Digital Signal Processing

Digital signal processing, principles, Algorithms and Applications.

Course Outline

- *Fundamentals of DSP*
- *Sampling of continuous Time Signals*
- *Z - transform*
- *Discrete Systems (Convolution & Correlation)*
- *Fourier Transform , its inverse*
- *Fast Fourier Transform (FFT)*
- *Finite Impulse Response (FIR) filters*
- *Infinite Impulse Response (IIR) filters*

Chapter1 :

Fundamentals of DSP

1.1

SIGNALS, SYSTEMS, SIGNAL PROCESSING

Signal

- **Signal :** any physical quantity that varies with time, space or any other independent variable or variables
- Mathematically we describe a signal as a function of one or more dependent variables
- **Example :**

$$\begin{aligned} s_1(t) &= 5t \\ s_2(t) &= 20t^2 \end{aligned} \qquad (1.1.1)$$

  - Describe two signals one that vary linearly with time t and the other vary quadratically with t

Signal

- **Another Example:**

$$s(x, y) = 3x + 2xy + 10y^2 \qquad (1.1.2)$$

  - Signal that describe two independent variables x,y that could represent two spatial coordinates in the plane
- The previous examples show signals that are precisely defined by specifying the functional dependence on the independent variables.

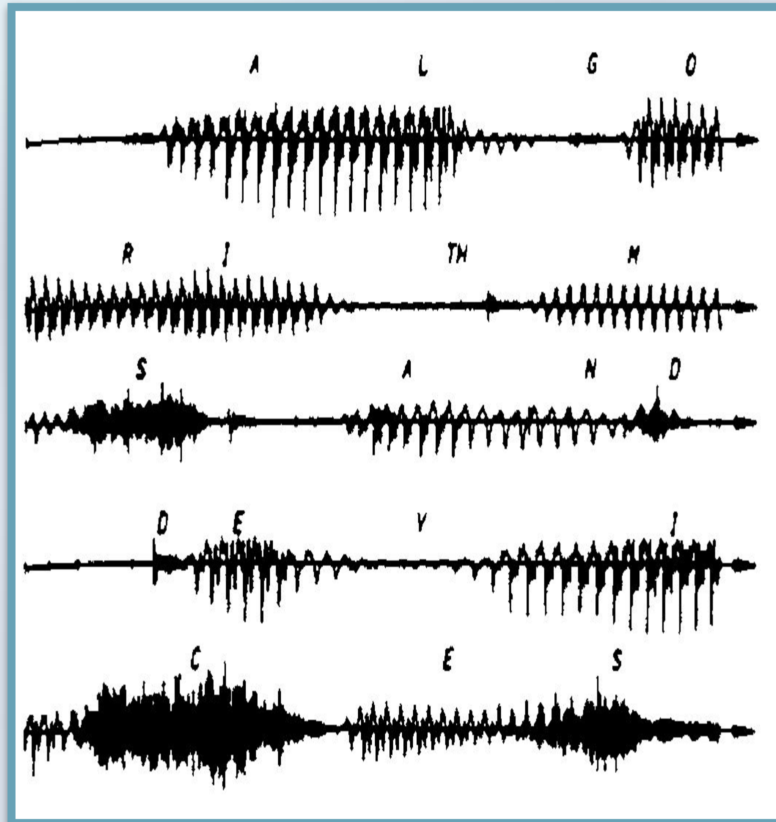
Are all signal can be represented mathematical? No

Ex: Speech

Signal

Cases where functional relationships are unknown or highly complicated

- Example:
  - Speech signal can not be described functionally using expressions it can be represented as sum of several sinusoids of different amplitudes, frequencies and phases



System

- Physical device that perform operations on the signal
- Example :
  - Noise filter for speech
  - When signal passed through the system the signal is processed(filtered)
- System definition include not only physical devices but also software realization of operations on signal
  - The operations performed on signal consist of number of mathematical equations (programs)
  - Provide more flexbility

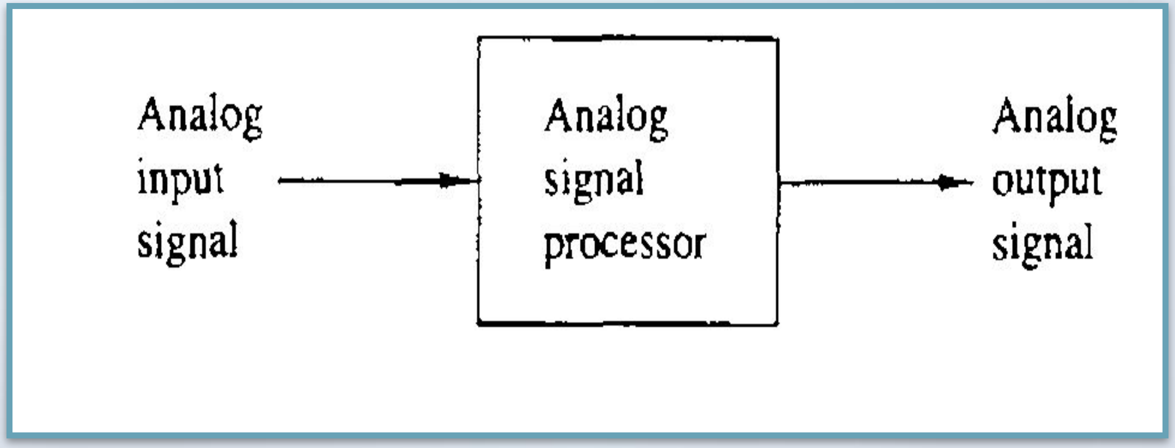
Signal Processing

- The system is characterized by the type of operations it performs on the signal
  - If the operation is Linear the system is called Linear
  - If the operation is Nonlinear the system is called Nonlinear
- These operations are referred as Signal Processing

Basic elements of DSP system

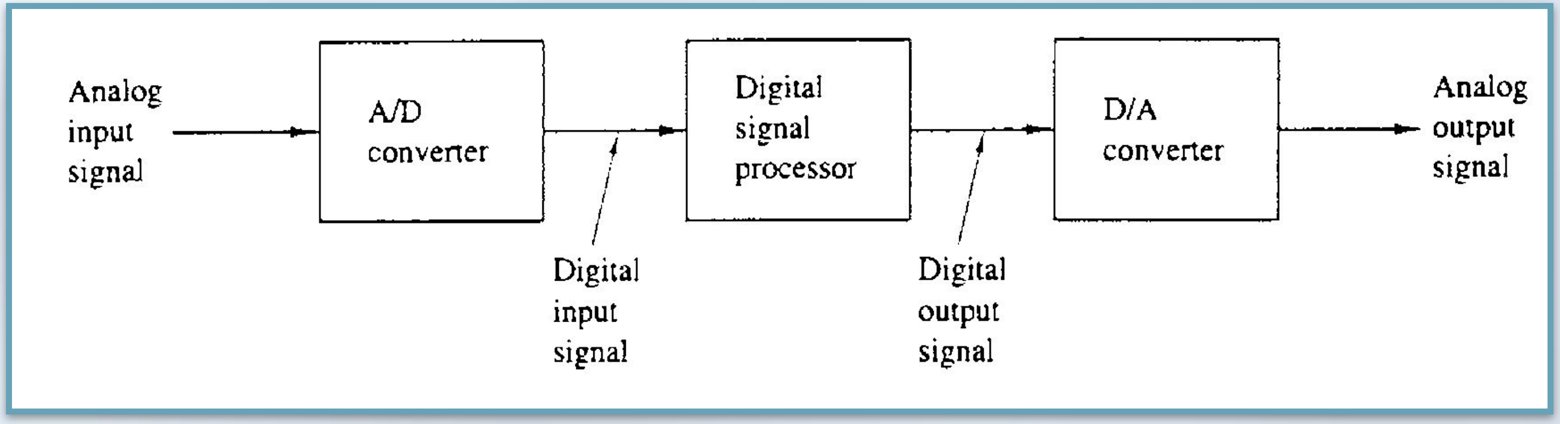
- Most of signals are analog in nature
- The signals are functions of continuous variable as time, space,…
- They usually takes values on continuous range.
- **Signal Processing**
  - Direct Signal Processing
  - Digital Signal Processing

Direct Signal Processing



- Signals are processed directly by appropriate analog systems for the purpose of changing their characteristics or extract information
- **Examples:** filters, frequency analyzers, frequency multipliers,…
- Both input and output are in analog form

Digital Signal Processing



- There is a need for interface between analog system and digital processor “Analog to Digital Converter” (A/D)
- The Output of A/D converter is a digital signal appropriate as input for digital processor
- Digital signal processor
  - Large programmable digital computer
  - Small microprocessor designed for specified purpose

Advantage of Digital over Analog Signal Processing

- Easy to reprogram
- More accurate
- Results can be stored to be reprocessed offline
- Cheaper

1.2

CLASSIFICATION OF SIGNALS

MULTI CHANNEL AND MULTI DIMENSION SIGNAL

If the signal is generated by multiple sources or sensors, it is called multi-channel signal. On the other hand, if the signal is dependent on more than one independent variable it is called multi-dimensional signal.

S(X, Y) is a multi-dimension signal S (S can be a 2D image),

12 lead ECG recording: multi-channel Signal

Continuous time Vs. Discrete time Signals

- **Continuous time signals:**
  - Defined for each value of time and take on values in continuous interval (a,b) where a can be -∞ and b is ∞
  - Can be described mathematically by functions of continuous variables
  - Example : ,

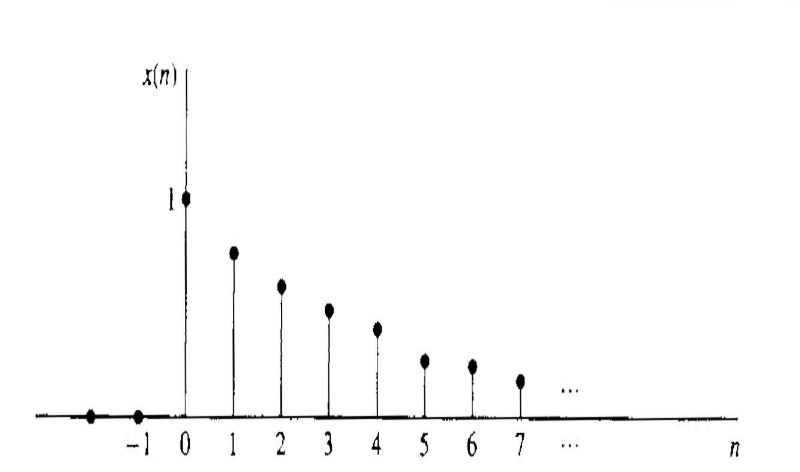
$$x_1(t) = \cos \pi t$$

$$x_2(t) = e^{-|t|}, \quad -\infty < t < \infty$$

Continuous time Vs. Discrete time Signals

- **Discrete-time signals:**
- Defined only at a certain specific values of time

$$x(n) = \begin{cases} 0.8^n, & \text{if } n \geq 0 \\ 0, & \text{otherwise} \end{cases}$$



- To emphasize the discrete-time nature of the system we denote the signal as x(n) instead of x(t)

Continuous valued vs. Discrete valued signals

- **Continuous valued signal:**
  - If the signal can take all possible values on a finite or infinite ranges.
- **Discrete valued signal:**
  - The signal take values from finite set of possible values

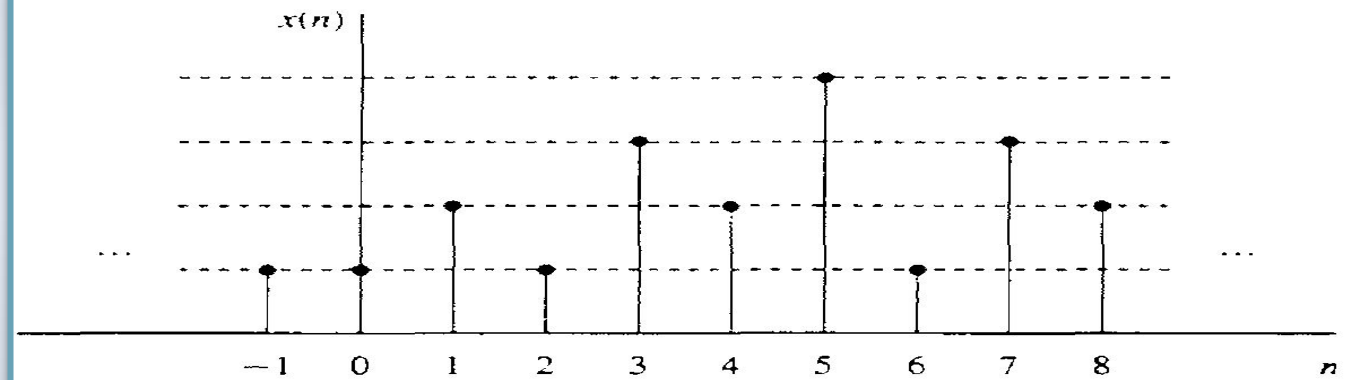
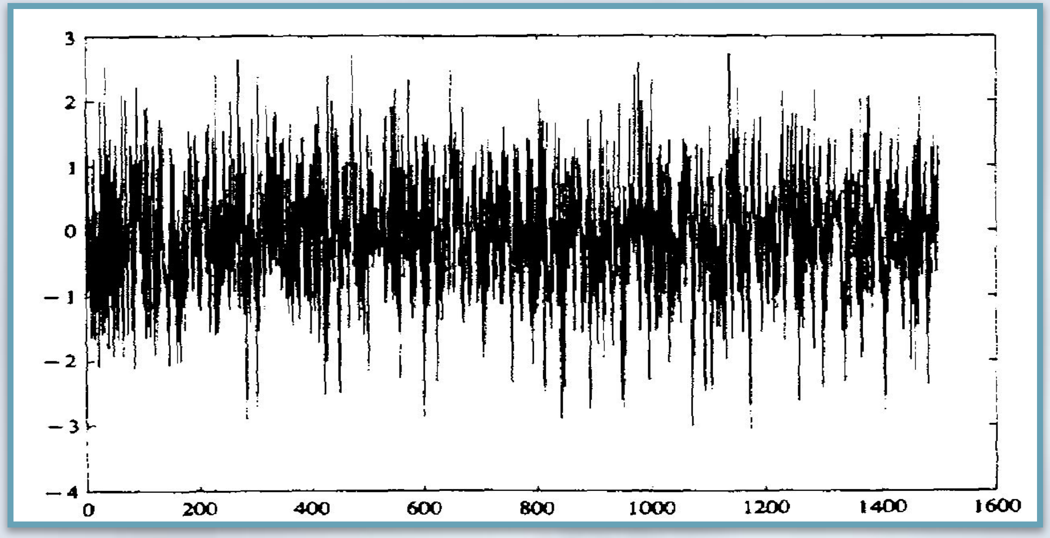


Figure 1.8 Digital signal with four different amplitude values.

Deterministic Vs. Random signals

- **Deterministic signal:**
  - Signal can be described by any explicit mathematical expression, a table of data, or a well defined rule
- **Random signal:**
  - Can not be described to any reasonable degree of accuracy by explicit mathematical formulas



Discrete time signals and Systems

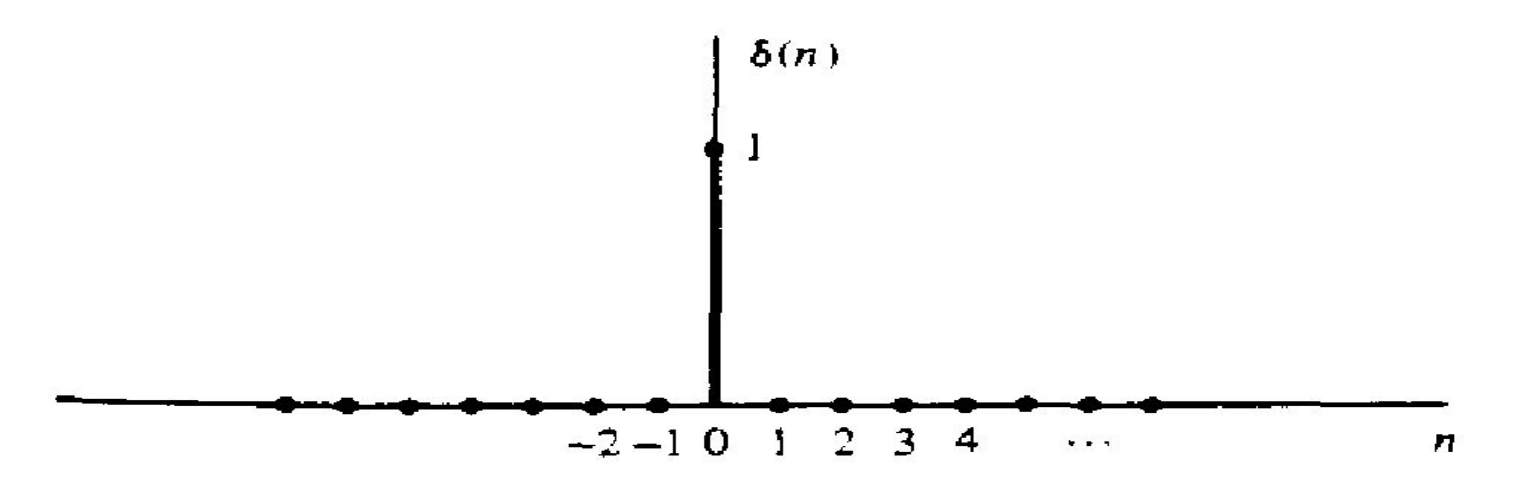
2.1

DISCRETE TIME SIGNALS

Discrete time signals

- **Unit sample sequence:**

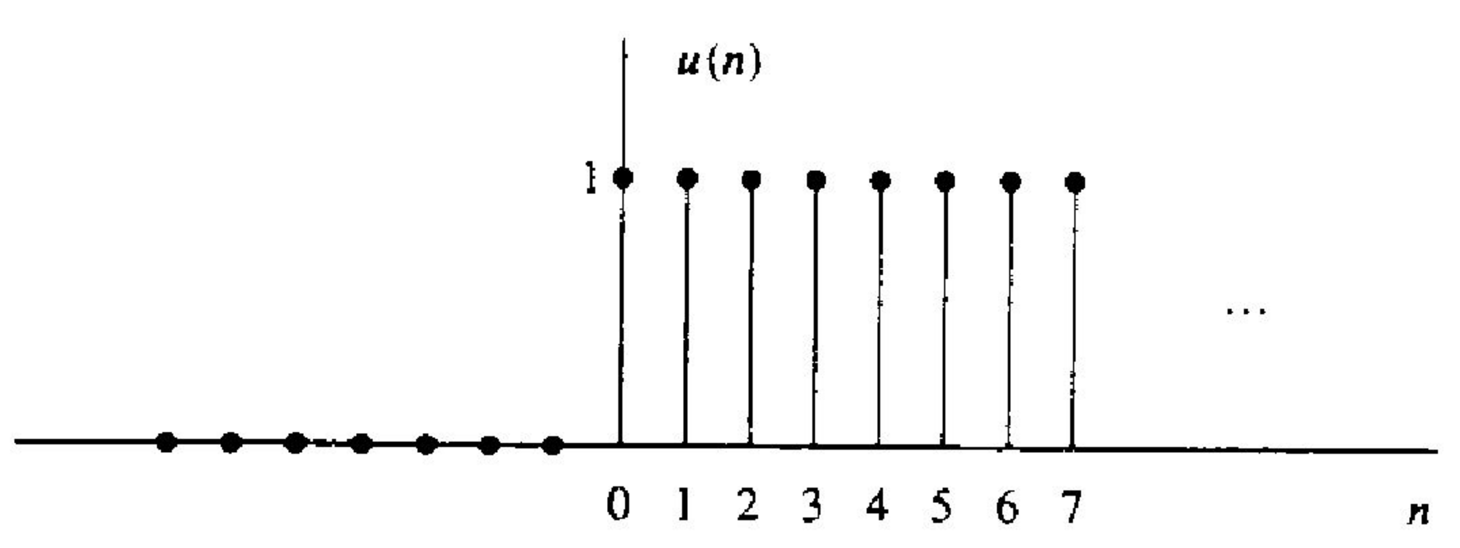
$$\delta(n) \equiv \begin{cases} 1, & \text{for } n = 0 \\ 0, & \text{for } n \neq 0 \end{cases}$$



Discrete time signals

- **Unit step signal:**

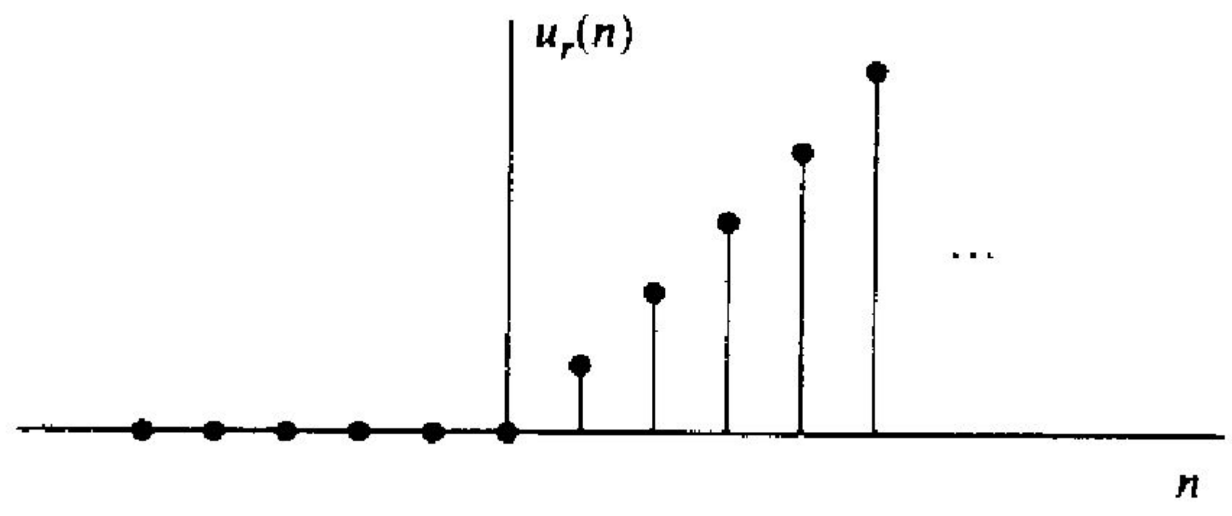
$$u(n) \equiv \begin{cases} 1, & \text{for } n \geq 0 \\ 0, & \text{for } n < 0 \end{cases}$$



Discrete time signals

- **Unit Ramp:**

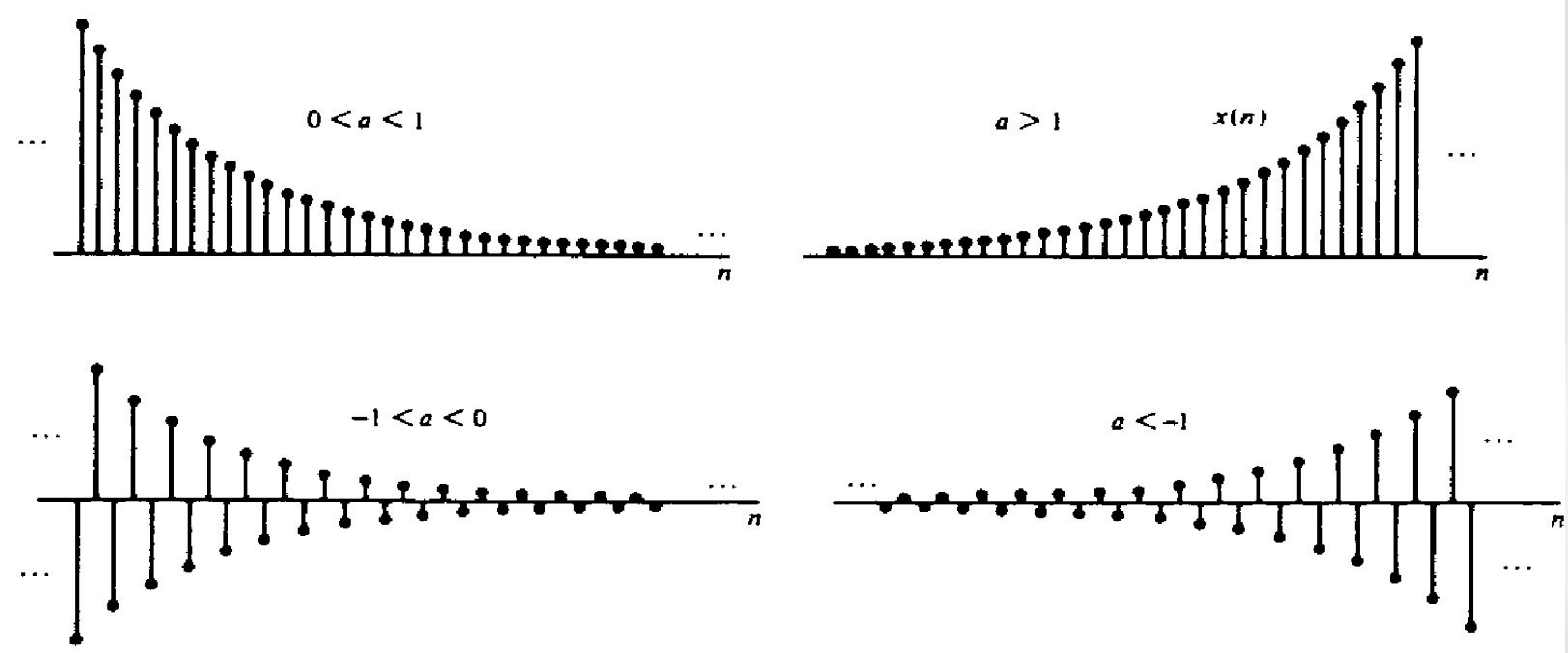
$$u_r(n) \equiv \begin{cases} n, & \text{for } n \geq 0 \\ 0, & \text{for } n < 0 \end{cases}$$



Discrete time signals

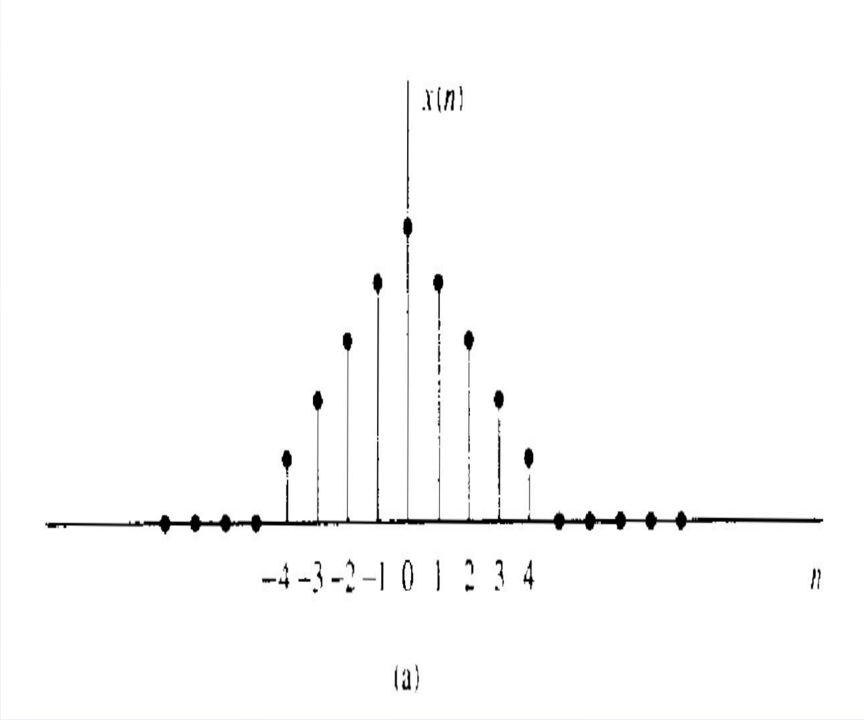
- **Exponential signal:**

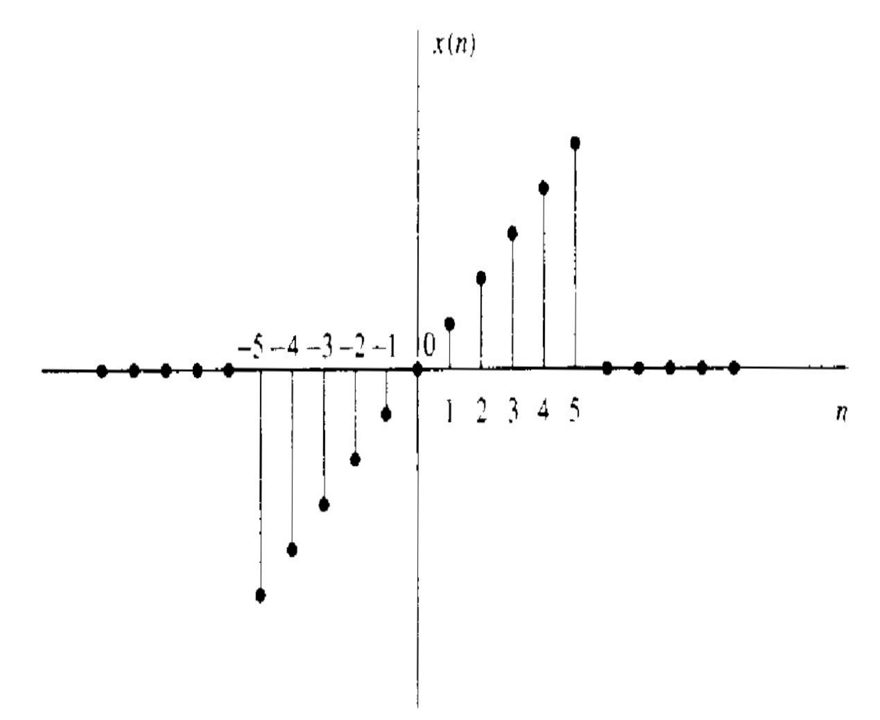
$$x(n) = a^n \qquad \text{for all } n$$



Classification of Discrete Time Signals

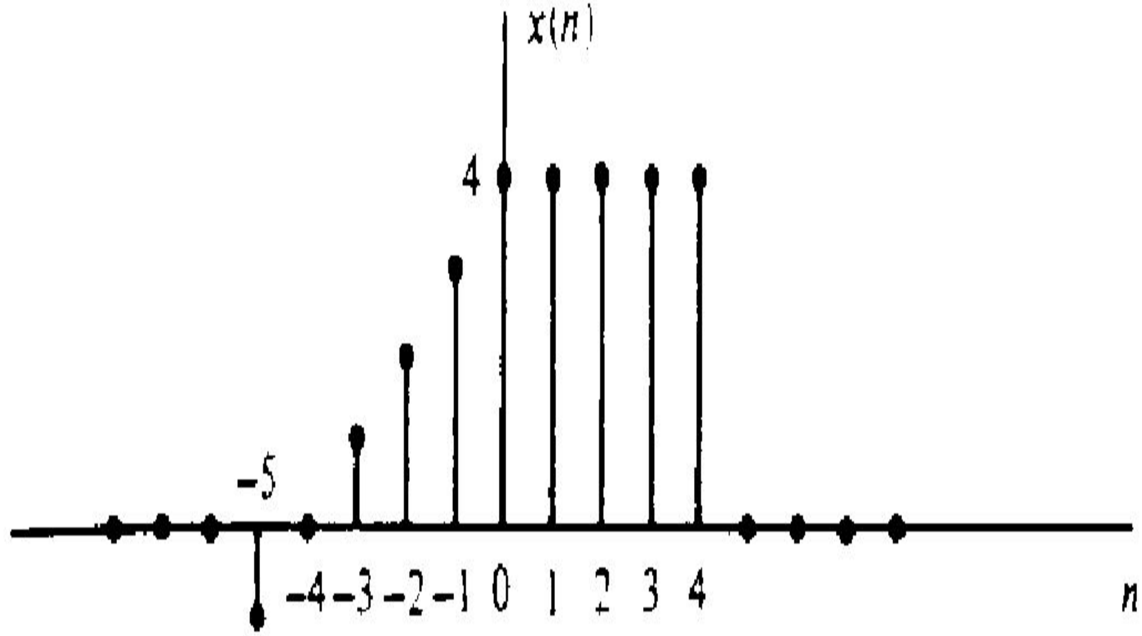
- **Symmetric and antisymmetric:**





Manipulations of Discrete time signals

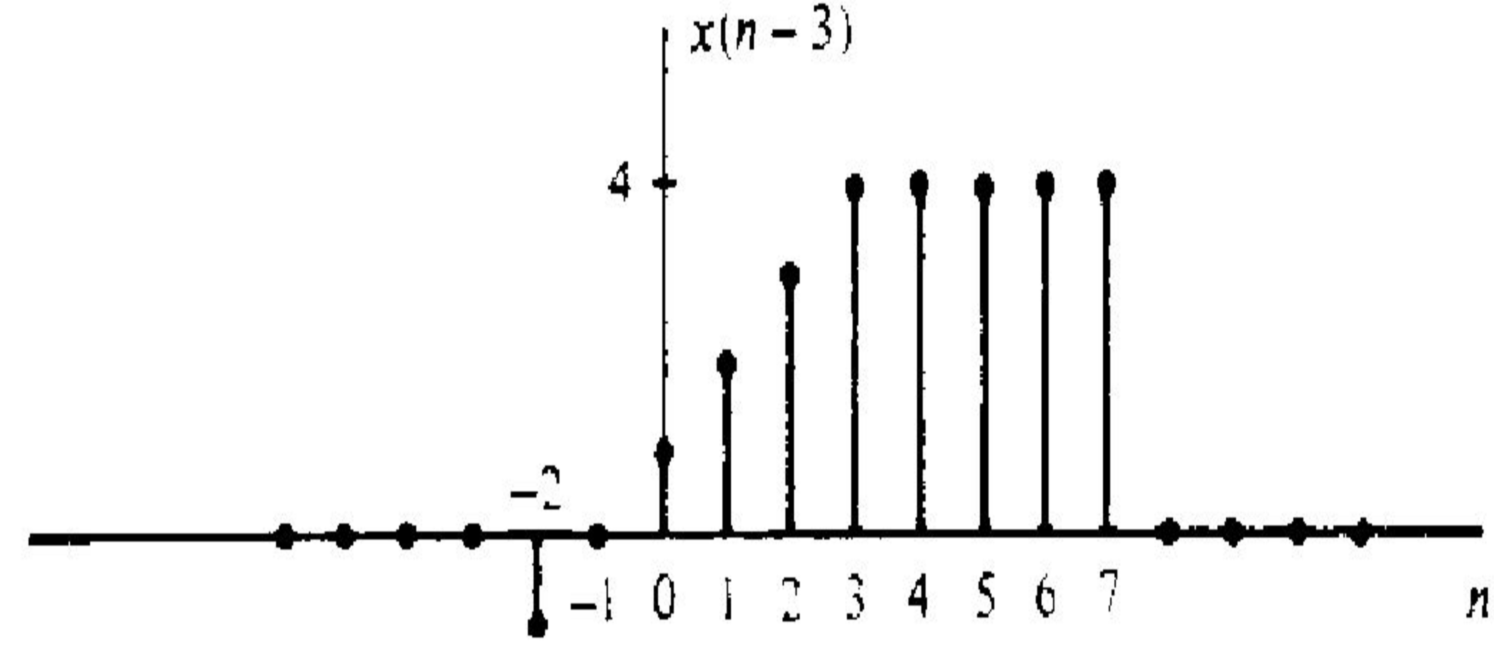
- Original Signal



(a)

Manipulations of Discrete time signals

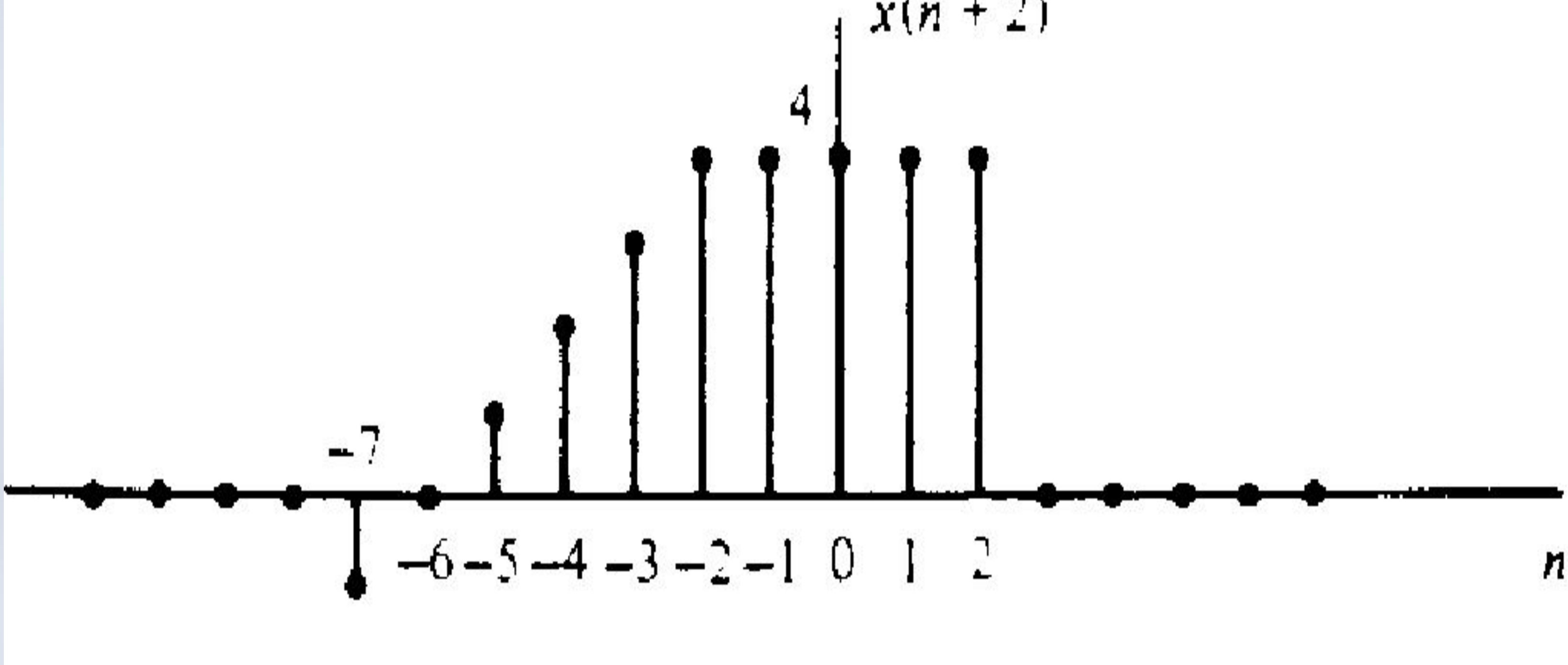
- Delayed



(b)

Manipulations of Discrete time signals

- Advanced



(c)

2.2

DISCRETE TIME SYSTEMS

Discrete time systems

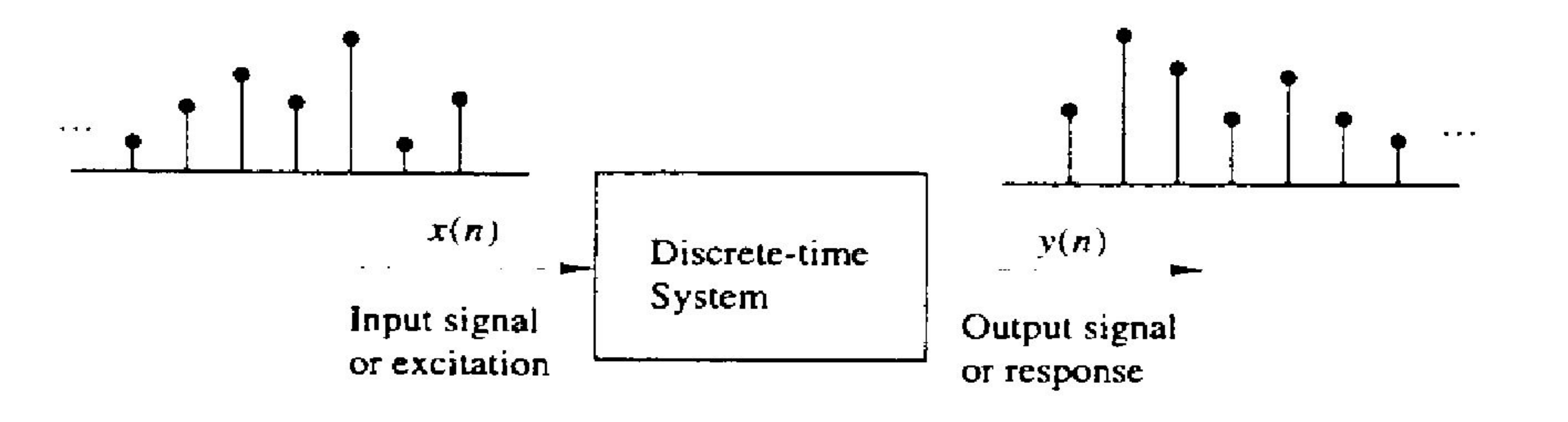
- Device or algorithm that perform some prescribed operations on a discrete time signal
- Discrete time system is a device or algorithm that operate on discrete time signal called the input or excitation according to some well defined rules to produce another discrete time signal called output or response of the system

Discrete time systems Input/output description

- The input signal x(n) is transformed by the system into a signal y(n) expressed as:

$$y(n) \equiv \mathcal{T}[x(n)]$$

- T denotes the transformation(operator) or processing performed by the system on x(n) to produce y(n)



Discrete time systems Input/output description

- Example: determine the response of the following systems to the input signal

$$x(n) = \begin{cases} |n|, & -3 \le n \le 3 \\ 0, & \text{otherwise} \end{cases}$$

(a) $y(n) = x(n)$

(b) $y(n) = x(n-1)$

(c) $y(n) = x(n+1)$

(d) $y(n) = \frac{1}{3}[x(n+1) + x(n) + x(n-1)]$

(e) $y(n) = max\{x(n+1), x(n), x(n-1)\}$

(f) $y(n) = \sum_{k=-\infty}^{n} x(k) = x(n) + x(n-1) + x(n-2) + \cdots$

Discrete time systems Input/output description

- Sol: $x(n) = \{\ldots, 0, 3, 2, 1, \underset{\uparrow}{0}, 1, 2, 3, 0, \ldots\}$

a) identity system.

b) system delays input by one sample.

Y(n) $= \{\ldots, 0, 3, 2, \underset{\uparrow}{1}, 0, 1, 2, 3, 0, \ldots\}$

c) Advances the input: y(n) $= \{\ldots, 0, 3, 2, 1, 0, \underset{\uparrow}{1}, 2, 3, 0, \ldots\}$

d) $y(0) = \frac{1}{3}[x(-1) + x(0) + x(1)] = \frac{1}{3}[1 + 0 + 1] = \frac{2}{3}$

And then $y(n) = \{\ldots, 0, 1, \frac{5}{3}, 2, 1, \underset{\uparrow}{\frac{2}{3}}, 1, 2, \frac{5}{3}, 1, 0, \ldots\}$

e) $y(n) = \{0, 3, 3, 3, 2, \underset{\uparrow}{1}, 2, 3, 3, 3, 0, \ldots\}$

f) accumulator $y(n) = \{\ldots, 0, 3, 5, 6, \underset{\uparrow}{6}, 7, 9, 12, 0, \ldots\}$

2.3

CLASSIFICATION OF DISCRETE TIME SYSTEMS

Static vs. dynamic systems

- **Static:** or called memory less, if the output at any instant n depends at most on input sample at the same time, but no past or future samples of input.
- Any other case the system is said to be dynamic or have memory
- **Examples**
  - Static

$$y(n) = nx(n) + bx^3(n)$$

  - Dynamic

$$y(n) = x(n) + 3x(n-1)$$

$$y(n) = \sum_{k=0}^{n} x(n-k)$$

Time Invariant Vs. Time Variant systems

- **Time Invariant systems-1:** if the input-output characteristics don't change with time

- System in relaxed state: we have system T in relaxed which when excited by an input signal *x(n)* produce output signal *y(n)*

$$y(n) = T[x(n)]$$

Time Invariant Vs. Time Variant systems

- **Time Invariant systems-2:**

Definition:

A relaxed system T is time invariant or shift invariant if and only if

$$x(n) \xrightarrow{\mathcal{T}} y(n)$$

implies that

$$x(n-k) \xrightarrow{\mathcal{T}} y(n-k)$$

for every input signal *x(n)* and every time shift k

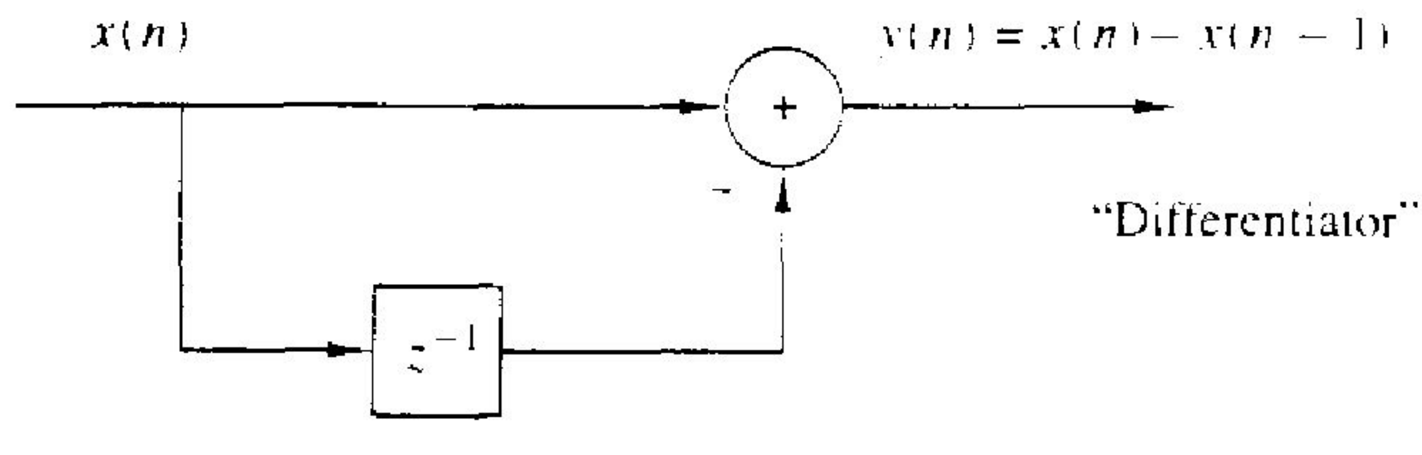
Time Invariant Vs. Time Variant systems

- **Time Invariant systems-3:**

Example1-1:

given the system

$$y(n) = T[x(n)] = x(n) - x(n-1)$$



(a)

Time Invariant Vs. Time Variant systems

- **Time Invariant systems-4:**

<u>Example1-2:</u>

delaying input with k units

$$y(n,k) = x(n-k) - x(n-k-1)$$

delaying output with k units

$$y(n-k) = x(n-k) - x(n-k-1)$$

since the RHS of (2) & (3) are identical

therefore y(n,k)=y(n-k) ,system is time invariant

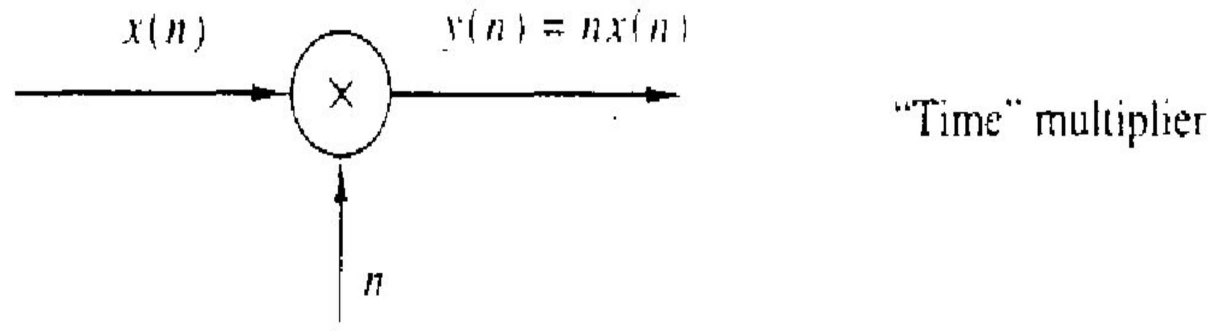
Time Invariant Vs. Time Variant systems

- **Time Invariant systems-5:**

Example2-1:

given the system

$$y(n) = T[x(n)] = nx(n)$$



(b)

Time Invariant Vs. Time Variant systems

- **Time Invariant systems-6:**

Example2-2:

delaying input with k units

$$y(n,k) = nx(n-k)$$

delaying output with k units

$$\begin{aligned} y(n-k) &= (n-k)x(n-k) \\ &= nx(n-k) - kx(n-k) \end{aligned}$$

since the RHS of (2) & (3) are not identical

therefore y(n,k)≠y(n-k) ,system is time variant

Linear Vs. Non Linear Systems

- **Linear Systems-1:**

  A system is called linear if and only if

$$T[a_1 x_1(n) + a_2 x_2(n)] = a_1 T[x_1(n)] + a_2 T[x_2(n)]$$

  For any arbitrary input sequences $x_1(n)$ and $x_2(n)$ and any arbitrary constants a_1 and a_2

Linear Vs. Non Linear Systems

- **Linear Systems-2:**

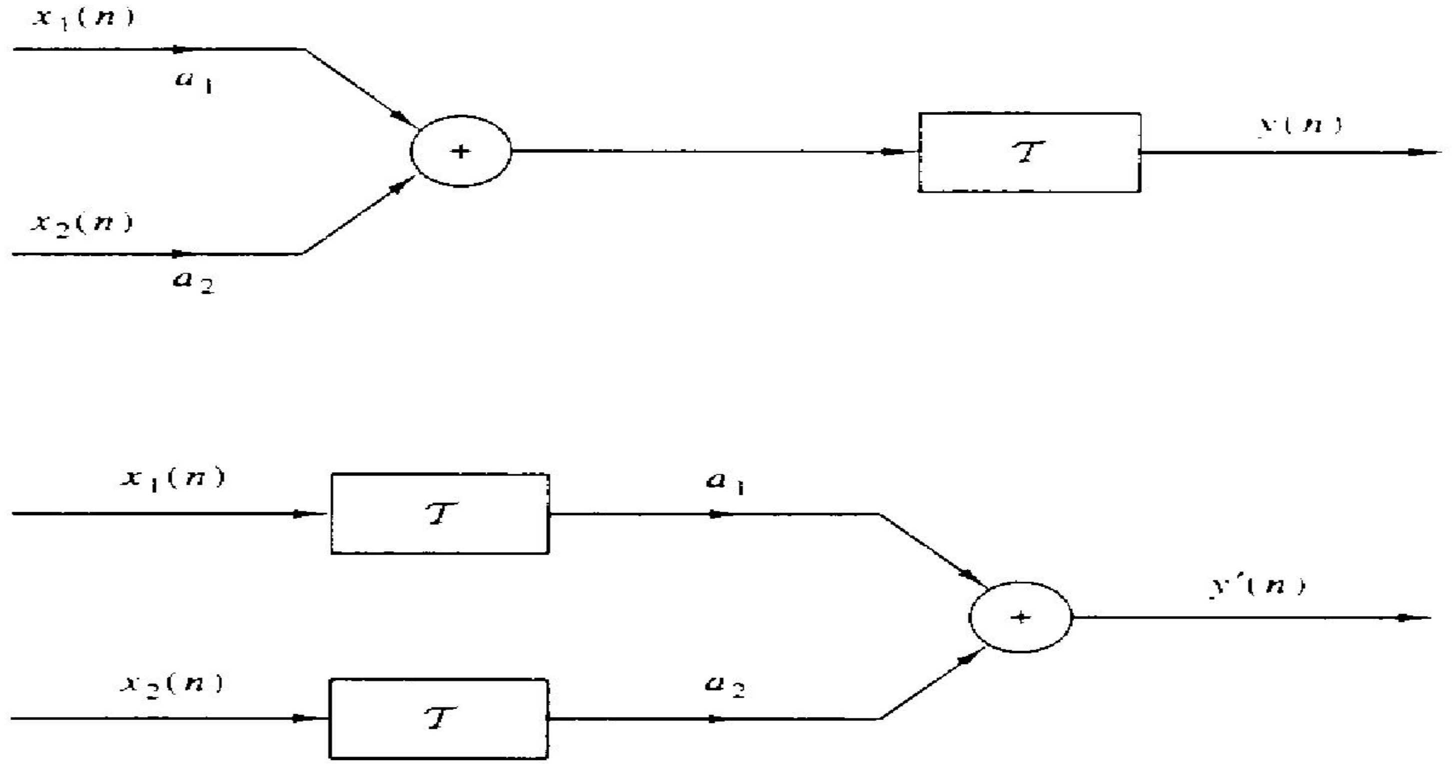


Figure 2.20 Graphical representation of the superposition principle. $\mathcal{T}$ is linear if and only if $y(n) = y'(n)$.

Linear Vs. Non Linear Systems

- **Linear Systems-3:**

Linear System properties:

A. ***Multiplicative or scaling property:***

$$\mathcal{T}[a_1 x_1(n)] = a_1 \mathcal{T}[x_1(n)] = a_1 y_1(n)$$

Where

$$y_1(n) = \mathcal{T}[x_1(n)]$$

If the response of the system to the input $x_1(n)$ is $y_1(n)$ then the response to $a_1x_1(n)$ is $a_1y_1(n)$ Any scaling of input results is an identical scaling of corresponding output

Linear Vs. Non Linear Systems

- **Linear Systems-3:**

Linear System properties:

B. ***Additivity property:***

Suppose a_1=a_2=1

$$\mathcal{T}[x_1(n) + x_2(n)] = \mathcal{T}[x_1(n)] + \mathcal{T}[x_1(n)]$$
$$= y_1(n) + y_2(n)$$

Example

Determine if the systems described by the following input–output equations are linear or nonlinear.

(a) $y(n) = nx(n)$ **(b)** $y(n) = x(n^2)$ **(c)** $y(n) = x^2(n)$
(d) $y(n) = Ax(n) + B$ **(e)** $y(n) = e^{x(n)}$

Solution

(a) For two input sequences $x_1(n)$ and $x_2(n)$, the corresponding outputs are

$$\begin{aligned} y_1(n) &= nx_1(n) \\ y_2(n) &= nx_2(n) \end{aligned} \tag{2.2.31}$$

A linear combination of the two input sequences results in the output

$$\begin{aligned} y_3(n) &= \mathcal{T}[a_1x_1(n) + a_2x_2(n)] = n[a_1x_1(n) + a_2x_2(n)] \\ &= a_1nx_1(n) + a_2nx_2(n) \end{aligned} \tag{2.2.32}$$

On the other hand, a linear combination of the two outputs in (2.2.31) results in the output

$$a_1y_1(n) + a_2y_2(n) = a_1nx_1(n) + a_2nx_2(n) \tag{2.2.33}$$

Since the right-hand sides of (2.2.32) and (2.2.33) are identical, the system is linear.

Solution Continued

c)

The responses of the system to two separate input signals are

$$\begin{aligned} y_1(n) &= x_1^2(n) \\ y_2(n) &= x_2^2(n) \end{aligned} \tag{2.2.37}$$

The response of the system to a linear combination of these two input signals is

$$\begin{aligned} y_3(n) &= \mathcal{T}[a_1x_1(n) + a_2x_2(n)] \\ &= [a_1x_1(n) + a_2x_2(n)]^2 \\ &= a_1^2x_1^2(n) + 2a_1a_2x_1(n)x_2(n) + a_2^2x_2^2(n) \end{aligned} \tag{2.2.38}$$

On the other hand, if the system is linear, it would produce a linear combination of the two outputs in (2.2.37), namely,

$$a_1y_1(n) + a_2y_2(n) = a_1x_1^2(n) + a_2x_2^2(n) \tag{2.2.39}$$

Since the actual output of the system, as given by (2.2.38), is not equal to (2.2.39), the system is nonlinear.

Solution continued

(d) Assuming that the system is excited by $x_1(n)$ and $x_2(n)$ separately, we obtain the corresponding outputs

$$\begin{aligned} y_1(n) &= Ax_1(n) + B \\ y_2(n) &= Ax_2(n) + B \end{aligned} \tag{2.2.40}$$

A linear combination of $x_1(n)$ and $x_2(n)$ produces the output

$$\begin{aligned} y_3(n) &= T[a_1x_1(n) + a_2x_2(n)] \\ &= A[a_1x_1(n) + a_2x_2(n)] + B \\ &= Aa_1x_1(n) + a_2Ax_2(n) + B \end{aligned} \tag{2.2.41}$$

On the other hand, if the system were linear, its output to the linear combination of $x_1(n)$ and $x_2(n)$ would be a linear combination of $y_1(n)$ and $y_2(n)$, that is,

$$a_1y_1(n) + a_2y_2(n) = a_1Ax_1(n) + a_1B + a_2Ax_2(n) + a_2B \tag{2.2.42}$$

Clearly, (2.2.41) and (2.2.42) are different and hence the system fails to satisfy the linearity test.

Causal Vs. Non Causal Systems

- **Casual Systems:**

  A system is called causal if the output [y(n)] of the system at any time n depends only on the present and past inputs ex[x(n-1),x(n-2),….]

  But does not depend on future inputs ex[x(n+1),x(n+2),….]

$$y(n) = F[x(n), x(n-1), x(n-2), \ldots]$$

  Where F[.] is any arbitrary function

  Any system that don’t satisfy the definition is non causal system

Stable Vs. non stable systems

- **Stable Systems:**

  An arbitrary relaxed system is said to be bounded input-bounded output (BIBO) stable if and only if every bounded input produces bounded output

$$|x(n)| \le M_x < \infty \qquad |y(n)| \le M_y < \infty$$

Otherwise system is unstable

EXample

A discrete-time system can be

(1) Static or dynamic
(2) Linear or nonlinear
(3) Time invariant or time varying
(4) Causal or noncausal
(5) Stable or unstable

Examine the following systems with respect to the properties above.

(a) $y(n) = \cos[x(n)]$

(b) $y(n) = \sum_{k=-\infty}^{n+1} x(k)$

(c) $y(n) = x(n)\cos(\omega_0 n)$

(d) $y(n) = x(-n+2)$

Solution

(a) Static, nonlinear, time invariant, causal, stable.

(b) Dynamic, linear, time invariant, noncausal and unstable. The latter is easily proved. For the bounded input $x(k) = u(k)$, the output becomes

$$y(n) = \sum_{k=-\infty}^{n+1} u(k) = \begin{cases} 0, & n < -1 \\ n+2, & n \geq -1 \end{cases}$$

since $y(n) \to \infty$ as $n \to \infty$, the system is unstable.

(c) Static, linear, timevariant, causal, stable.

(d) Dynamic, linear, time invariant, noncausal, stable.