

$$(|v| + 1) - |e| + n = 2 \tag{3}$$

multiplying equation(3) by 3

$$3(|v| + 1) - 3|e| + 3n = 6$$

$$3(|v| + 1) = 3|e| - 3n + 6$$

$$2|e| \geq 3|e| - 3n + 6$$

$$3n - 6 \geq |e| \Rightarrow |e| \leq 3n - 6$$

multiplying equation(3) by 2

$$2(|v| + 1) - 2|e| + 2n = 4$$

and from equation(1) and equation(3)

$$2|e| = 2(|v| + 1) + 2n - 4 \geq 3(|v| + 1)$$

$$2n - 4 \geq (|v| + 1)$$

$$|v| \leq 2N - 5$$

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