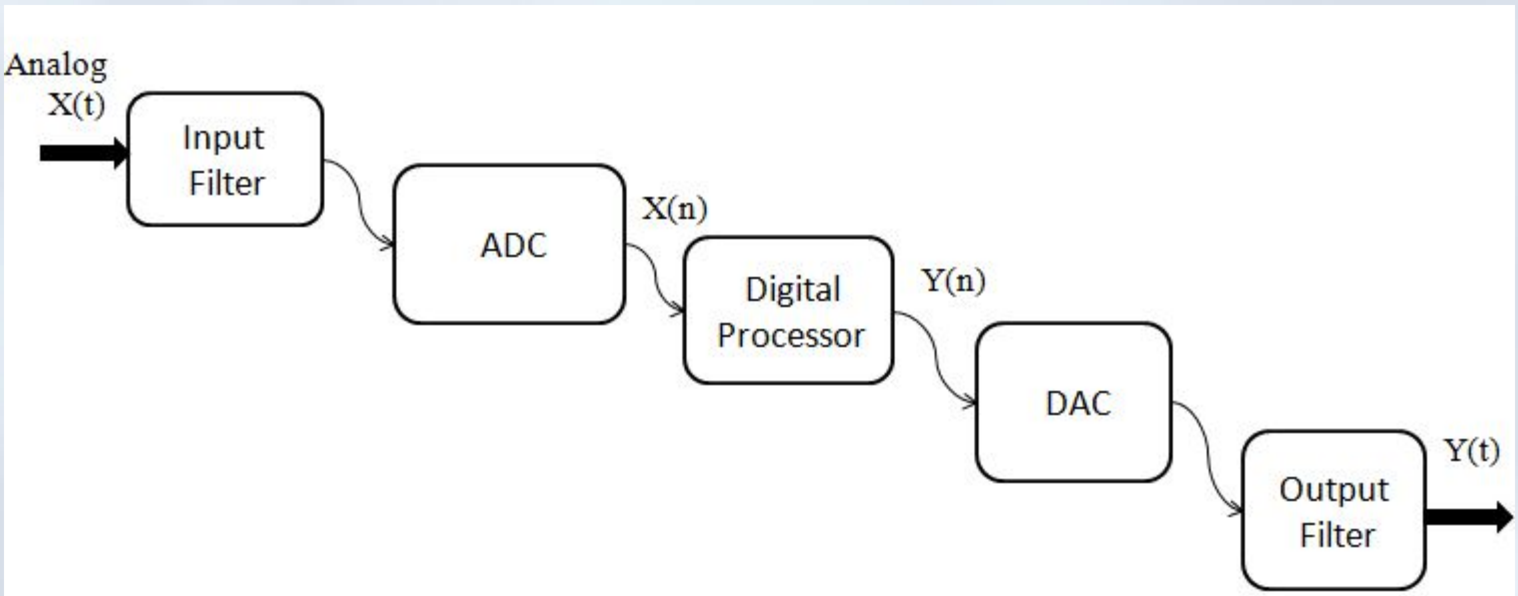
Digital Signal Processing

Digital signal processing, principles, Algorithms and Applications.

Lect 2

ANALOG TO DIGITAL **&** DIGITAL TO ANALOG **CONVERSION**

Typical Real-time DSP Systems



Typical Real-time DSP Systems

Input Filter *:* is used to band limit the analog input signal prior to digitization to reduce aliasing.

- ***ADC*** *:* most signals in real world are in analog form so an ADC is needed to convert input signal to digital form.

Typical Real-time DSP Systems

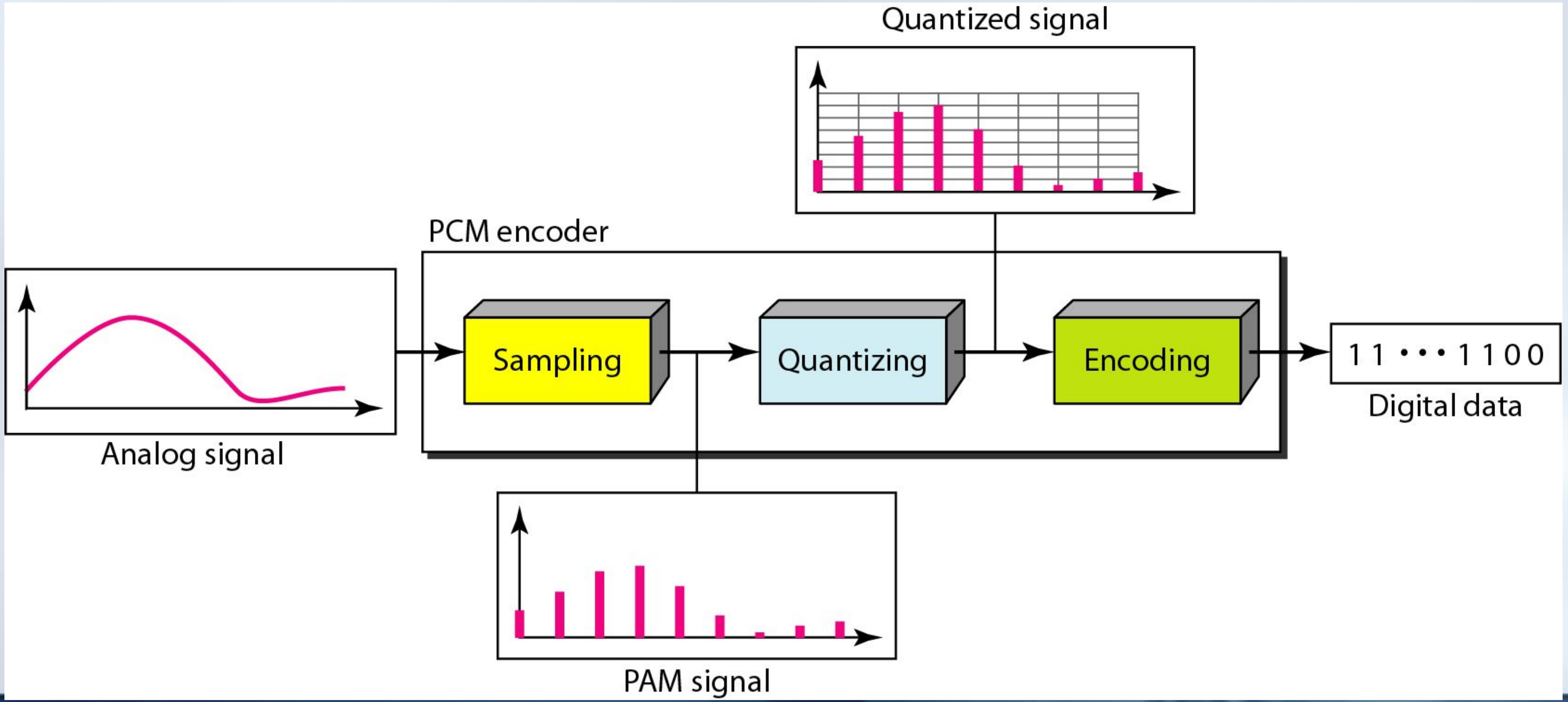
- *Digital Processor* is the core of the system, it applies one of the several DSP algorithms on the digital input according to needed task.
- • *DAC* used to return the processed signal back to its analog form.
- • *Output Filter* smoothes out the outputs of the DAC a nd removes unwanted high frequency components.

Analog to Digital Conversion

- Pulse code Modulation Technique
- Delta Modulation Technique

Analog to Digital Conversion

- **A/D (PCM technique) is done on three steps:**
  - Sampling
  - Quantization
  - Coding



Sampling

- Conversion of continuous time signal ☐ discrete time signal
- Obtained by taking samples of continuous time signal at discrete time instances
- **Uniform Sampling:**

$$x(n) = x_a(nT), \qquad -\infty < n < \infty$$

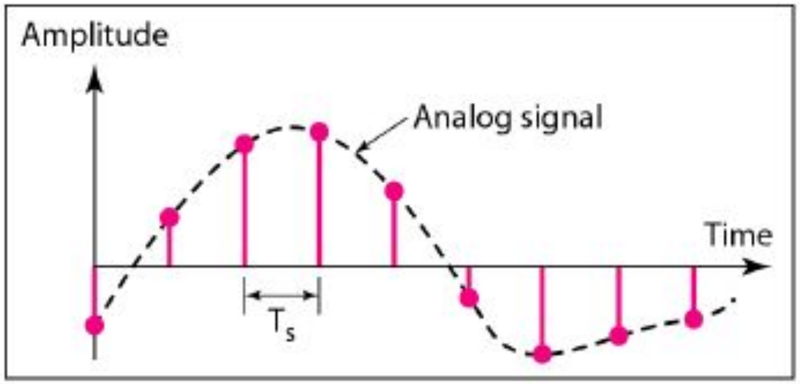
  - X(n) is discrete time signal obtained by taking samples of the analog signal $x_o(t)$ at each T seconds
- **Sampling Rate:** $F_s = 1/T$

Sampling

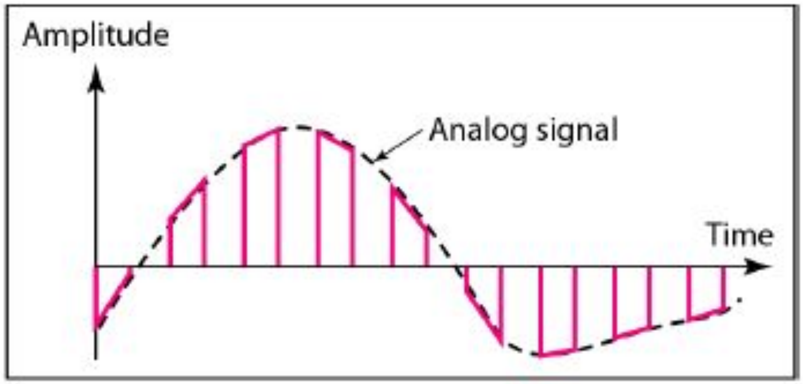
There are 3 sampling methods:

- Ideal - an impulse at each sampling instant
- Natural - a pulse of short width with varying amplitude
- Flattop - sample and hold, amplitude value

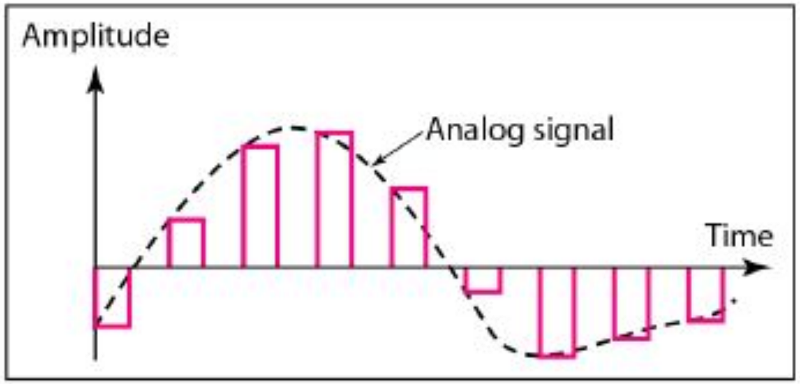
Sampling Methods



a. Ideal sampling

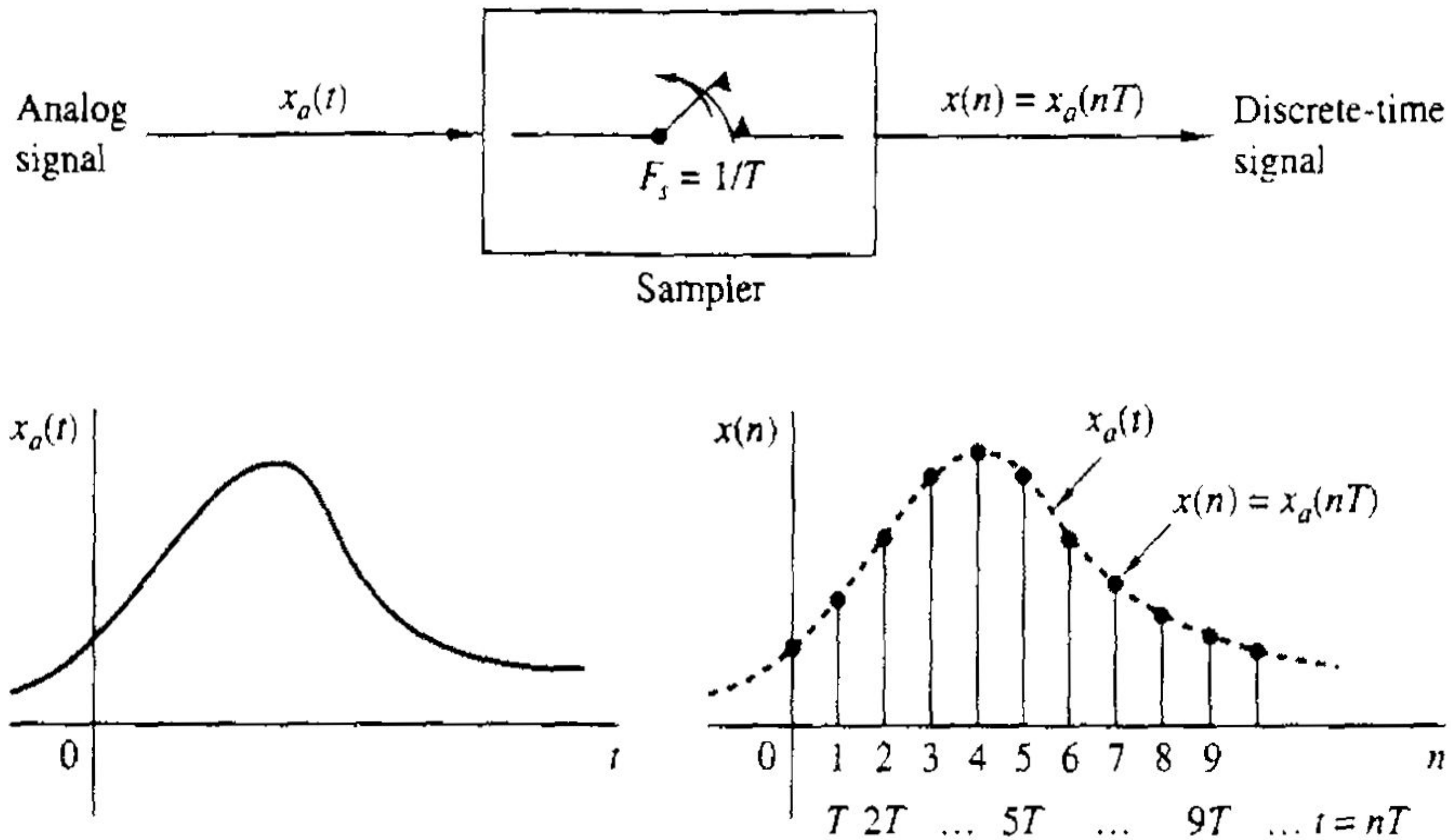


b. Natural sampling



c. Flat-top sampling

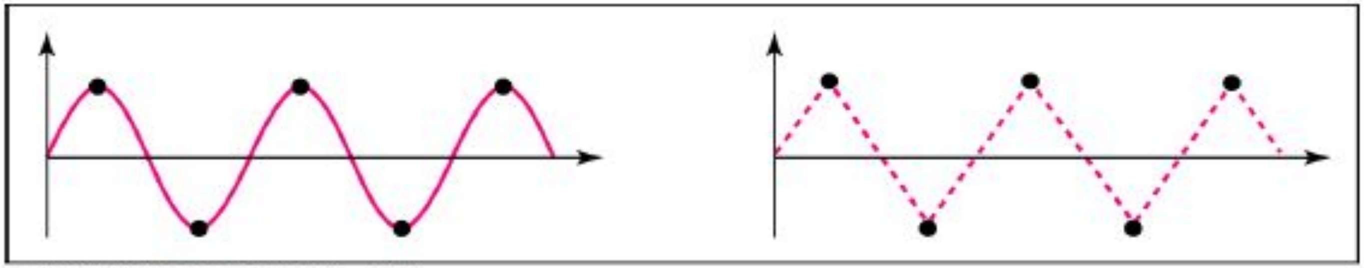
Ideal Sampling



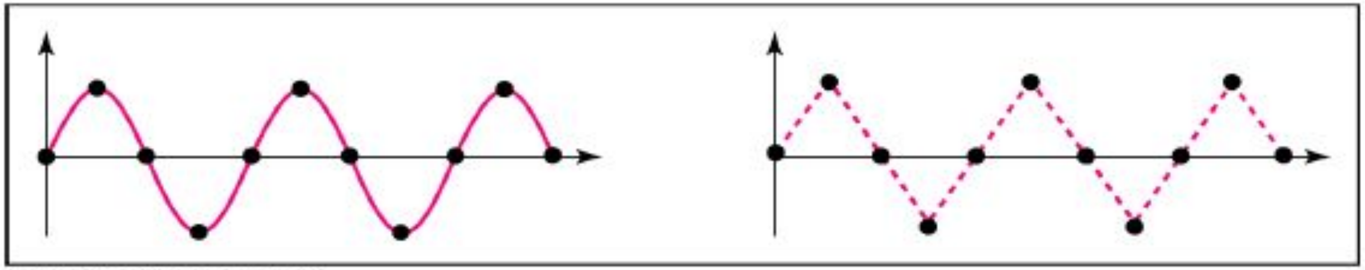
Uniform (ideal sampling)

- Analog signal is sampled every T secs. T is referred to as the sampling interval. $F_s = 1/T$ is called the sampling rate or sampling frequency
- The analog signal can be reconstructed from the samples without any distortion provided that the sampling frequency is sufficiently high to avoid the aliasing problem.

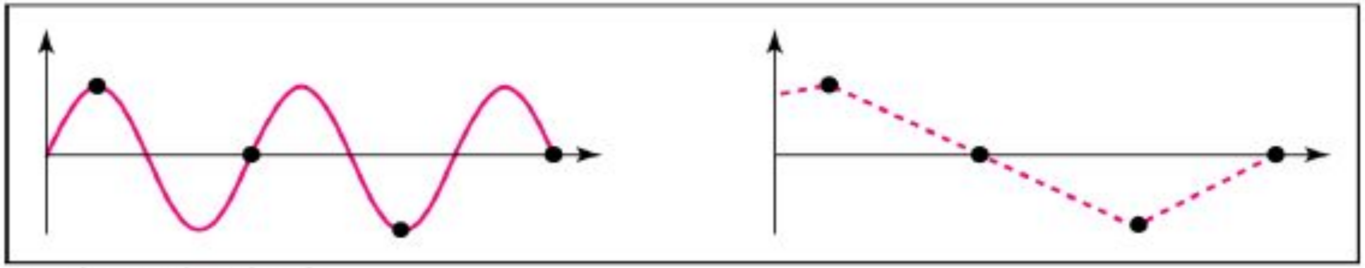
Recovery of a Sampled Sine wave for different Sampling Rates



a. Nyquist rate sampling: $f_s = 2f$



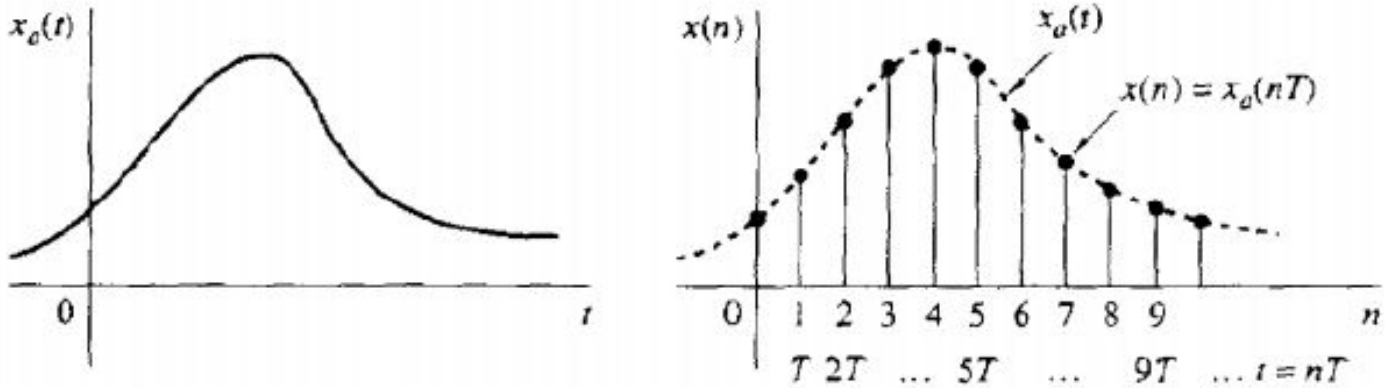
b. Oversampling: $f_s = 4f$



c. Undersampling: $f_s = f$

Uniform (Ideal) Sampling

- Relation between F (analog frequency) &f (discrete frequency)



$$x(n) = x_a(nT), \qquad -\infty < n < \infty$$

$$t = nT = \frac{n}{F_s}$$

Uniform (ideal) Sampling

Relation between Analog Frequency F, discrete frequency f (normalized frequency)

$$x_a(t) = A\cos(2\pi F t + \theta)$$

$$x_a(nT) \equiv x(n) = A\cos(2\pi F n T + \theta)$$

$$= A\cos\left(\frac{2\pi n F}{F_s} + \theta\right)$$

$$f = \frac{F}{F_s}$$

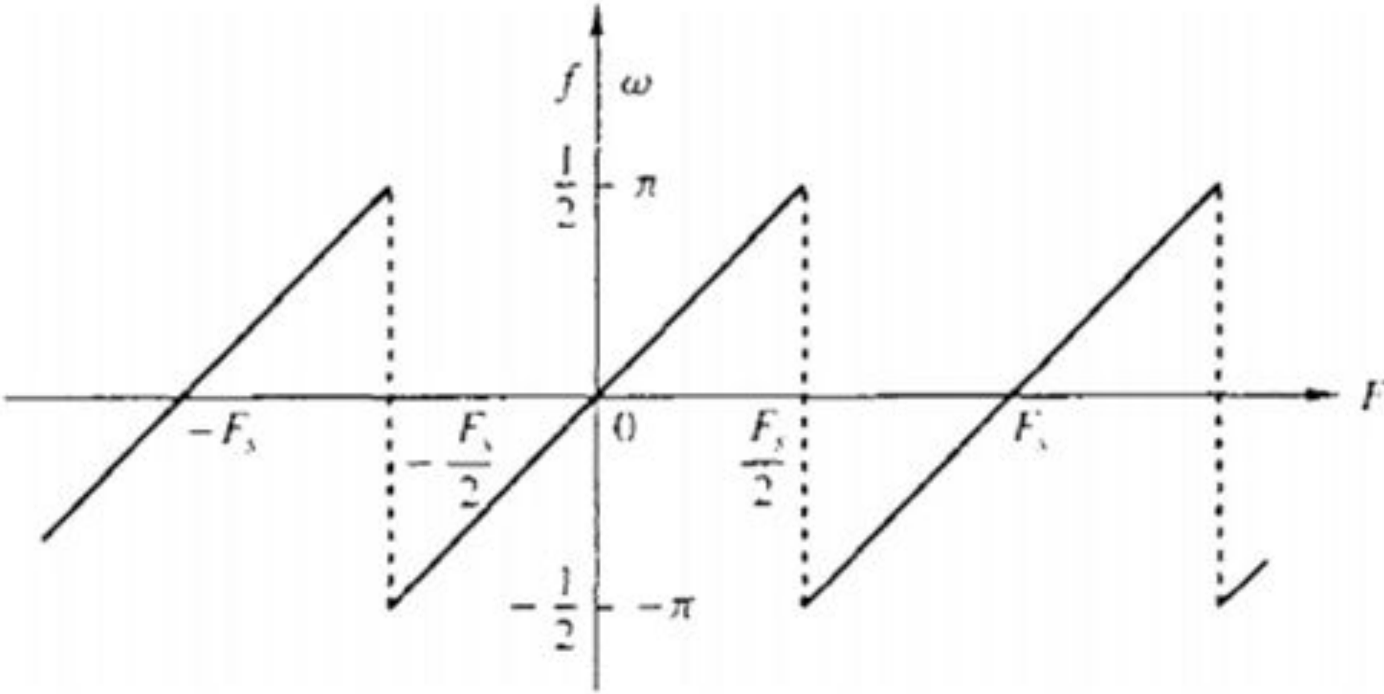
$$\omega = \Omega T$$

Where w = 2Πf is the discrete angular frequency

Then x(n) = A cos (wn+ Ɵ)

Uniform (Ideal) Sampling

- Relation between F (analog frequency) &f (discrete frequency(



The frequency Fs/2 (or w= π) is called the ***folding frequency.***

Uniform (Ideal) Sampling

- Sampling introduces an ambiguity, since the highest frequency in a continuous- time signal that can be uniquely distinguished when such a signal is sampled at a rate $F_s = 1/T$ is $F_{max} = F_s/2$

Ex: $X_1 = \cos 2\pi (10) t$, $X_2 = \cos 2\pi (50) t$

Both of them are sampled at $F_s = 40$.

Find $X_1(n)$ and $X_2(n)$?

$$x_1(n) = \cos 2\pi \left(\frac{10}{40}\right) n = \cos \frac{\pi}{2} n$$

$$x_2(n) = \cos 2\pi \left(\frac{50}{40}\right) n = \cos \frac{5\pi}{2} n$$

$$= \cos\left(2\pi n + \frac{\pi}{2} n\right) = \cos\frac{\pi}{2} n = x_1(n)$$

Sampling Theorem

According to the Sampling theorem, the sampling rate must be at least 2 times the highest frequency (F_{max}) contained in the signal.

$$\boldsymbol{F_s > 2\,F_{max}}$$

The sampling rate ***Fs*** which is equals to 2 $\boldsymbol{F_{max}}$ is called ***Nyquist rate***.

Sampling Theorem Example

A signal has a bandwidth of 200 kHz, beginning from 0. What is the minimum sampling rate for this signal? ***Fs = 400***

A signal has a bandwidth of 300 kHz. What is the minimum sampling rate for this signal?

We cannot find the minimum sampling rate in this case because we do not know where the bandwidth starts or ends. We do not know the maximum frequency in the signal.

Sampling theorem problem

Consider the analog signal

$$x_a(t) = 3\cos 50\pi t + 10\sin 300\pi t - \cos 100\pi t$$

What is the Nyquist rate for this signal?

Solution: Nyquist rate Fs= 2 Fmax = 2*150 = 300 samples/s

Quantization

- Conversion of

  discrete time continuous valued signal to

  discrete time discrete valued signal
- The value of each signal sample is represented by a value selected from a finite set of possible values
- The error introduced is called *quantization error* or *quantization noise*.

Quantization

• Sampling results in a series of pulses of varying amplitude values ranging between two limits: a min and a max.

• The amplitude values are infinite between the two limits.

We need to map the infinite amplitude values into a finite set of known values.

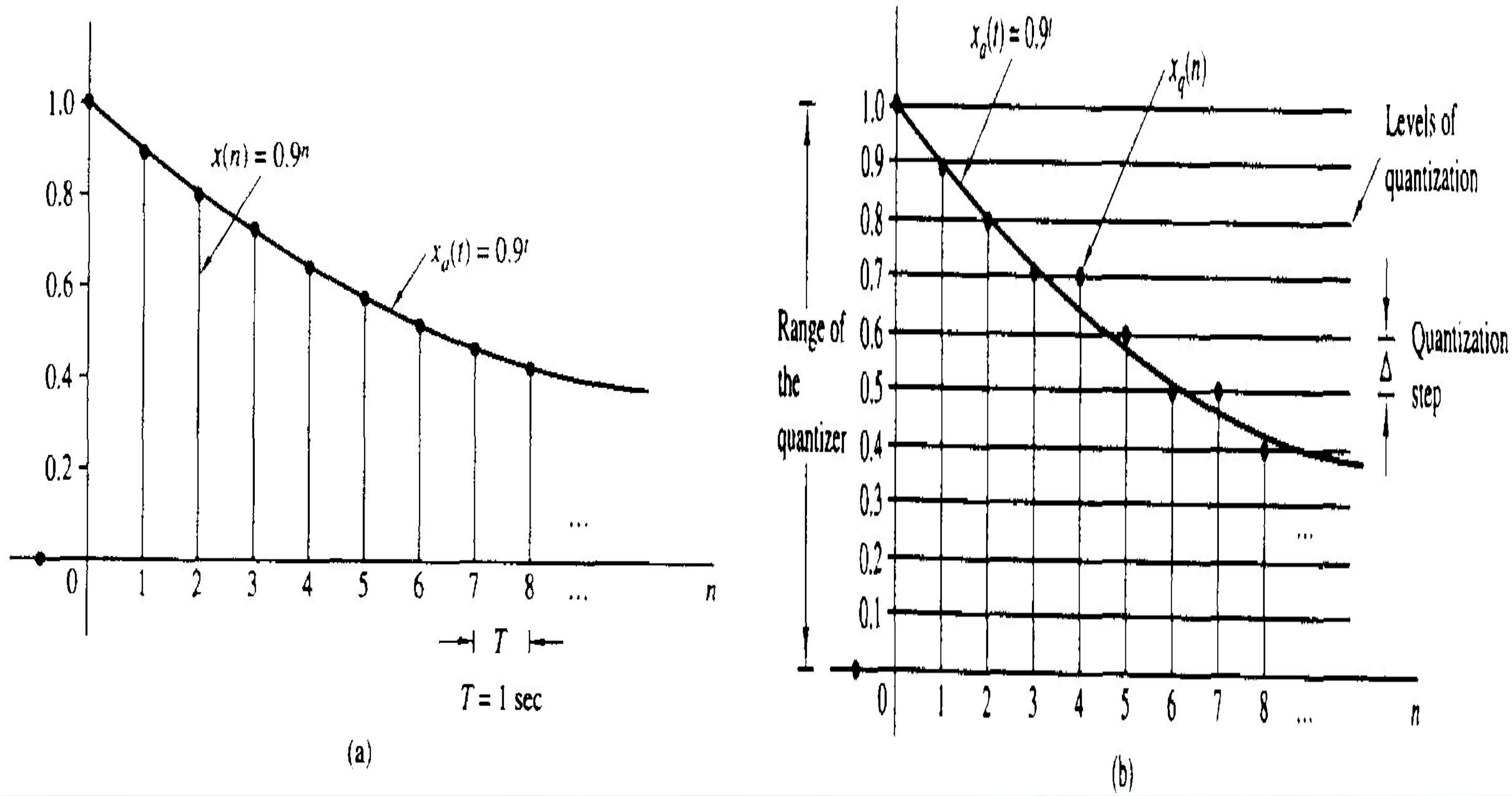
• This is achieved by dividing the distance between min and max into L -1 zones, each of height ⌂

⌂ =(max - min)/L-1

Each zone separated between 2 levels•

• Each sample falling in a zone is then approximated to the midpoint

Quantization



Quantization Error

- Quantization is a nonreversible process that results in signal distortion.
- When a signal is quantized, we introduce an error - the coded signal is an approximation of the actual amplitude value.
- The difference between actual and the quantized value is referred to as the quantization error.

$$e_q(n) = x_q(n) - x(n) \qquad -\frac{\Delta}{2} \le e_q(n) \le \frac{\Delta}{2}$$

the average quantized error power:

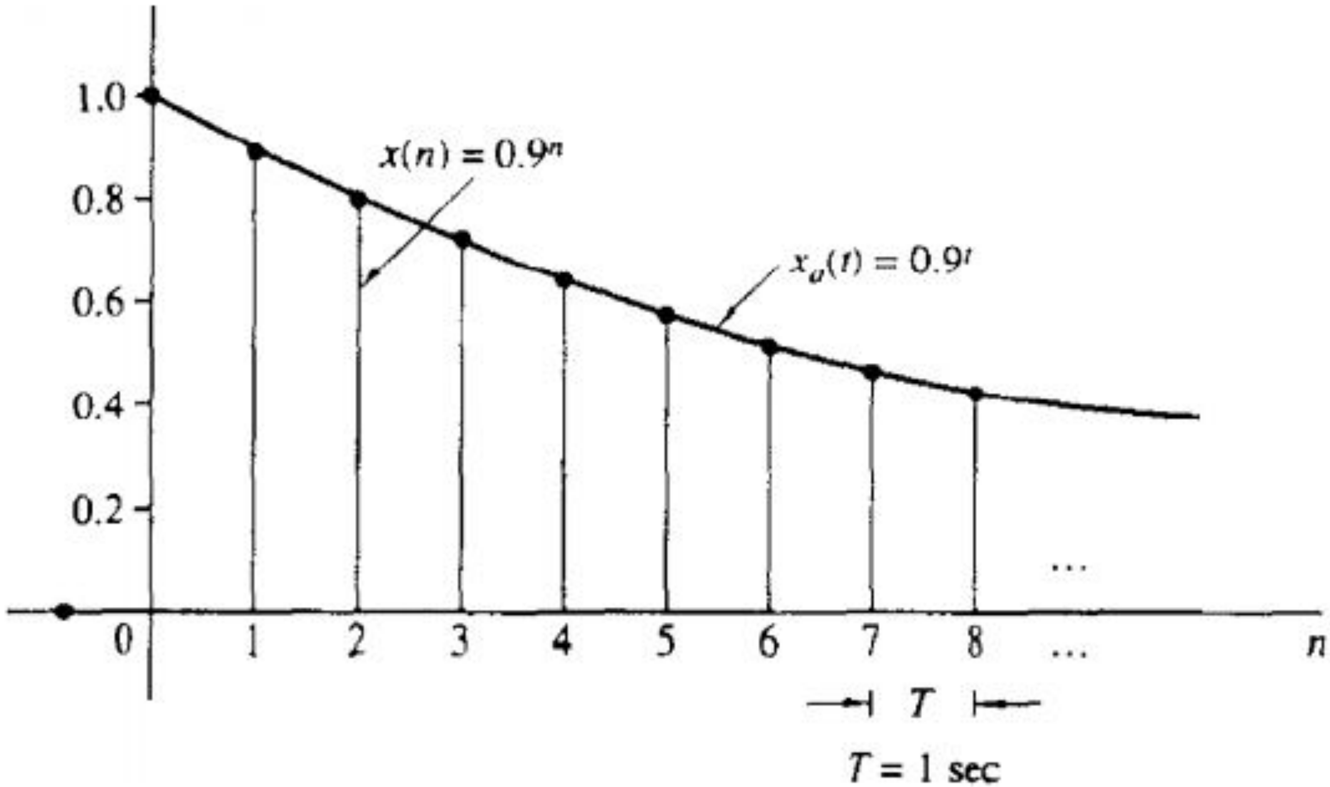
$$P_q = \frac{1}{N}\sum_{i=1}^{N} e_q^2$$

N is number of samples

- The more zones, the smaller Δ which results in smaller errors. BUT, the more zones the more bits required to encode the samples -> higher bit rate.

Quantization Error Example

$$x(n) = \begin{cases} 0.9^n, & n \geq 0 \\ 0, & n < 0 \end{cases}$$



Quantization Error Example

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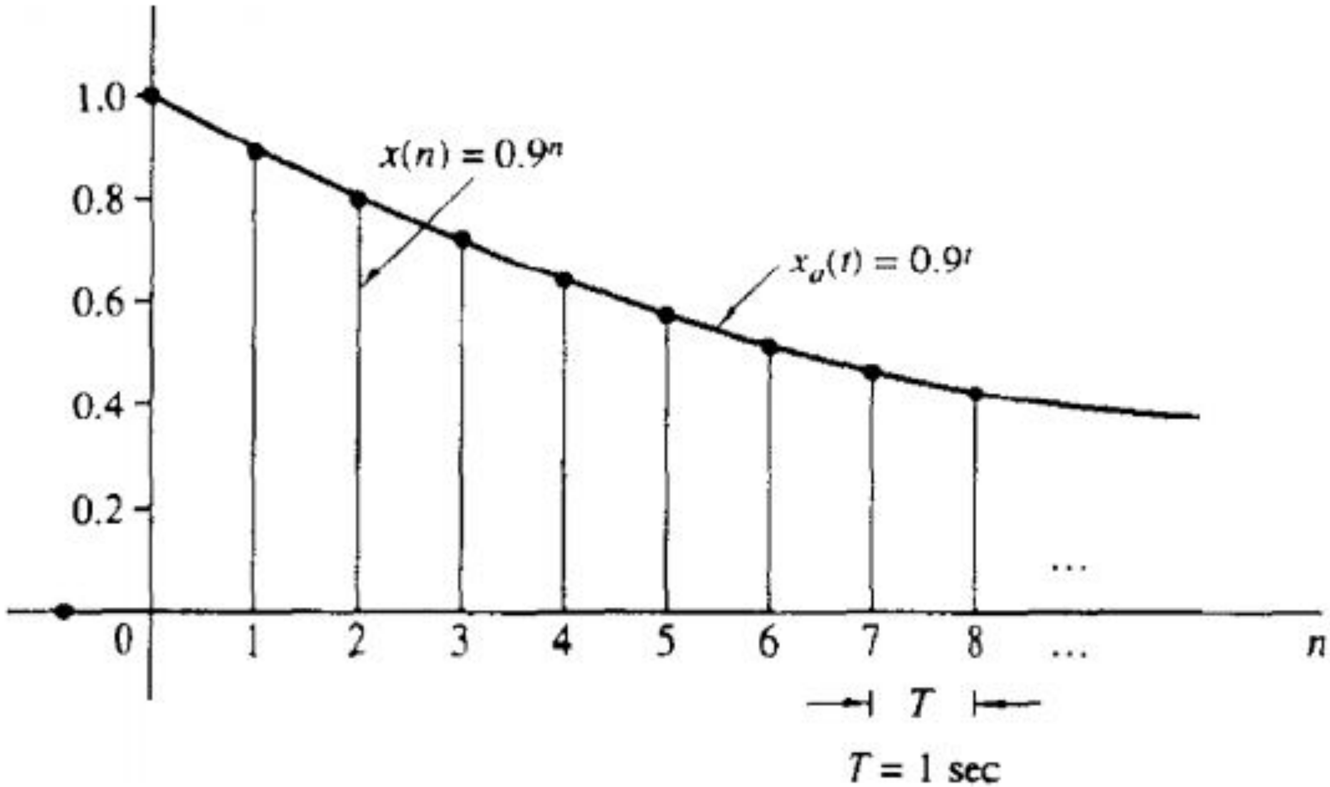

Quantization Error Example

n	$x(n)$ Discrete-time signal	$x_q(n)$ (Truncation)	$x_q(n)$ (Rounding)	$e_q(n) = x_q(n) - x(n)$ (Rounding)
0	1	1.0	1.0	0.0
1	0.9	0.9	0.9	0.0
2	0.81	0.8	0.8	−0.01
3	0.729	0.7	0.7	−0.029
4	0.6561	0.6	0.7	0.0439
5	0.59049	0.5	0.6	0.00951
6	0.531441	0.5	0.5	−0.031441
7	0.4782969	0.4	0.5	0.0217031
8	0.43046721	0.4	0.4	−0.03046721
9	0.387420489	0.3	0.4	0.012579511

Encoding

- Each Quantization Level is then assigned a binary code.
- The number of bits required to encode the levels is obtained using the following eq:

$$n_b = \log_2 L$$

For example:

- If we have 8 levels L= 8 from 0 to 7 then we have 7 separated zones
- Then nb = 3 bites
- Code 000 mapped to L=0   001 to L=1 ,……, and code 111 to L=7

- Thus zone 1 between L=0 to L=1, Zone 2 between L=1 to L=2 and so on.

Quantization and Encoding Example 1:

- Quantize and encode the following sampled signal x(n) using 8 quantization levels and minimum number of bits. Compute the average error power.

X(n)= { 0.387, 0.43, 0.47, 0.52, 0.63, 0.72, 0.81, 0.9, 1, 0.3}

Example 2

1.13 The discrete-time signal $x(n) = 6.35\cos(\pi/10)n$ is quantized with a resolution (a) $\Delta = 0.1$ or (b) $\Delta = 0.02$. How many bits are required in the A/D converter in each case?

(a)

$$\begin{aligned} \text{Range} &= x_{\max} - x_{\min} = 12.7. \\ m &= 1 + \frac{\text{range}}{\Delta} \\ &= 127 + 1 = 128 \Rightarrow log_2(128) \\ &= 7 \text{ bits.} \end{aligned}$$

(b) $m = 1 + \frac{127}{0.02} = 636 \Rightarrow log_2(636) \Rightarrow 10$ bit A/D.