Cross Correlation

Correlation

A mathematical operation that closely resembles convolution is correlation. Just as in the case of convolution, two signal sequences are involved in correlation. In contrast to convolution, however, our objective in computing the correlation between the two signals is to measure the degree to which the two signals are similar and thus to extract some information that depends to a large extent on the application. Correlation of signals is often encountered in radar, sonar, digital communications, geology, and other areas in science and engineering.

correlation

To be specific, let us suppose that we have two signal sequences $x(n)$ and $y(n)$ that we wish to compare. In radar and active sonar applications, $x(n)$ can represent the sampled version of the transmitted signal and $y(n)$ can represent the sampled version of the received signal at the output of the analog-to-digital (A/D)

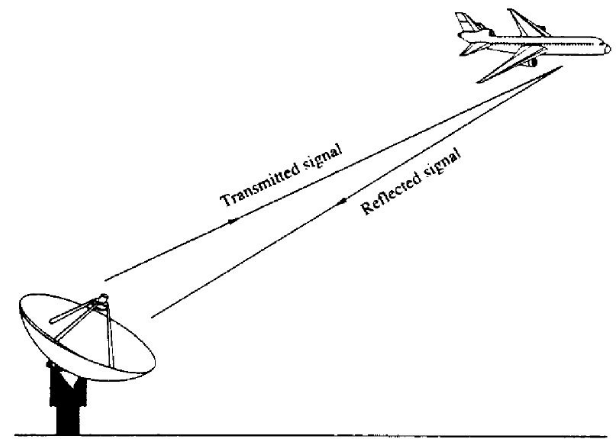

Figure 2.38 Radar target detection.

We can represent the received signal sequence as

$$y(n) = \alpha x(n - D) + w(n)$$

where α is some attenuation factor representing the signal loss involved in the round-trip transmission of the signal $x(n)$, D is the round-trip delay, which is assumed to be an integer multiple of the sampling interval, and $w(n)$ represents the additive noise that is picked up by the antenna and any noise generated by the electronic components and amplifiers contained in the front end of the receiver.

Cross Correlation

Suppose that we have two real signal sequences $x(n)$ and $y(n)$ each of which has finite energy. The *crosscorrelation* of $x(n)$ and $y(n)$ is a sequence $r_{xy}(l)$, which is defined as

$$r_{xy}(l) = \sum_{n=-\infty}^{\infty} x(n)y(n-l) \qquad l = 0, \pm1, \pm2, \ldots \tag{2.6.3}$$

or, equivalently, as

$$r_{xy}(l) = \sum_{n=-\infty}^{\infty} x(n+l)y(n) \qquad l = 0, \pm1, \pm2, \ldots \tag{2.6.4}$$

Cross correlation

$$r_{yx}(l) = \sum_{n=-\infty}^{\infty} y(n)x(n-l) \tag{2.6.5}$$

or, equivalently,

$$r_{yx}(l) = \sum_{n=-\infty}^{\infty} y(n+l)x(n) \tag{2.6.6}$$

By comparing (2.6.3) with (2.6.6) or (2.6.4) with (2.6.5), we conclude that

$$r_{xy}(l) = r_{yx}(-l) \tag{2.6.7}$$

Therefore, $r_{yx}(l)$ is simply the folded version of $r_{xy}(l)$,

Example 2.6.1

Determine the crosscorrelation sequence $r_{xy}(l)$ of the sequences

$$x(n) = \{\ldots, 0, 0, 2, -1, 3, 7, \underset{\uparrow}{1}, 2, -3, 0, 0, \ldots\}$$

$$y(n) = \{\ldots, 0, 0, 1, -1, 2, -2, \underset{\uparrow}{4}, 1, -2, 5, 0, 0, \ldots\}$$

Solution Let us use the definition in (2.6.3) to compute $r_{xy}(l)$. For $l = 0$ we have

$$r_{xy}(0) = \sum_{n=-\infty}^{\infty} x(n)y(n)$$

The product sequence $v_0(n) = x(n)y(n)$ is

$$v_0(n) = \{\ldots, 0, 0, 2, 1, 6, -14, \underset{\uparrow}{4}, 2, 6, 0, 0, \ldots\}$$

and hence the sum over all values of n is

$$r_{xy}(0) = 7$$

$Y(n\text{-}1)=\{0,0,1,-1,2,\underset{\uparrow}{-2},4,,-2,5,0,0,0\}$

$V1(n)= \{0,0,-1,-3,14,-2,8,-3,0,0\}$ $\quad r_{xy}(1)= \sum \; v1(n)$

Example Continued

For $l > 0$, we simply shift $y(n)$ to the right relative to $x(n)$ by l units, compute the product sequence $v_l(n) = x(n)y(n-l)$, and finally, sum over all values of the product sequence. Thus we obtain

$$r_{xy}(1) = 13, \quad r_{xy}(2) = -18, \quad r_{xy}(3) = 16, \quad r_{xy}(4) = -7$$

$$r_{xy}(5) = 5, \quad r_{xy}(6) = -3, \quad r_{xy}(l) = 0, \quad l \geq 7$$

For $l < 0$, we shift $y(n)$ to the left relative to $x(n)$ by l units, compute the product sequence $v_l(n) = x(n)y(n-l)$, and sum over all values of the product sequence. Thus we obtain the values of the crosscorrelation sequence

$$r_{xy}(-1) = 0, \quad r_{xy}(-2) = 33, \quad r_{xy}(-3) = -14, \quad r_{xy}(-4) = 36$$

$$r_{xy}(-5) = 19, \quad r_{xy}(-6) = -9, \quad r_{xy}(-7) = 10, \quad r_{xy}(l) = 0, \quad l \leq -8$$

Therefore, the crosscorrelation sequence of $x(n)$ and $y(n)$ is

$$r_{xy}(l) = \{10, -9, 19, 36, -14, 33, 0, \underset{\uparrow}{7}, 13, -18, 16, -7, 5, -3\}$$

Similarity to convolution

$$r_{xy}(l) = x(l) * y(-l)$$

Autocorrelation

In the special case where $y(n) = x(n)$, we have the *autocorrelation* of $x(n)$, which is defined as the sequence

$$r_{xx}(l) = \sum_{n=-\infty}^{\infty} x(n)x(n-l) \tag{2.6.9}$$

or, equivalently, as

$$r_{xx}(l) = \sum_{n=-\infty}^{\infty} x(n+l)x(n) \tag{2.6.10}$$

Finite limit

particular, if $x(n)$ and $y(n)$ are causal sequences of length N [i.e., $x(n) = y(n) = 0$ for $n < 0$ and $n \geq N$], the crosscorrelation and autocorrelation sequences may be expressed as

$$r_{xy}(l) = \sum_{n=i}^{N-|k|-1} x(n)y(n-l) \qquad (2.6.11)$$

and

$$r_{xx}(l) = \sum_{n=i}^{N-|k|-1} x(n)x(n-l) \qquad (2.6.12)$$

where $i = l$, $k = 0$ for $l \geq 0$, and $i = 0$, $k = l$ for $l < 0$.