



# Computer vision

# Computer Vision

## Lecture 4: Features III

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# Agenda

- Feature Alignment (RANSAC)

# Building Panorama

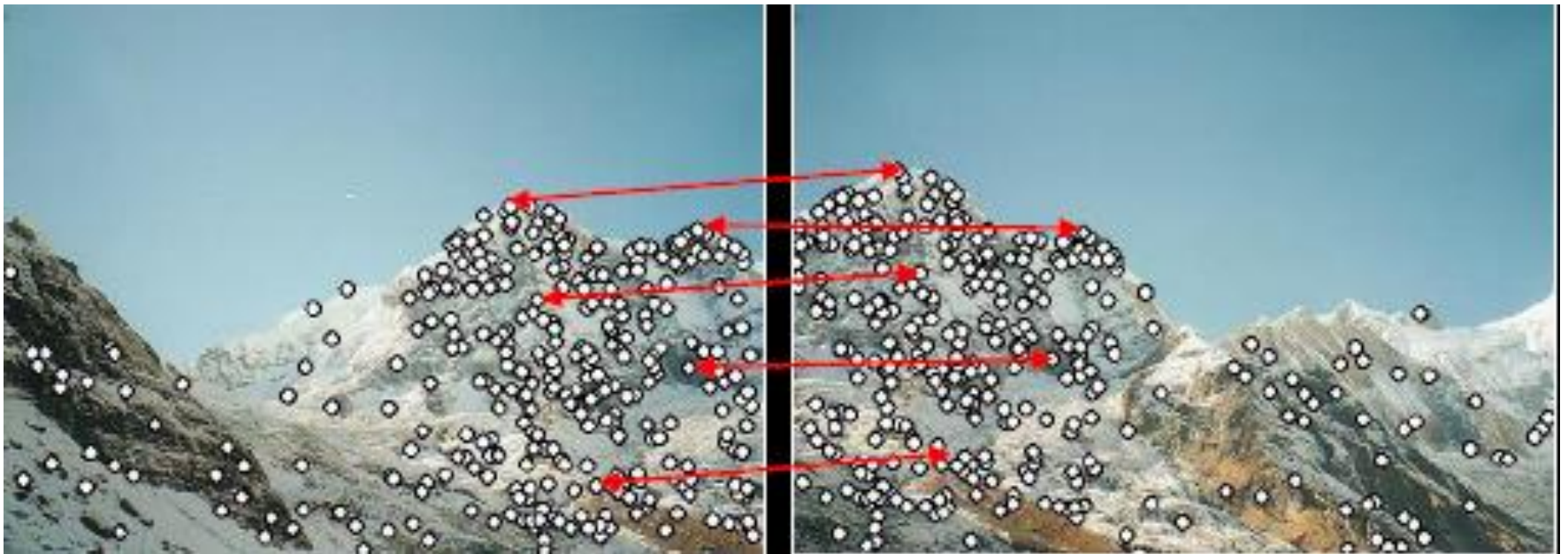
- **Detect** and **describe** in both images (Harris – SIFT- HOG - .. etc.).





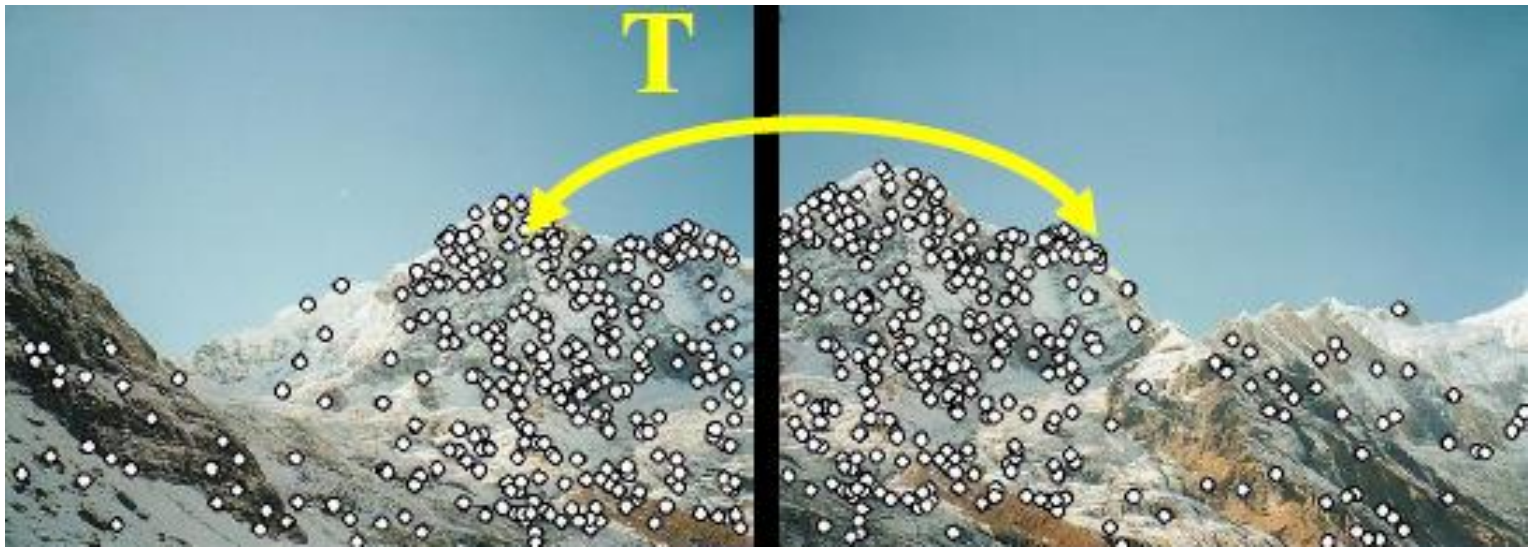
# Building Panorama

- Match features - find corresponding pairs



# Building Panorama

- Use these pairs to align images (**Compute** and apply the best transformation: *e.g. affine, translation, or rotation*)

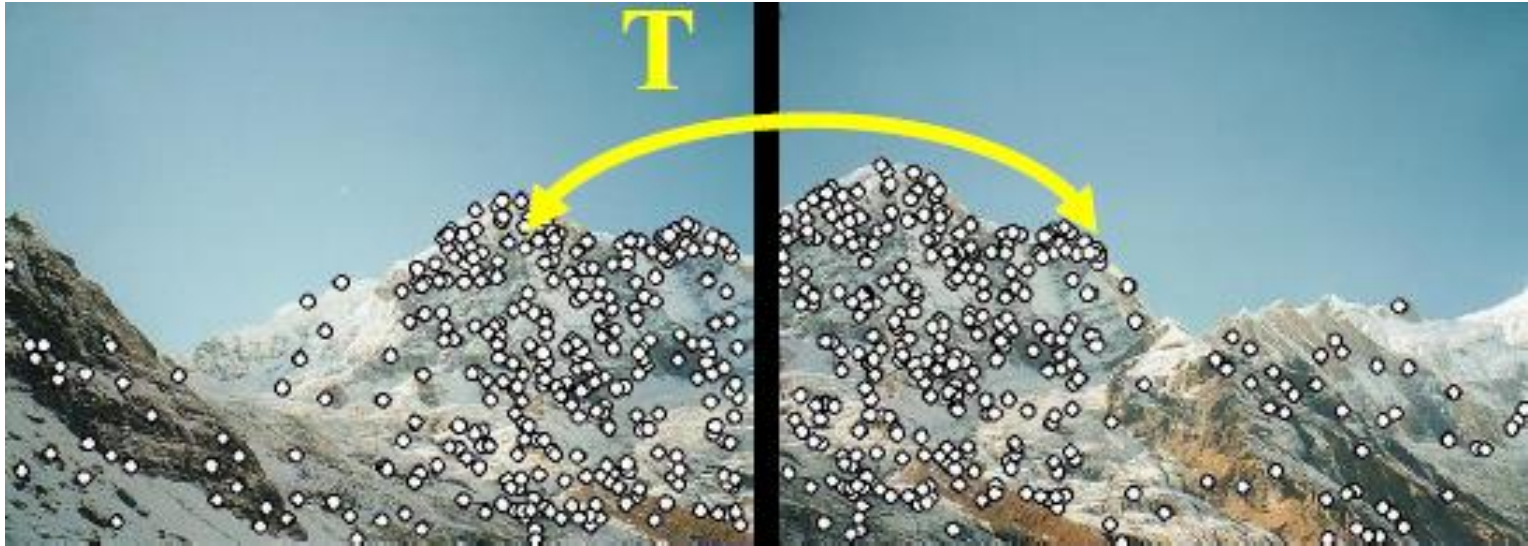


# Building Panorama

- Use these pairs to align images (Compute and **apply** the best transformation: *e.g. affine, translation, or rotation*)







# FEATURE-BASED ALIGNMENT

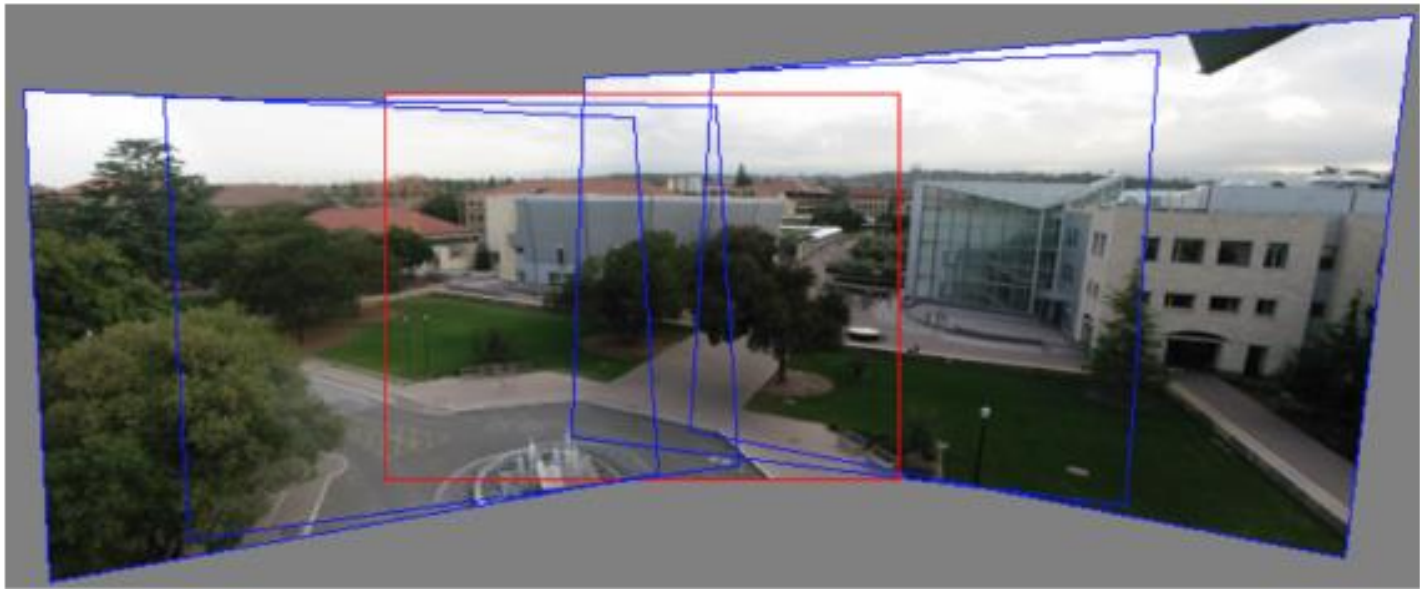
# First: Finding good matches



- Still we would get set of incorrect matches (**outliers**) ,  
so what we do?? [Compute ***closest matches***]

# Second: Compute Transformations

- Images from different **perspectives** need to be **transformed** to a common perspective before they can be stitched together.



# Second: Compute Transformations

- The transformation of an image from one projective plane to another may involve translation, scaling (up or down), rotation, shear, and changes in aspect ratio



translation



rotation



aspect



affine



perspective

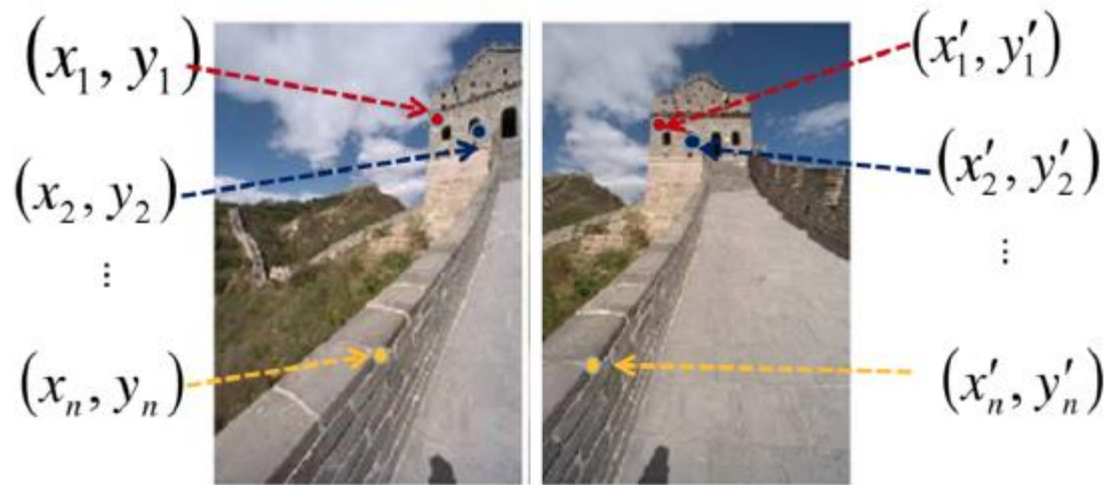


# Second: Compute Transformations

- Stitching together multiple images requires an understanding of projective **geometry** and **homography**.
- In addition to 3D Euclidean space, an additional dimension **W** (distance between camera and an image) is considered.
- The 4D space is called “**projective space**” and the coordinates in projective space are called “**homogeneous coordinates**”.

# Homography

- Homography/transformation matrix **H** describes the transformation between points in one picture and a related neighboring picture (different perspective).



- Homography matrix **H** can be estimated given N matched keypoint pairs using least squares.

$$\begin{bmatrix} wx' \\ wy' \\ 1 \end{bmatrix} = \begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

# Homography matrix for basic transformations

$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & t_x \\ 0 & 1 & t_y \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

Translate

$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} s_x & 0 & 0 \\ 0 & s_y & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

Scale

$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} \cos \Theta & -\sin \Theta & 0 \\ \sin \Theta & \cos \Theta & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

Rotate

$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & sh_x & 0 \\ sh_y & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

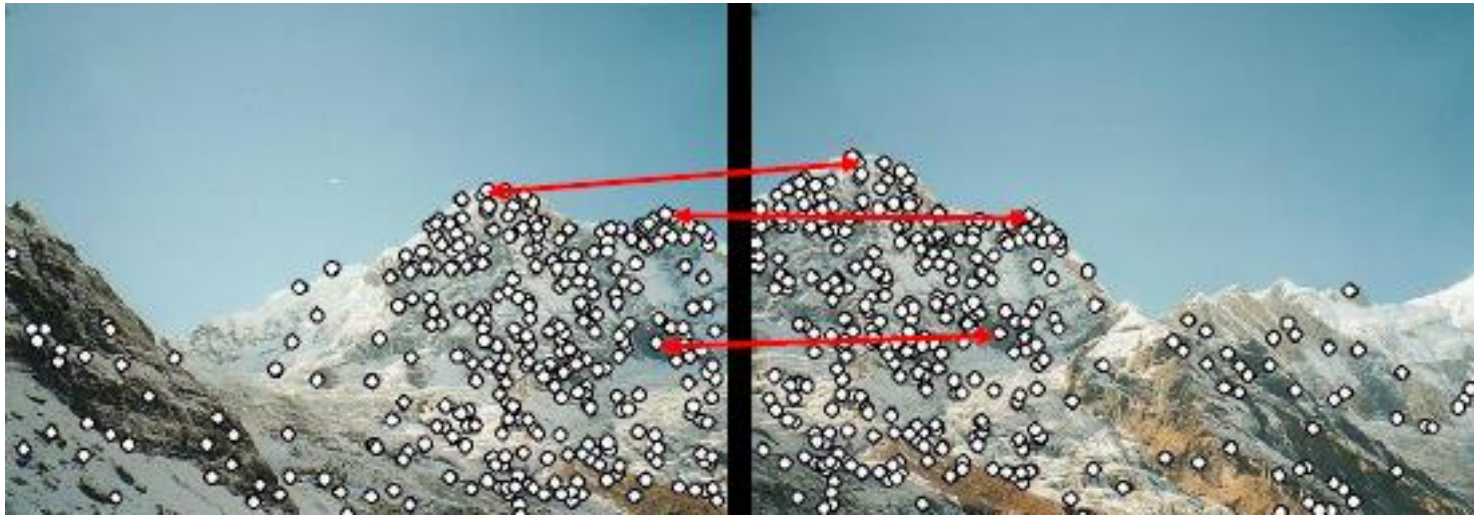
Shear

Finding good matches

# **RANDOM SAMPLE CONSENSUS** **(RANSAC)**

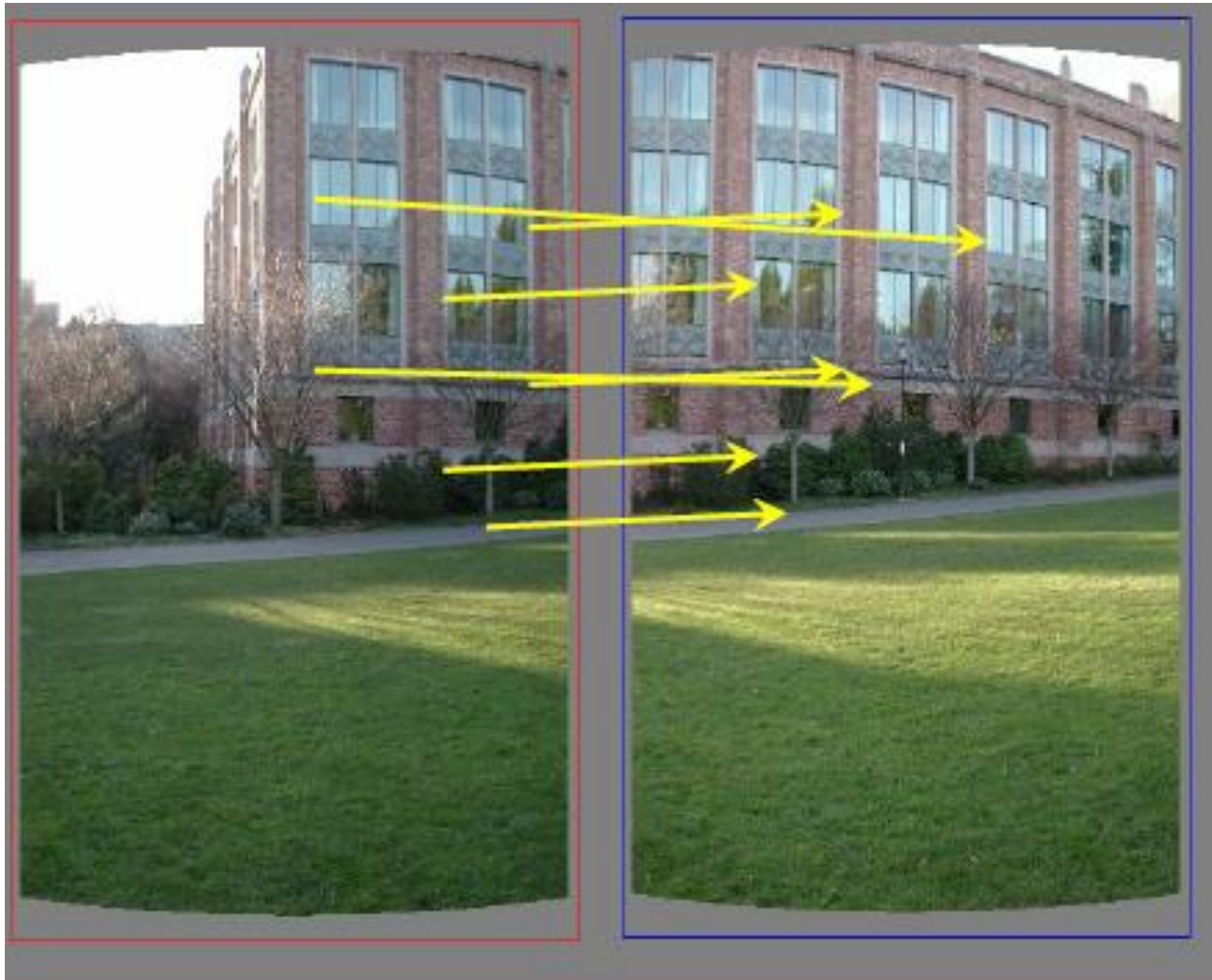


# Feature-based alignment to find transforms

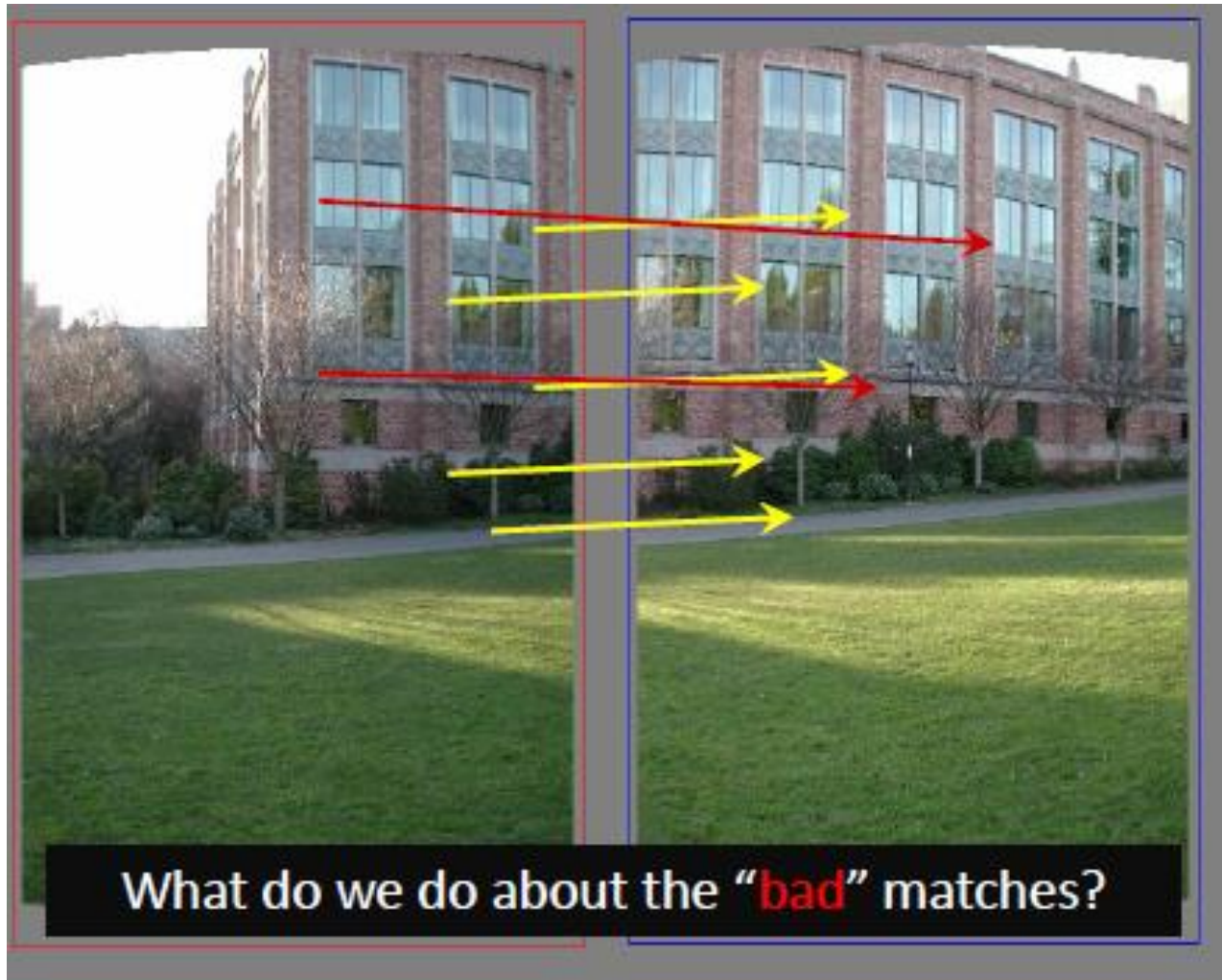


- Loop until happy:
  - *Hypothesize* transformation  $T$  from some matches.
  - *Verify* transformation (search for other matches consistent with  $T$ ) – mark best

# Matching features



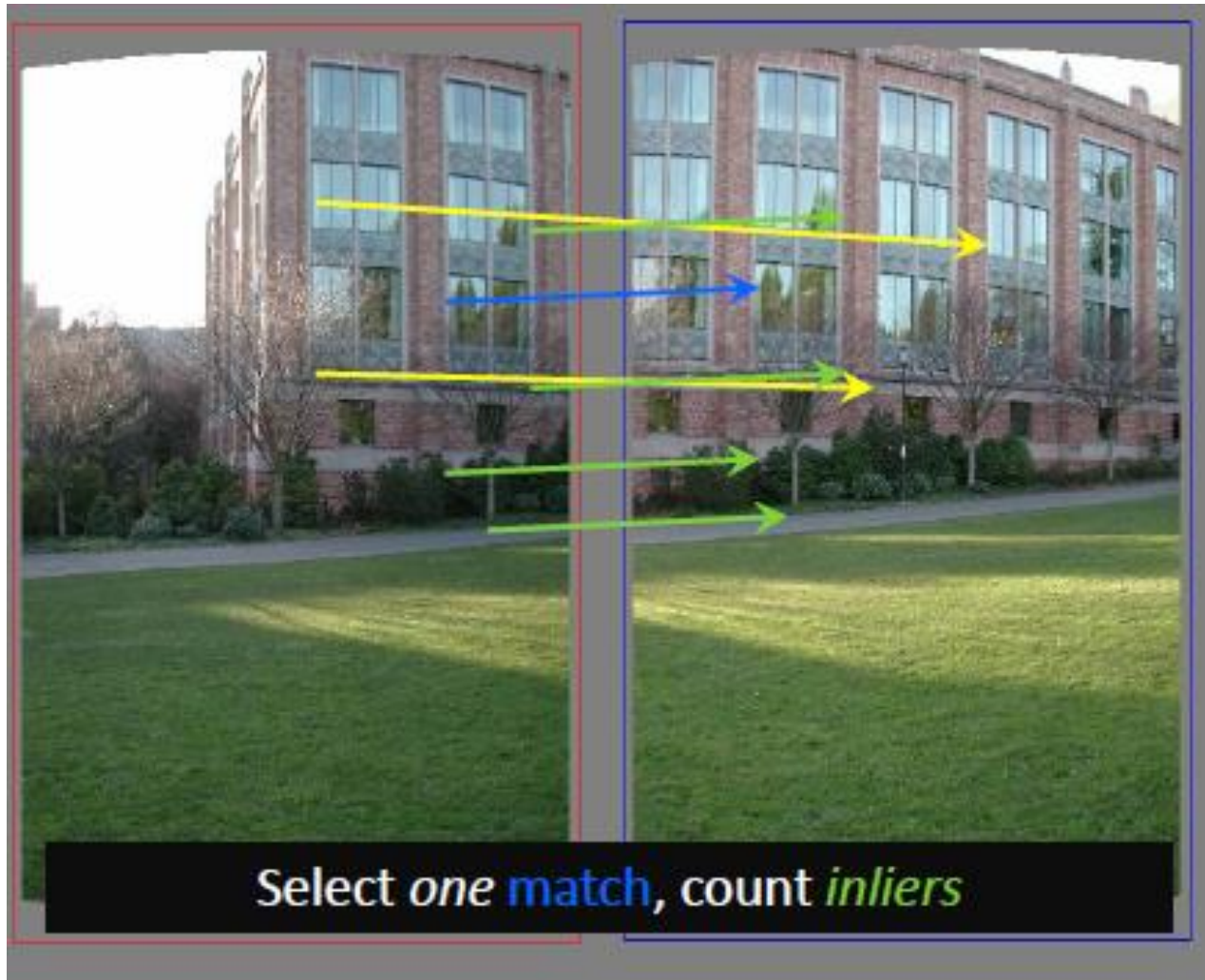
# Matching features





# RAndom SAmple Consensus (1)

## Select one

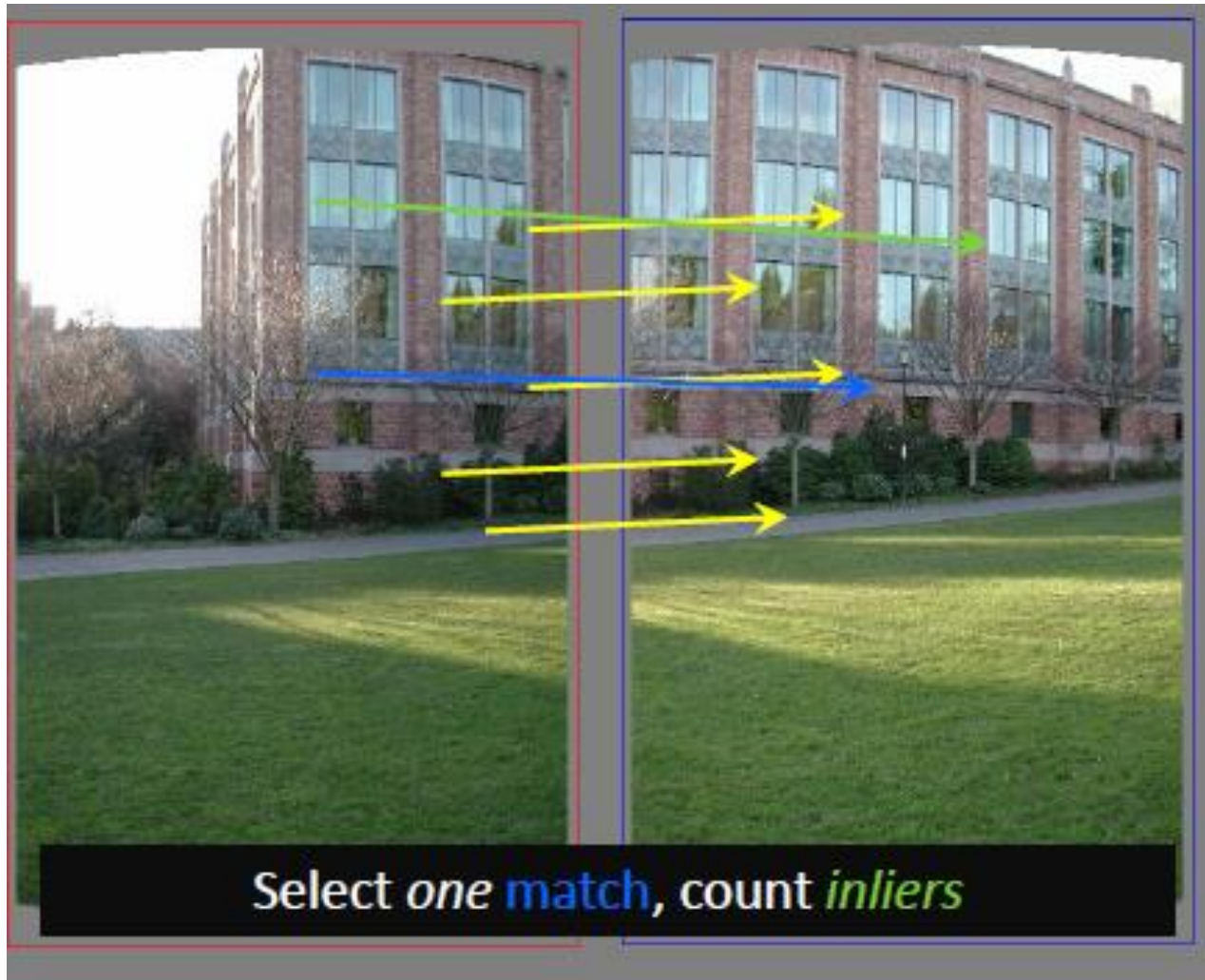


5 total  
inliers  
(including  
selected)



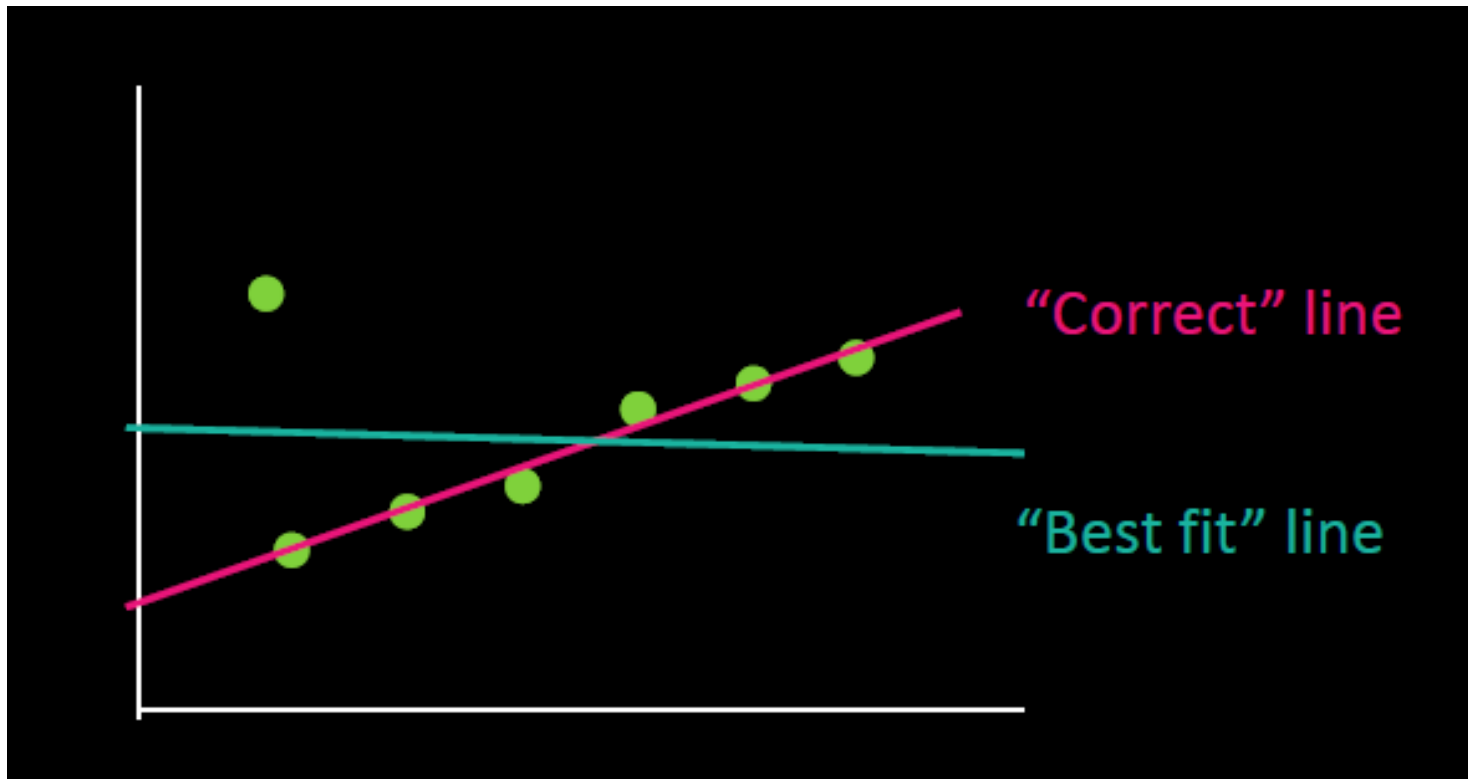
# RAndom SAmple Consensus (2)

## Select one



2 total  
inliers

# Simple Example: Fitting a line



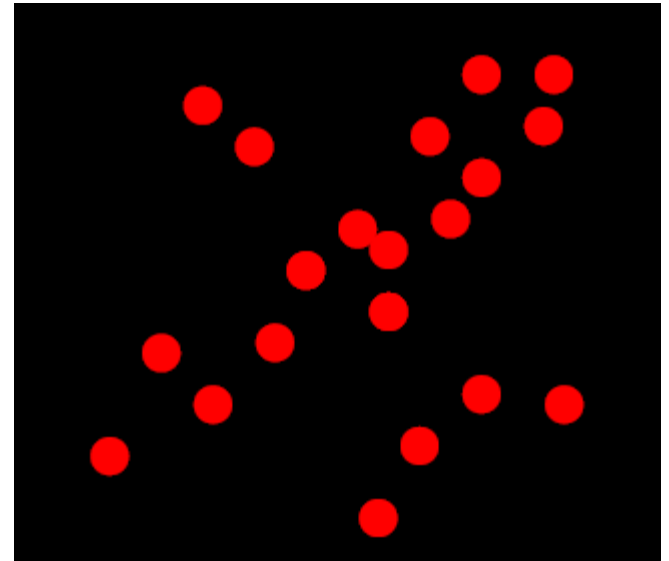
So question is how to find those **outliers**? Points don't fit to our model !

# RANSAC: Main idea

- RANdom SAmple Consensus:
  - Randomly pick some points to define your line (model).
  - Repeat enough times until you find a good line (model) – on with many inliers.

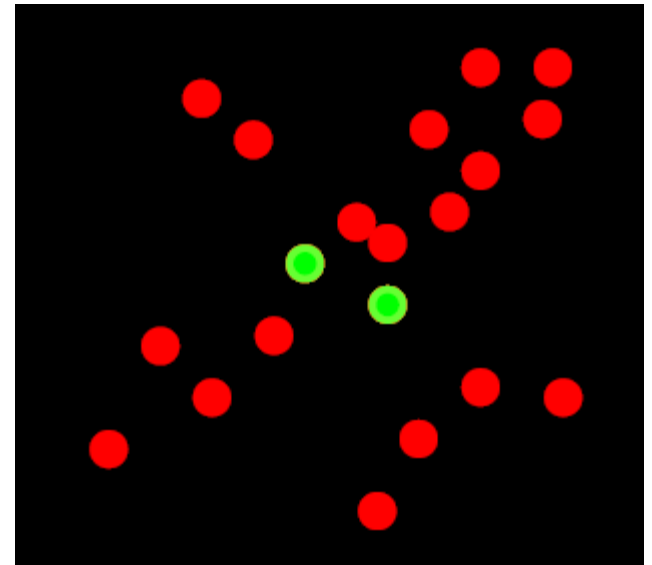
# RANSAC

1. **Sample** (randomly) the number of points required to fit the model.
2. **Solve** for model parameters using sample.
3. **Score** by the fraction of *inliers* within a preset threshold of the model.
4. **Repeat** 1-3 until the best model is found with high confidence.



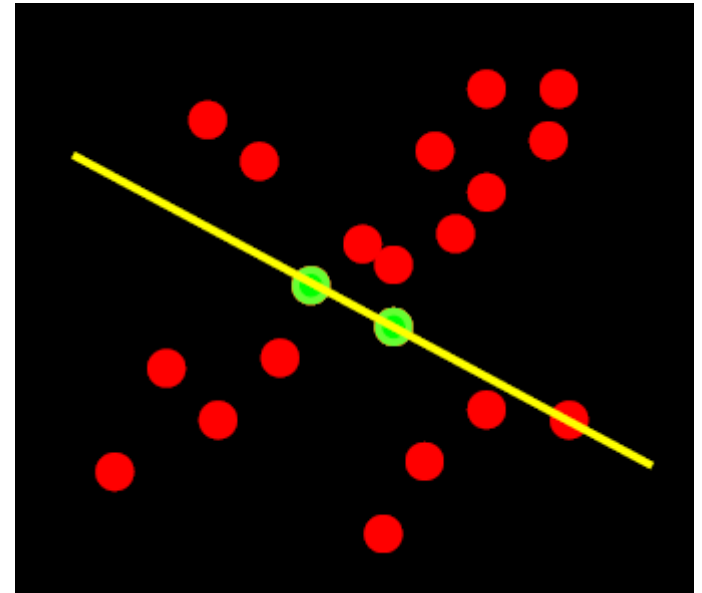
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# RANSAC

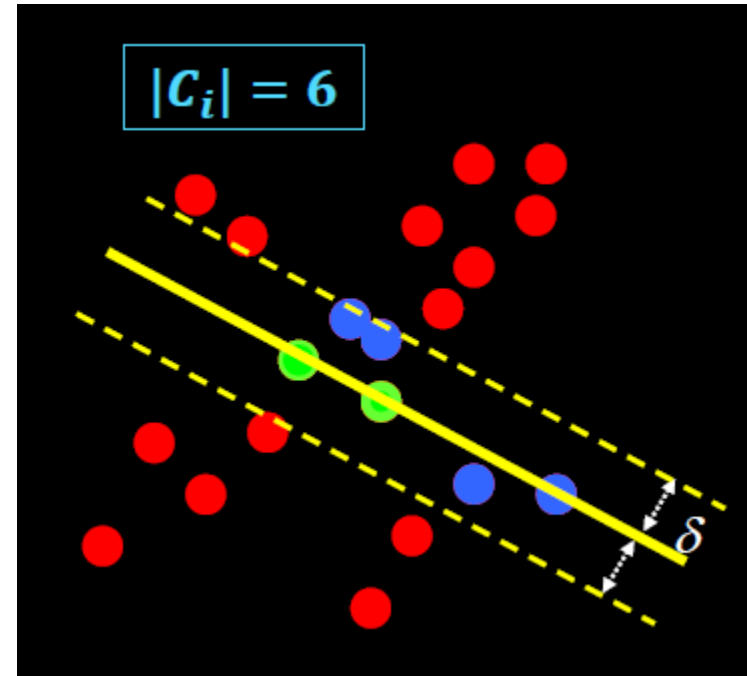
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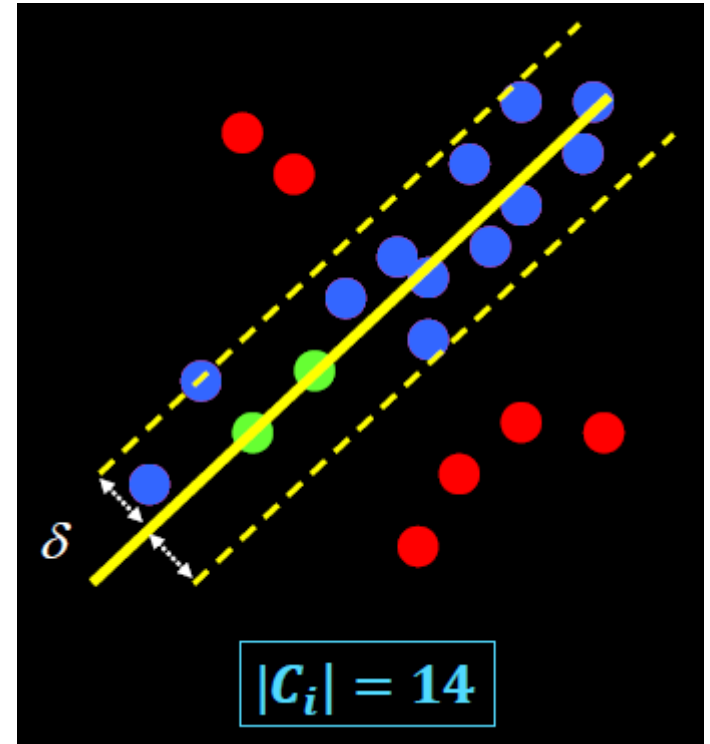
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# RANSAC

1. **Sample** (randomly) the number of points required to fit the model.
2. **Solve** for model parameters using sample.
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4. **Repeat** 1-3 until the best model is found with high confidence.



# RANSAC [Fischler & Bolles 1981]

1. Randomly select  $s$  points (or point pairs) to form a *sample*.
2. Instantiate the model.
3. Get consensus set  $C_i$ : The points within error bounds (distance threshold) of the model
4. If  $|C_i| > T$  (inliers), terminate and return model
5. Repeat for  $N$  trials, return model with max  $|C_i|$

# After RANSAC

- re-compute the model using a least-squares estimate using all of the inliers that were within the threshold distance.
- Keep this transformation as the model that best approximates the data.

# Choosing the parameters

- Initial number of points  $s$ 
  - Typically minimum number needed to fit the model
- Distance threshold  $T$ 
  - Choose  $T$  so probability for inlier is high (e.g. 0.95)
- Number of iterations  $N$ 
  - Choose  $N$  so that, with probability  $p$ , at least one random sample set is free from outliers (e.g.  $p = 0.99$ )

$$N > \log(1-p) / \log(1-(1-e)^s)$$

- Set  $p=0.99$  – chance of getting good sample

$s = 2, e = 5\%$   $\Rightarrow N=2$

$s = 2, e = 50\%$   $\Rightarrow N=17$

$s = 4, e = 5\%$   $\Rightarrow N=3$

$s = 4, e = 50\%$   $\Rightarrow N=72$

$s = 8, e = 5\%$   $\Rightarrow N=5$

$s = 8, e = 50\%$   $\Rightarrow N=1177$

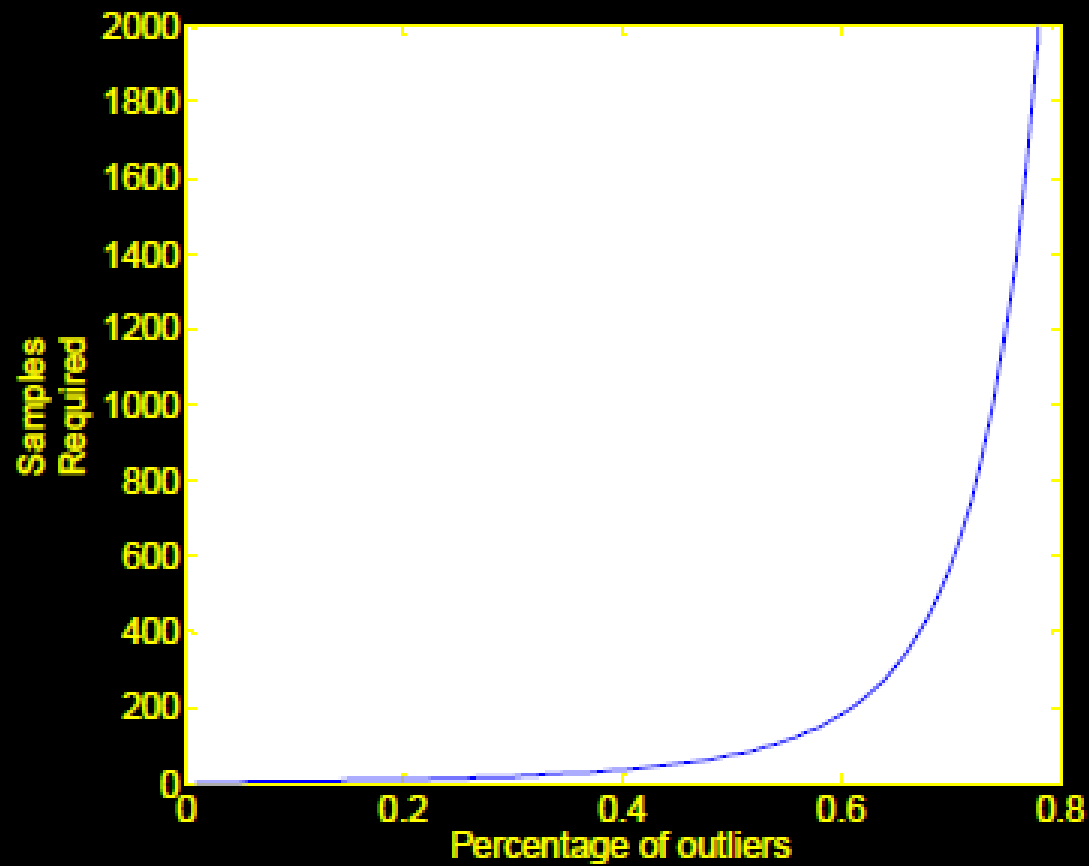
s	proportion of outliers $e$						
	5%	10%	20%	25%	30%	40%	50%
2	2	3	5	6	7	11	17
3	3	4	7	9	11	19	35
4	3	5	9	13	17	34	72
5	4	6	12	17	26	57	146
6	4	7	16	24	37	97	293
7	4	8	20	33	54	163	588
8	5	9	26	44	78	272	1177



- $N$  increases steeply with  $s$

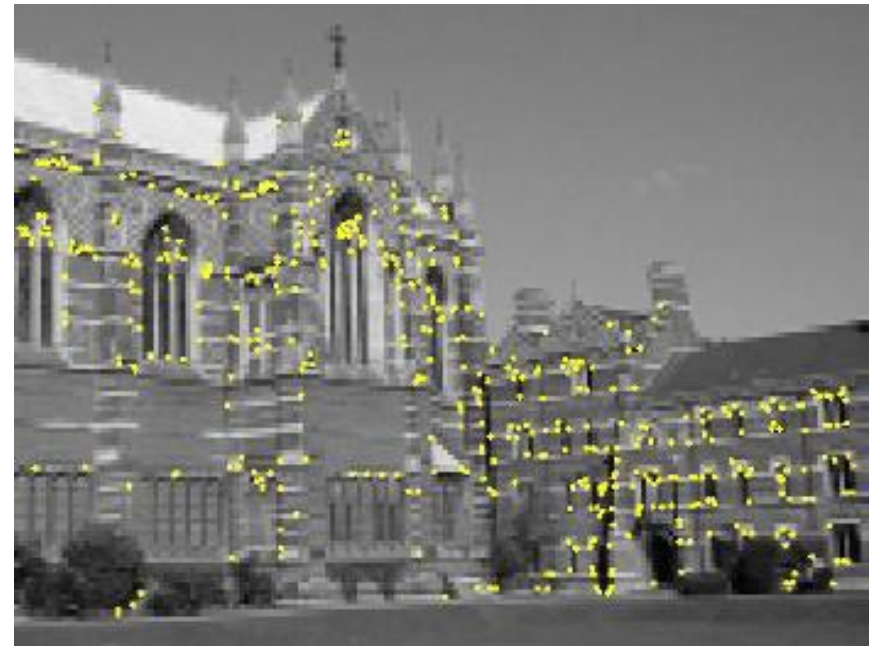
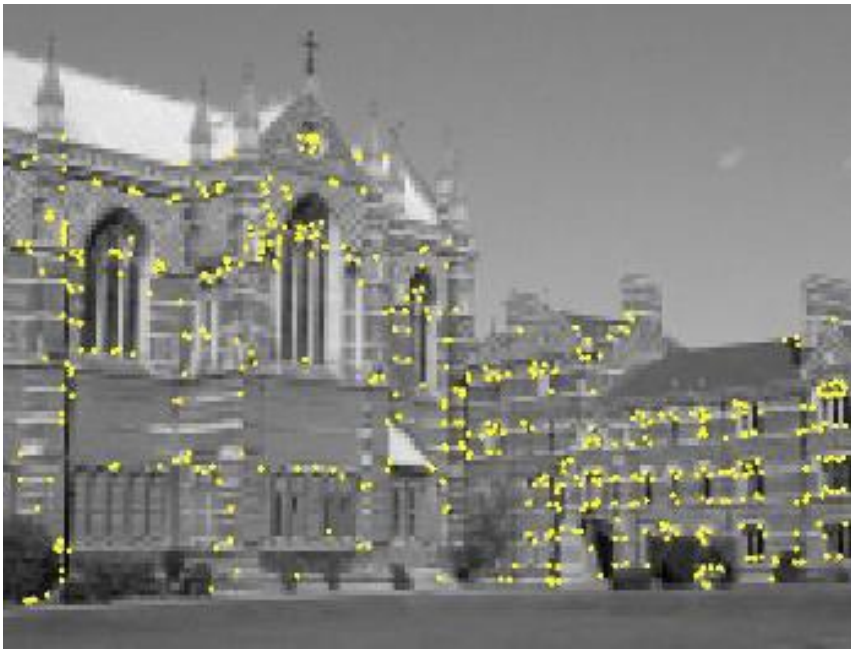


$S = 4$

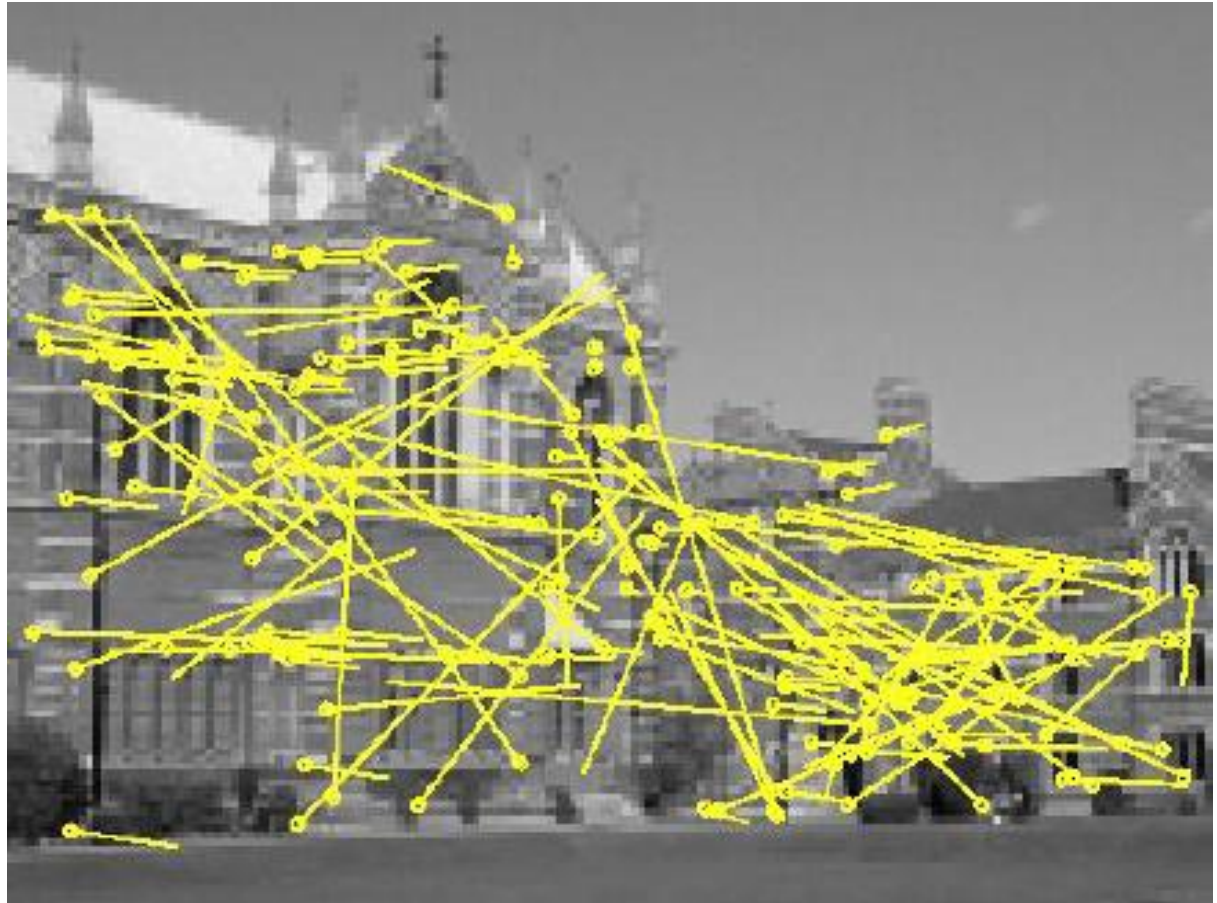


# Adaptive Procedure

- $N = \infty$ , count = 0,  $e = 1.0$
- While  $N > \text{count}$ 
  - Choose a sample and count the number of inliers
  - Set  $e_0 = 1 - \frac{\text{number of inliers}}{\text{total number of points}}$
  - If  $e_0 < e$  Set  $e = e_0$  and recompute  $N$  from  $e$ :  
$$N > \log(1-p) / \log(1-(1-e)^s)$$
  - Increment the count by 1



- 188 matches



inliers (99)



outliers(89)





# RANSAC: Conclusions

*The good...*

- Simple and general
- Applicable to many different problems, often works well in practice for model fitting in images (e.g., line/edge detection)
- Robust to large numbers of outliers
- Parameters are easy to choose

# RANSAC: Conclusions

*The not-so-good...*

- Computational time grows quickly with the number of model parameters
- Not as good for getting multiple fits

# RANSAC: Conclusions

## *Common applications*

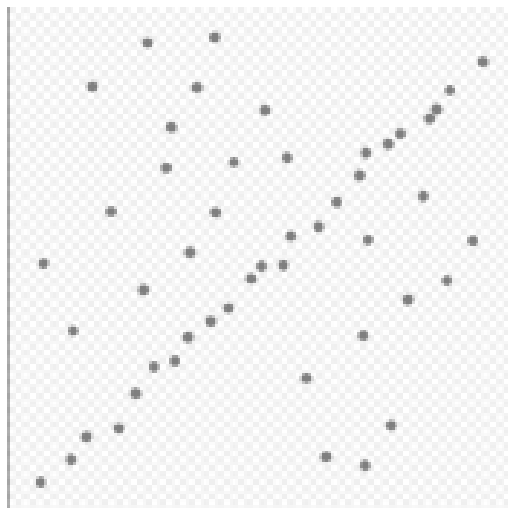
- Computing a homography (e.g., image stitching) or other image transform.
- Estimating fundamental matrix (relating two views).
- *Pretty much every problem in robot vision.*



**PRACTICE**

# Questions

- Given the following set of 2-D point cloud, assume that there are 60% inliers; compute the minimal number of RANSAC iterations needed to get, with probability 95%, at least one random sample that is free from outliers.





# solution

$$N > \log(1-p) / \log(1-(1-e)^s)$$

$$P = 0.95 \quad e = 0.4, \quad s = 2$$

$$N > \log(1-0.95) / \log(1-(1-0.4)^2)$$

$$N > \log(0.05) / \log(1-0.36)$$

$$N > \log(0.05) / \log(0.64)$$

$$N > 6.712$$

$$N = 7$$

# Credit for

*CS 4495 Computer Vision (Spring 2015)*

*A. Bob - College of Computing, Georgia Tech.*

*CS131 “Computer Vision: Foundations and Applications” by  
University of Stanford (Fall 2019)*

*CAP5415 “Computer Vision” University of Central Florida,  
Center of Research in Computer Vision (UCF CRCV), Fall  
2020*