

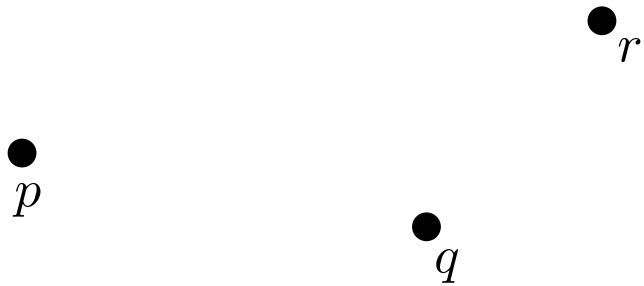
Basic tools: orientation tests

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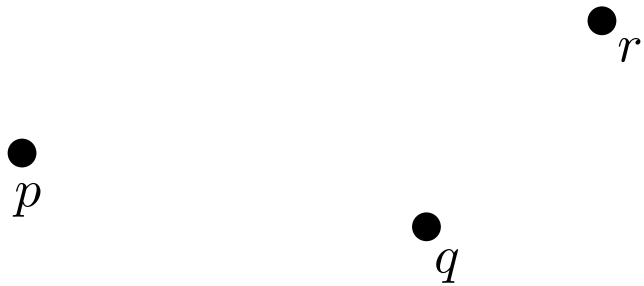
Turning right or turning left?

Consider a sequence of 3 points: p, q, r



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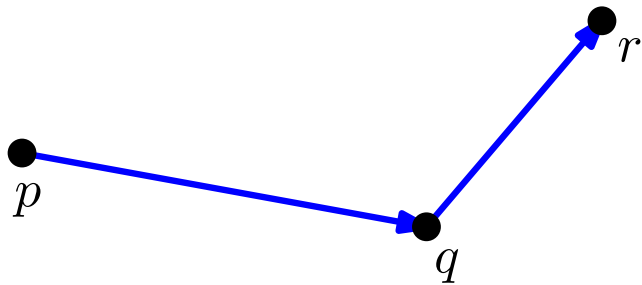
Basic question: do they make a right turn?

A left turn?

Or perhaps they are colinear?

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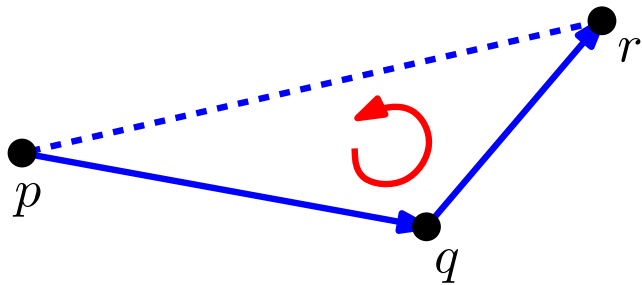
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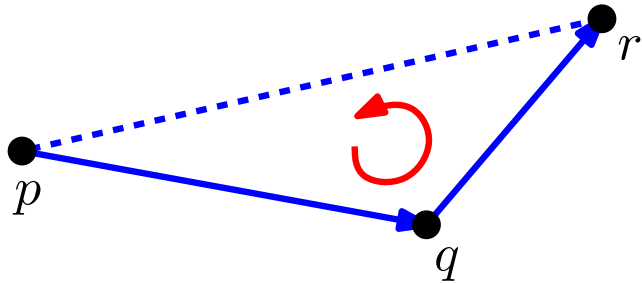
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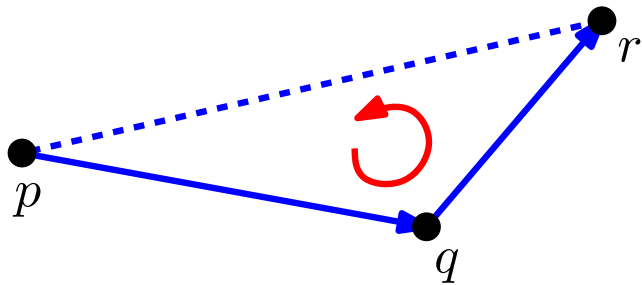
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- Efficient
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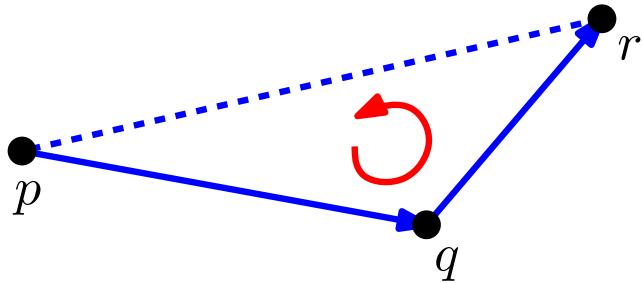
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Solution: **signed area of triangle** $\triangle pqr$

Signed area of triangle

Signed area of $\triangle pqr$

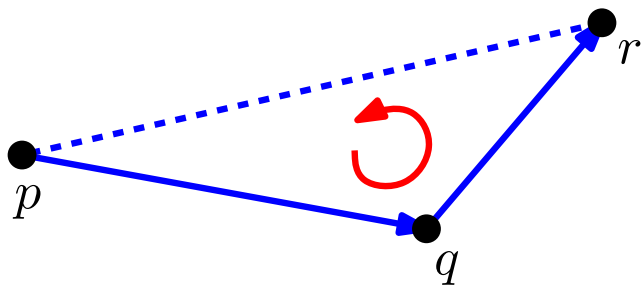


The signed area of $\triangle pqr$ tells us the area of the triangle, and the **sign** gives the orientation (left-turning or right-turning)

- Can the area be zero?

Signed area of triangle

Signed area of $\triangle pqr$

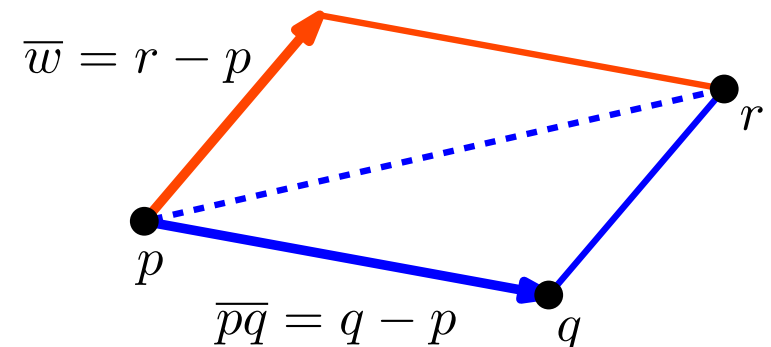


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Computing $A(\triangle pqr)$

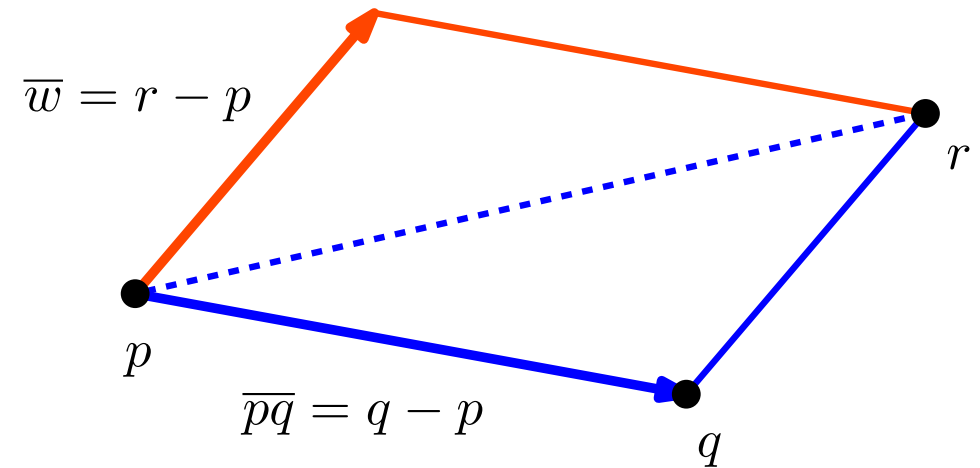
- $A(\triangle pqr)$ is twice the area of the parallelogram defined by vectors \overrightarrow{pq} and \overrightarrow{w}
- Recall: in 2-dimensions, the area of the parallelogram defined by two vectors is the determinant of a 2×2 matrix whose columns are the two vectors



Signed area of triangle

Signed area of $\triangle pqr$

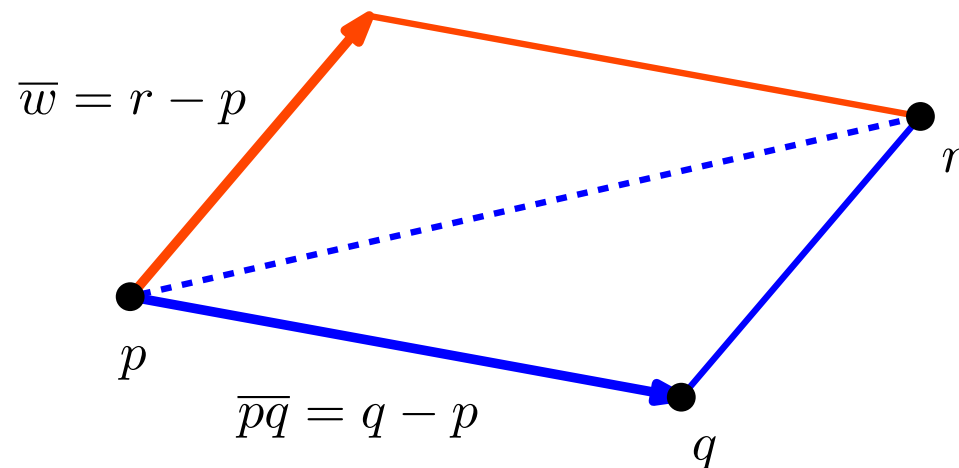
$$A(\triangle pqr) = \frac{1}{2} \begin{vmatrix} \overline{pq}_x & \overline{w}_x \\ \overline{pq}_y & \overline{w}_y \end{vmatrix}$$



Signed area of triangle

Signed area of $\triangle pqr$

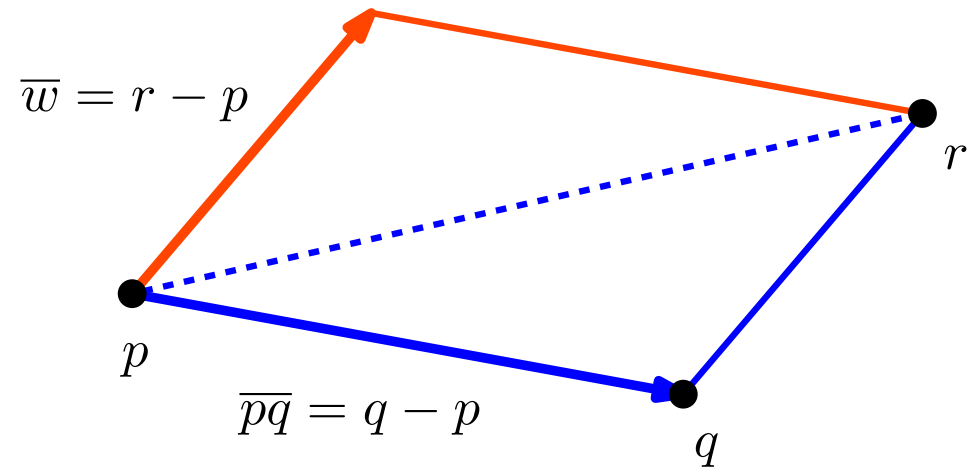
$$\begin{aligned} A(\triangle pqr) &= \frac{1}{2} \begin{vmatrix} \overline{pq}_x & \overline{w}_x \\ \overline{pq}_y & \overline{w}_y \end{vmatrix} \\ &= \frac{1}{2} \begin{vmatrix} (q-p)_x & (r-p)_x \\ (q-p)_y & (r-p)_y \end{vmatrix} \\ &= \frac{1}{2} \begin{vmatrix} q_x - p_x & r_x - p_x \\ q_y - p_y & r_y - p_y \end{vmatrix} \\ &= \frac{1}{2} \begin{vmatrix} p_x & q_x & r_x \\ p_y & q_y & r_y \\ 1 & 1 & 1 \end{vmatrix} \end{aligned}$$



Signed area of triangle

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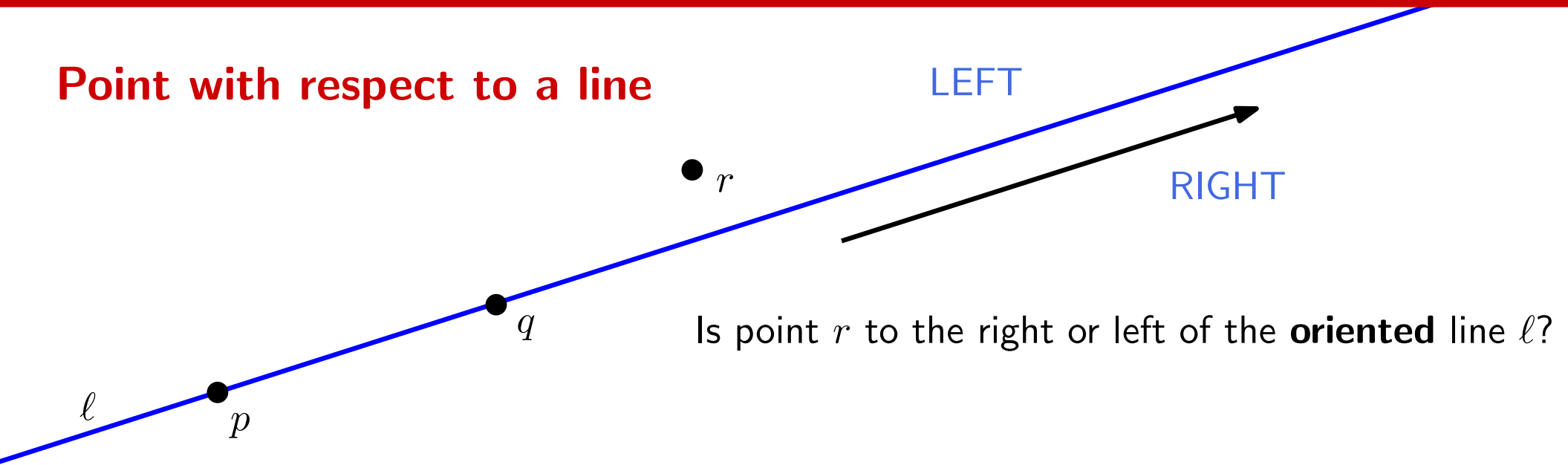
Does it turn right or left?

Fact: (check it yourself!)

$A(\triangle pqr) > 0$ if and only if p, q, r are in **counter-clockwise** order

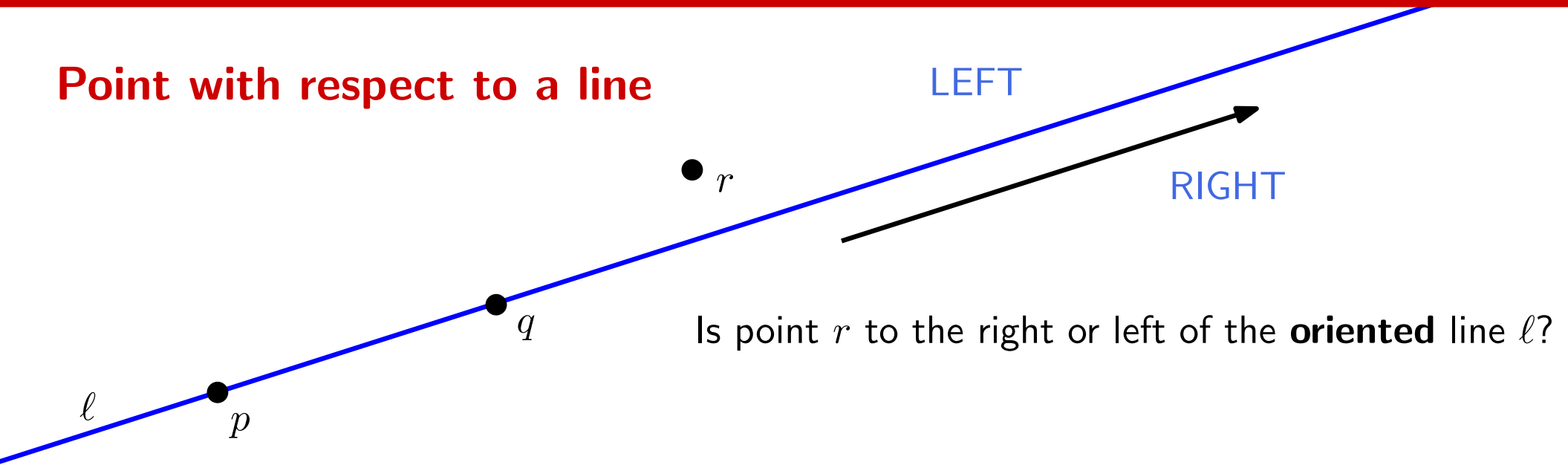
More things you can do with this test

Point with respect to a line



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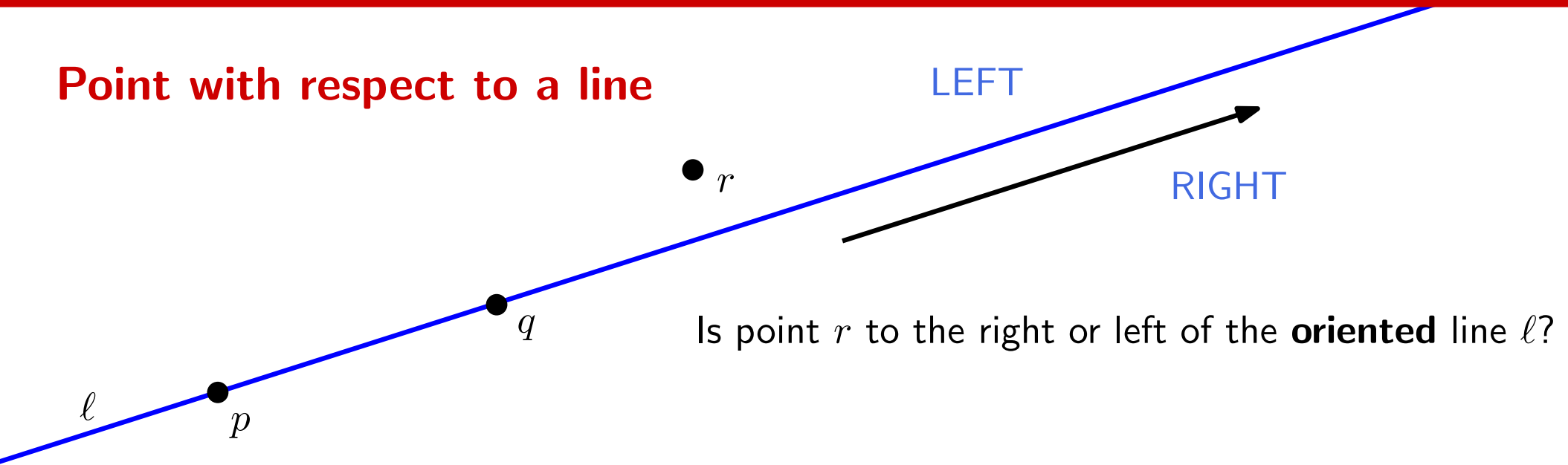


Is point r to the right or left of the **oriented** line ℓ ?

Equivalent question: do p, q, r turn right or left?

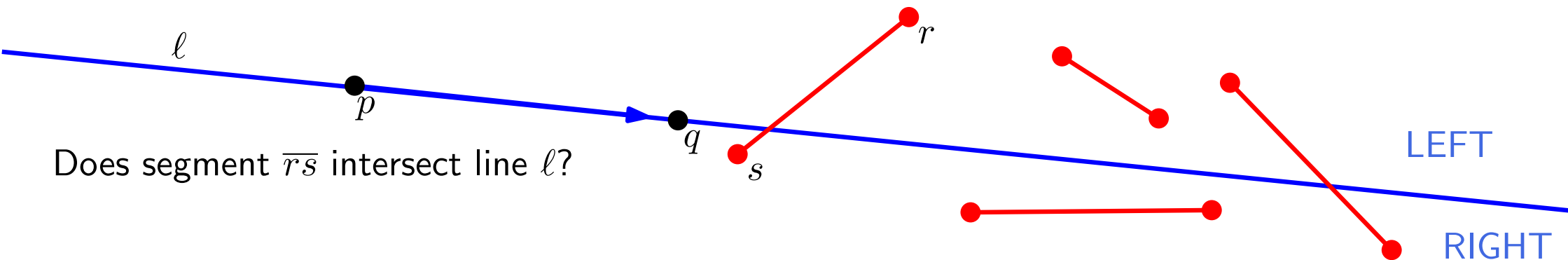
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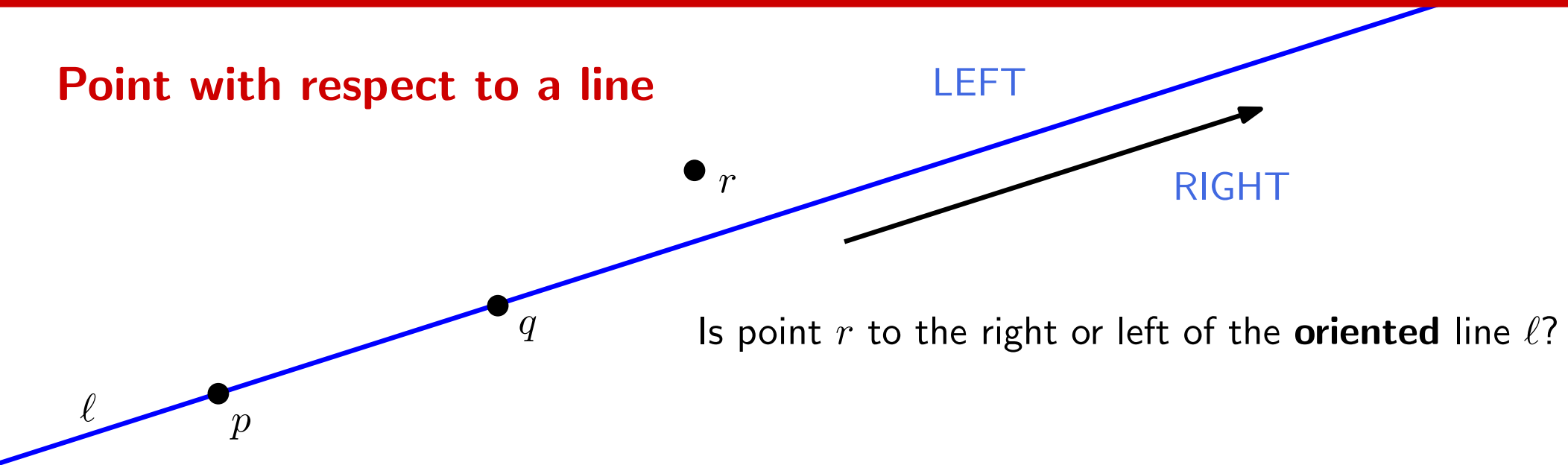
Equivalent question: do p, q, r turn right or left?

Line-segment intersection test



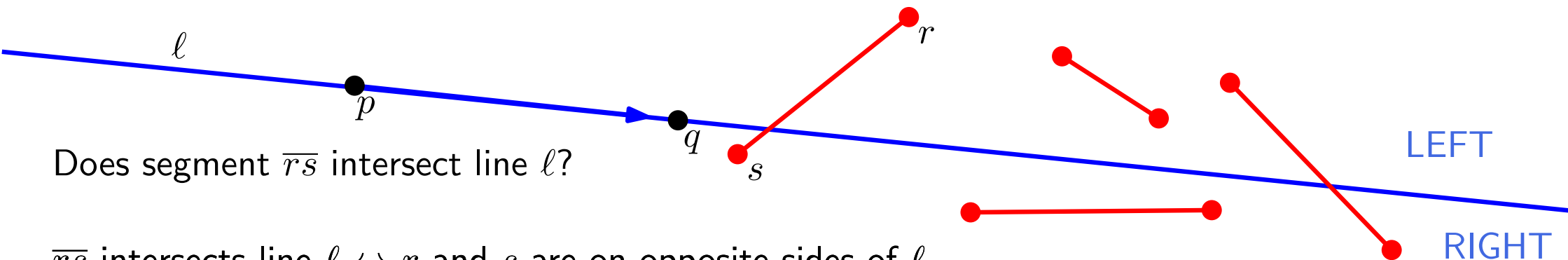
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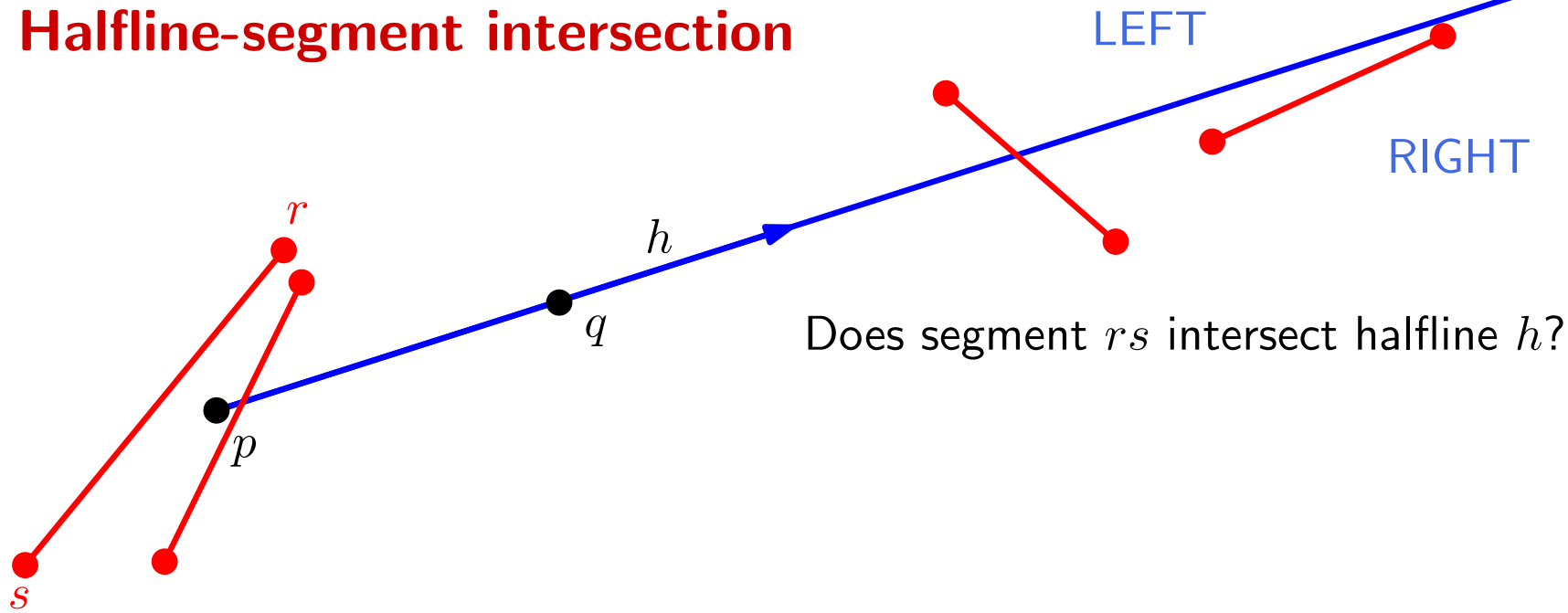


\overline{rs} intersects line $\ell \leftrightarrow r$ and s are on opposite sides of ℓ

$\leftrightarrow (r \text{ is to the left and } s \text{ is to the right of } \ell) \text{ OR } (r \text{ is to the right and } s \text{ is to the left of } \ell)$

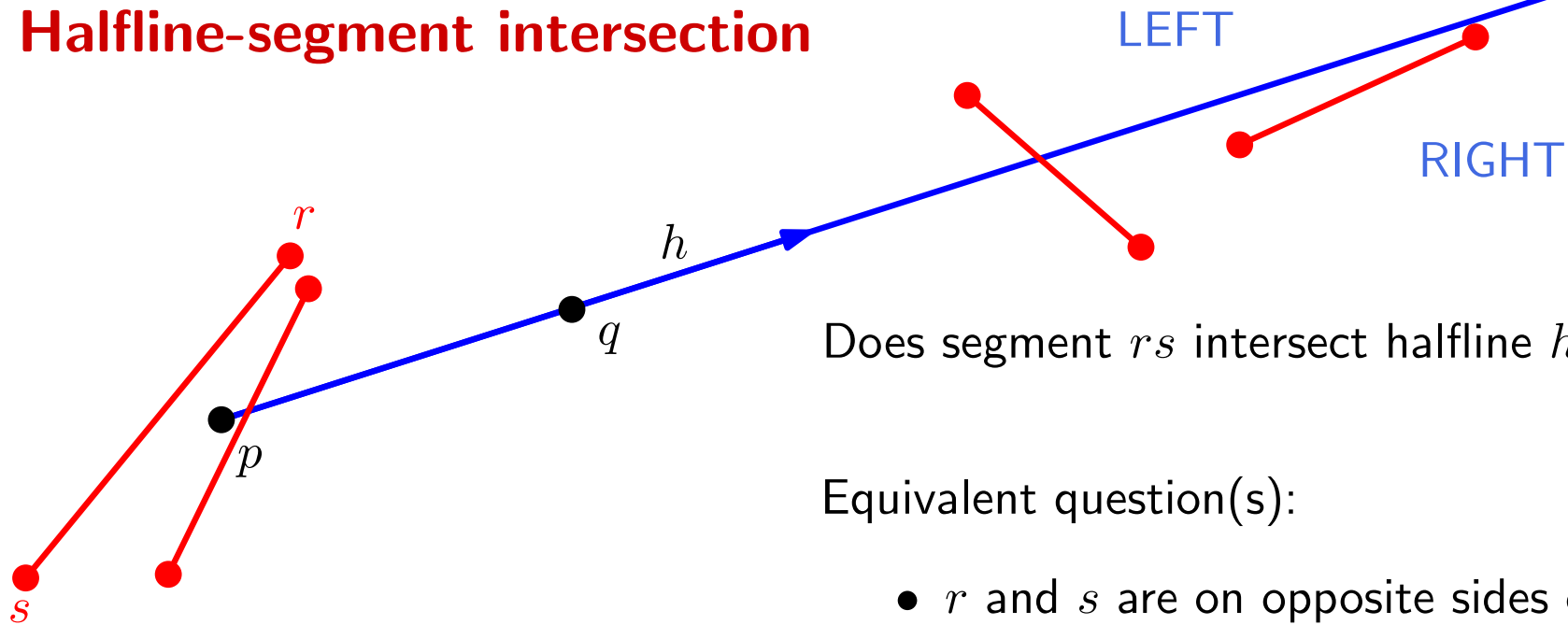
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Halfline-segment intersection



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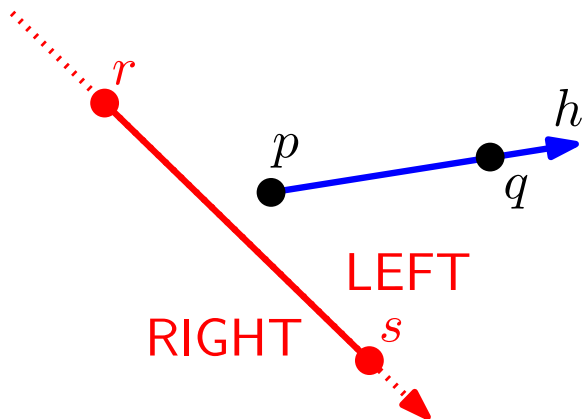
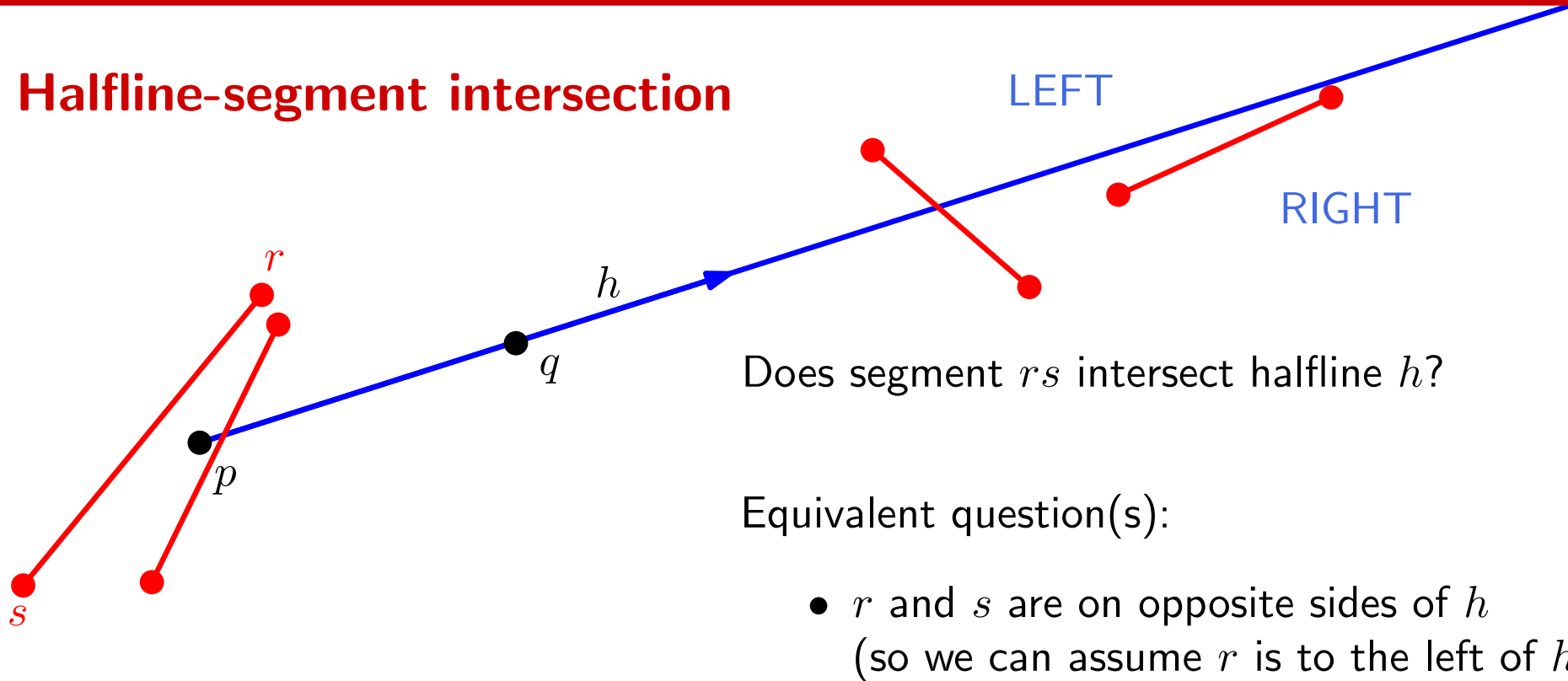
Does segment rs intersect halfline h ?

Equivalent question(s):

- r and s are on opposite sides of h
(so we can assume r is to the left of h)

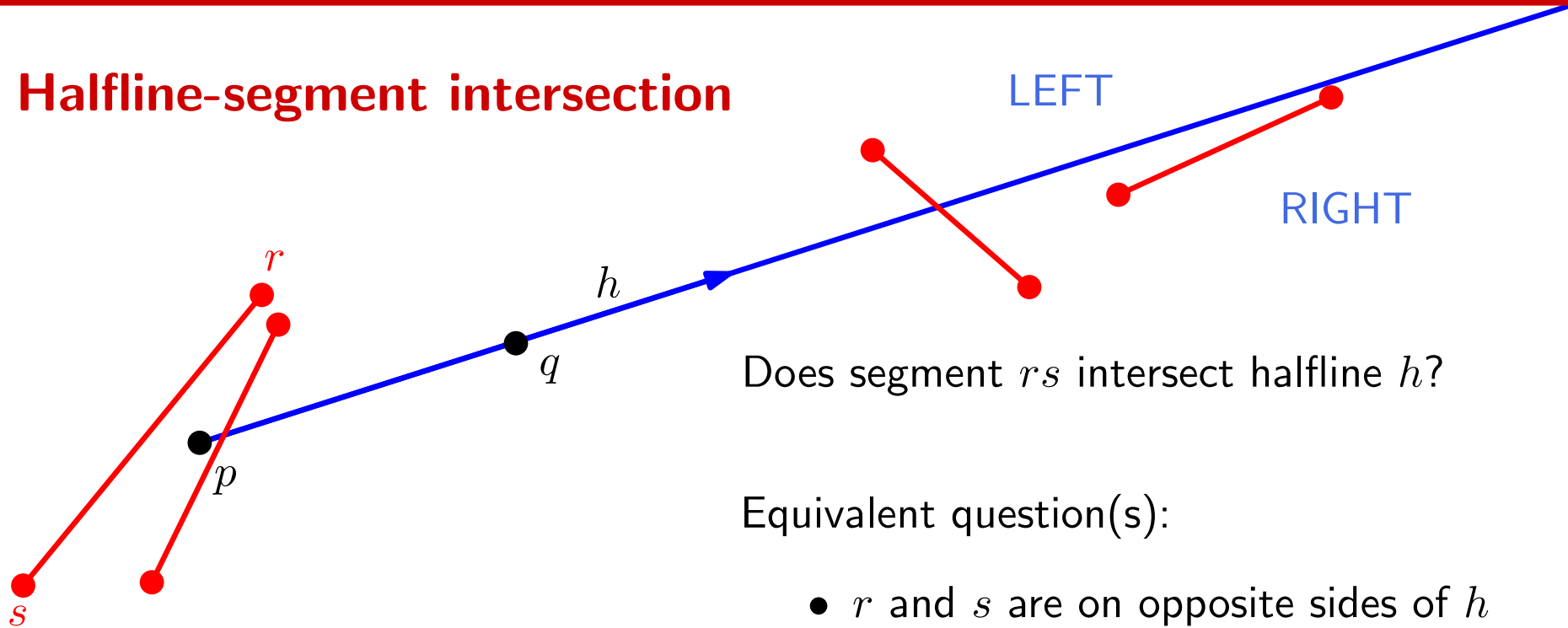
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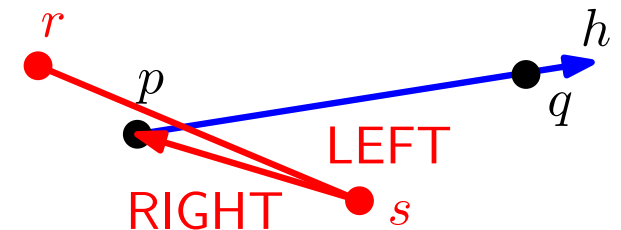
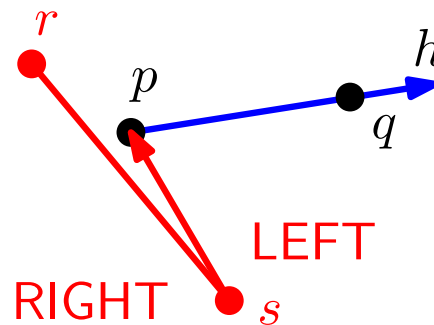
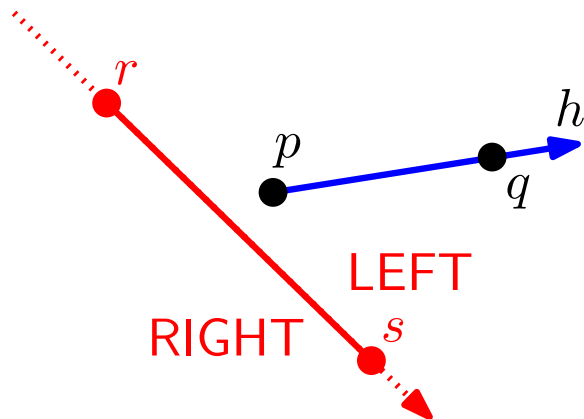
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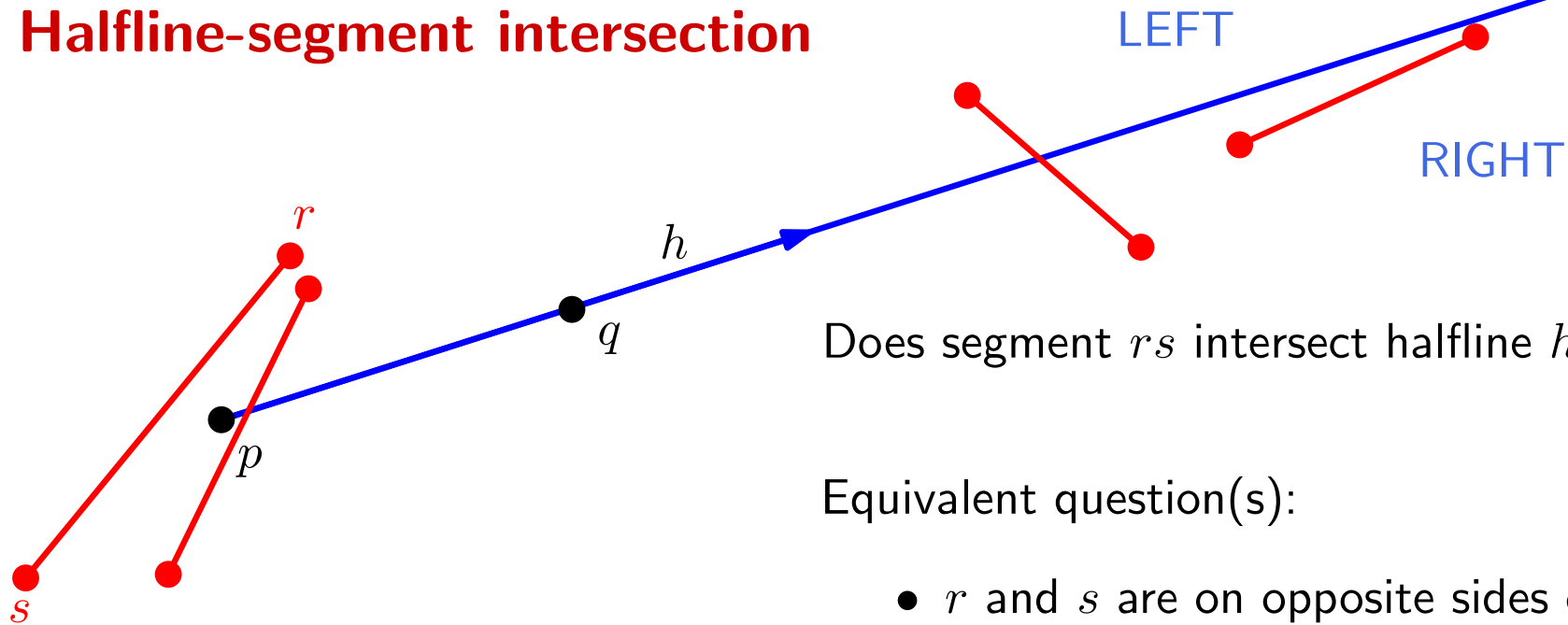
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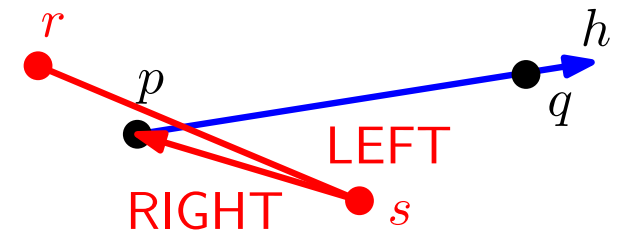
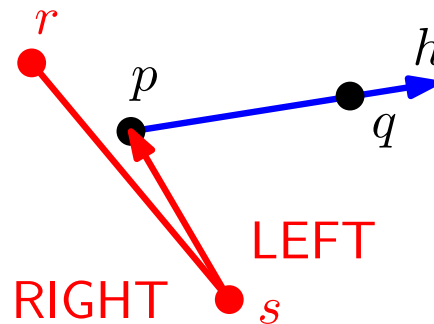
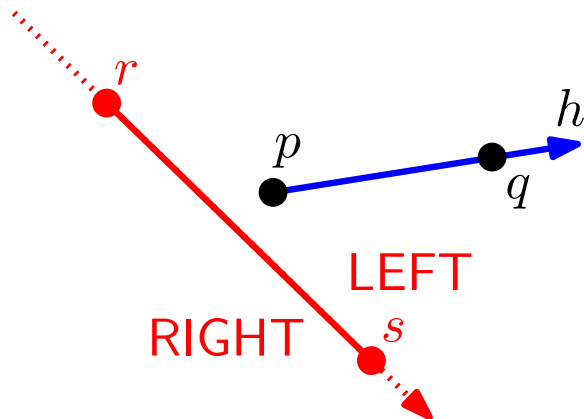
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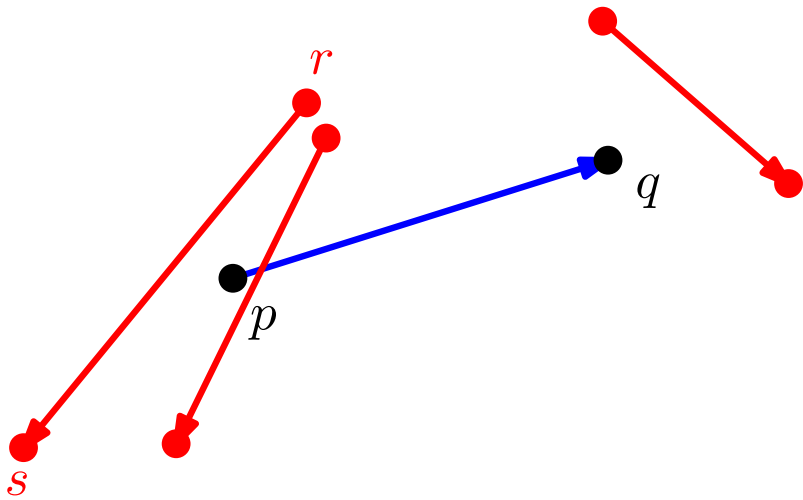
Equivalent question(s):

- r and s are on opposite sides of h
(so we can assume r is to the left of h)
- r, s, p makes a **right turn**



More things you can do with this test

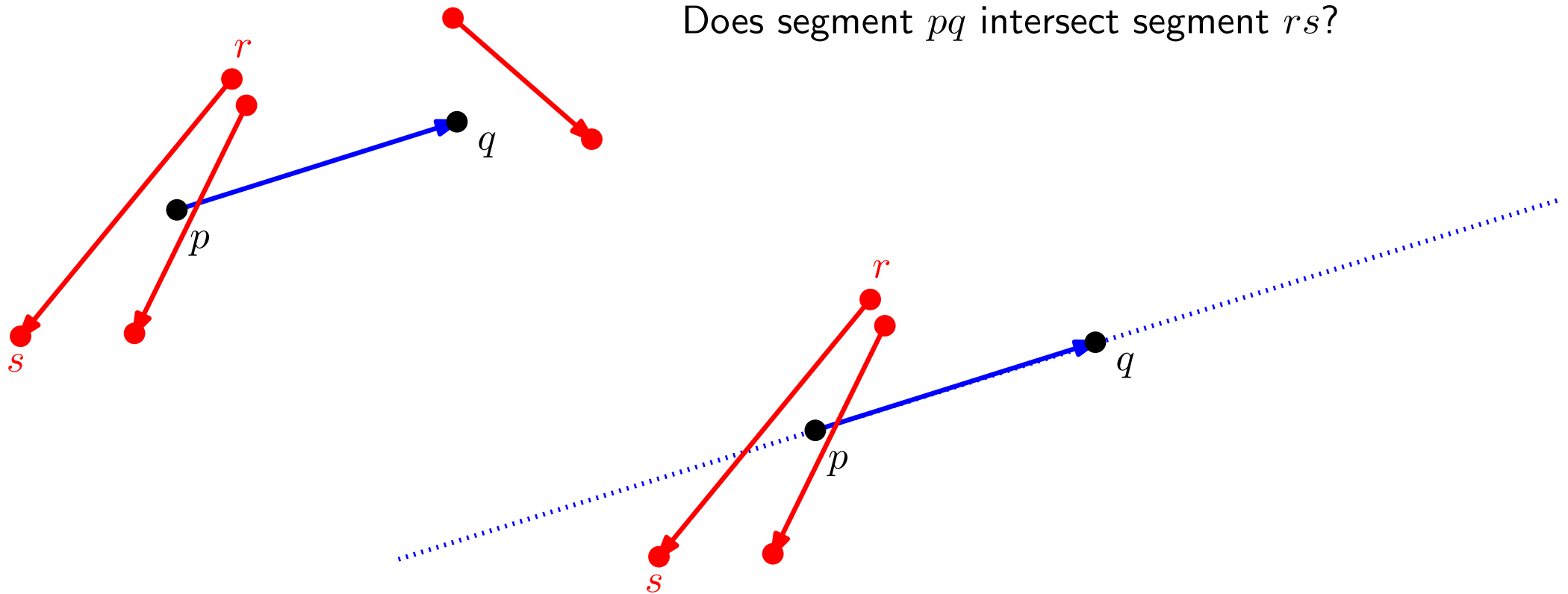
Segment-segment intersection



Does segment pq intersect segment rs ?

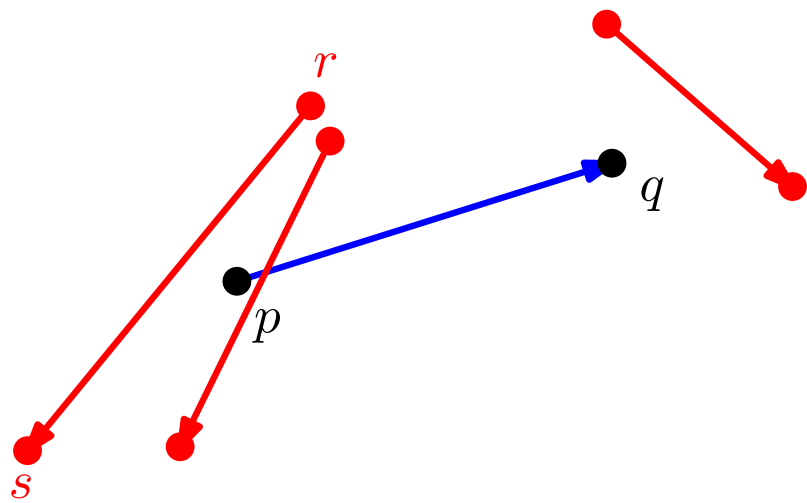
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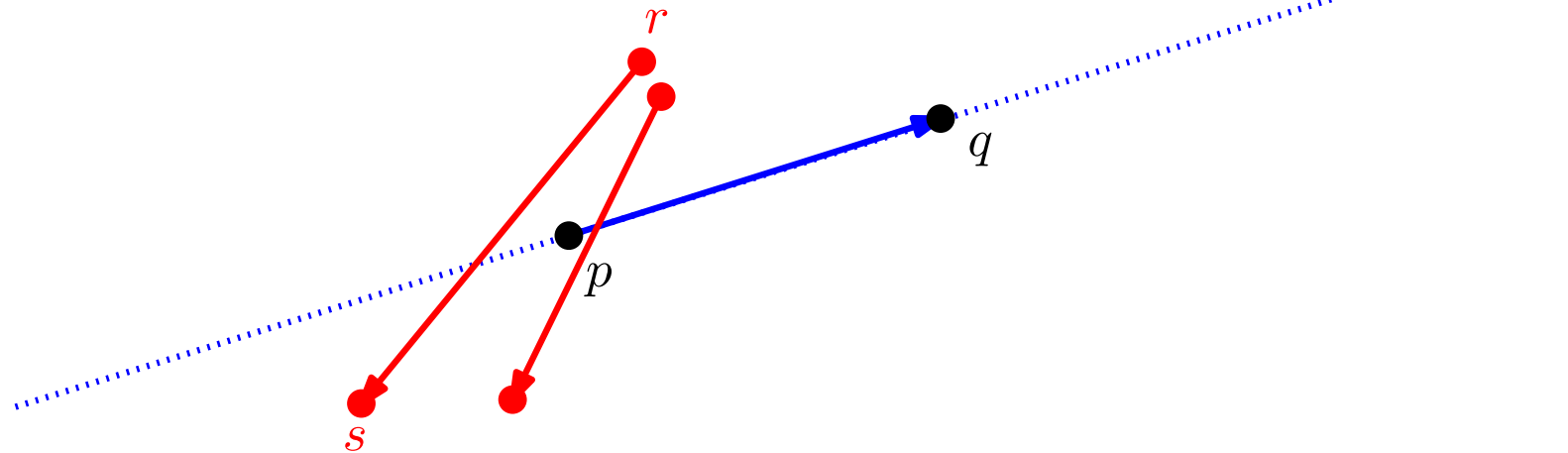


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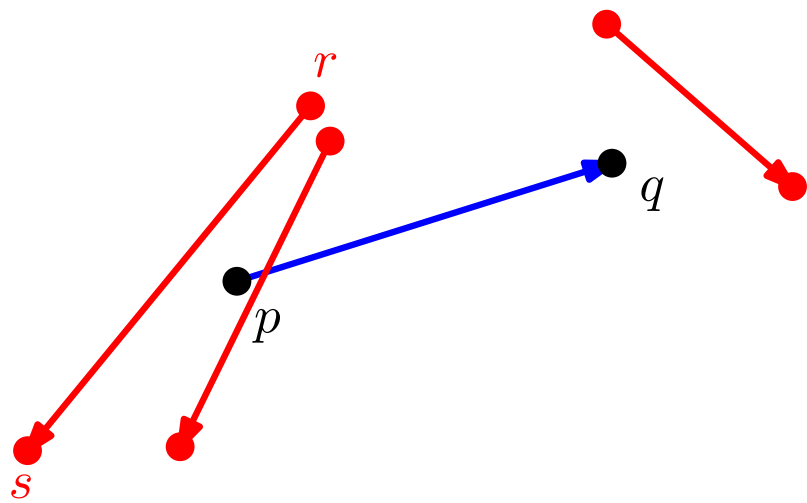


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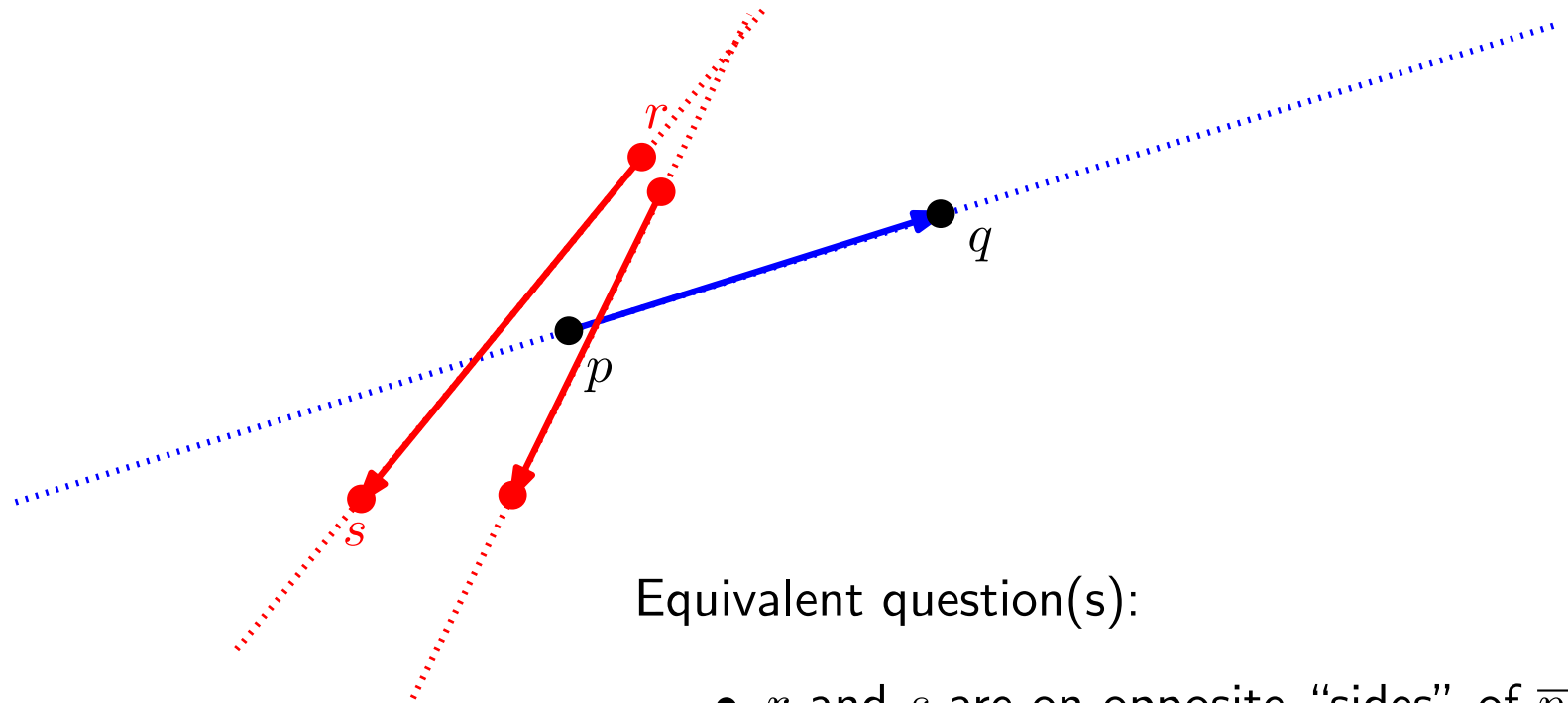
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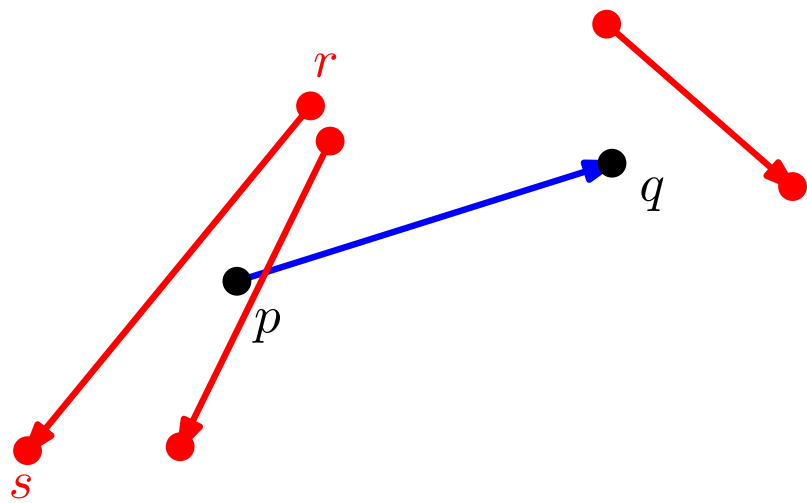


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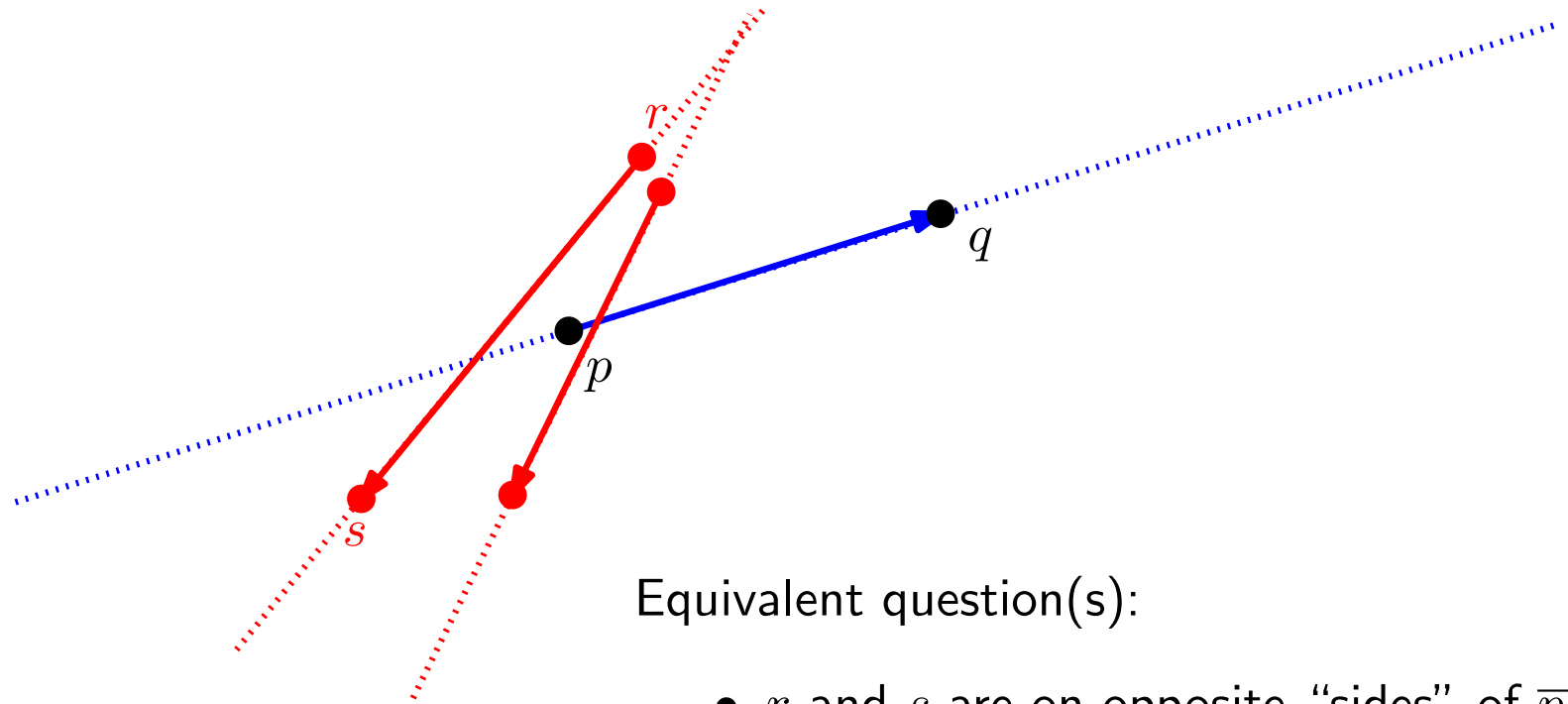
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- p and q are on opposite “sides” of \overline{rs}

Summary

Things we can do with the orientation test

- Test if p, q, r make a right turn
- Test if a point is to the left of a line
- Test if a segment and a line intersect
- Test if a segment and a halfline intersect
- Test if two segments intersect
- Test if a point is inside a triangle

All by evaluating the sign of a couple of determinants!

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Another useful test: point in circle

- See notes on course webpage

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