Digital Signal Processing

Design of IIR filter (part2)

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Step1: performance specifications

The frequency response specifications are often in the form of the tolerance scheme.

ε^2	passband ripple parameter
δ_p	passband deviation
δ_s	stopband deviation
f_{p1} and f_{p2}	passband edge frequencies
f_{s1} and f_{s2}	stopband edge frequencies

The passband attenuation in decibels is:

$$A_p = 10\log_{10}(1+\varepsilon^2) = -20\log_{10}(1-\delta_p)$$

And The Stopband attenuation in decibels is:

$$A_s = -20\log_{10}(\delta_s)$$

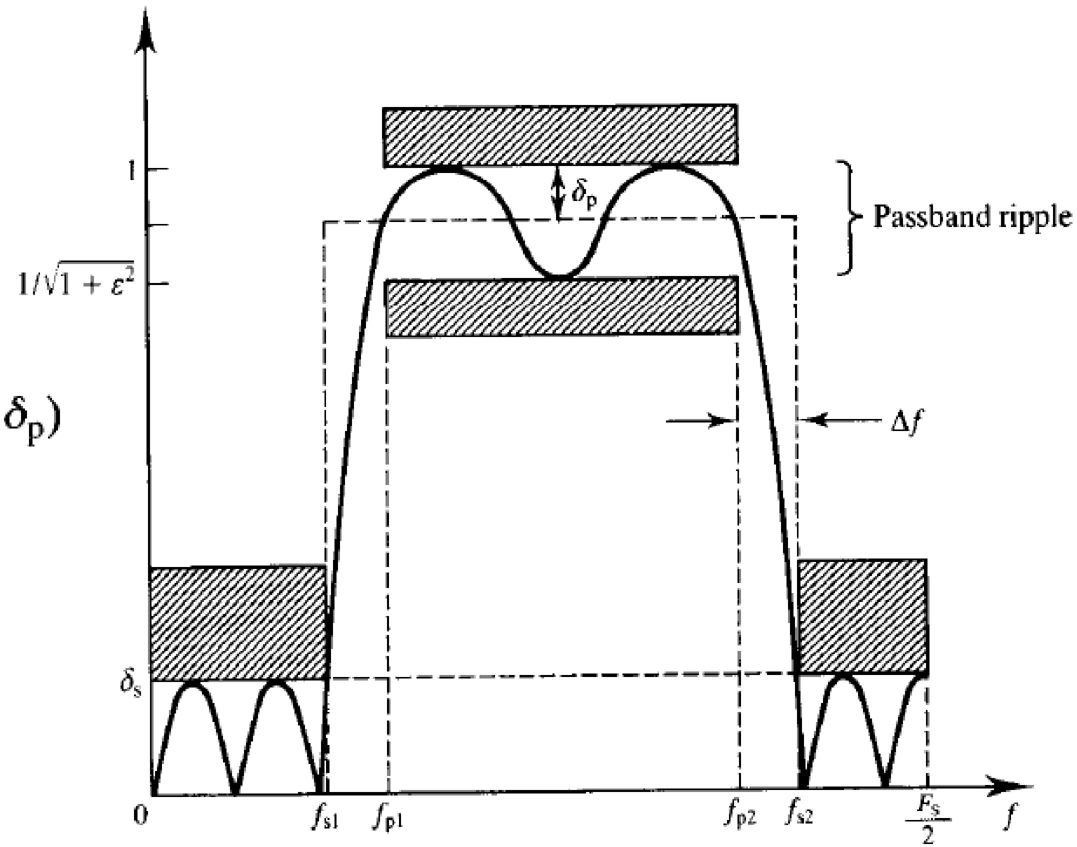

Figure 7.2 Tolerance scheme for an IIR bandpass filter.

Step2: Calculating the filter coefficient filter Using the BZT method

Summary of the procedure for calculating digital filter coefficients by the BZT method

(1) Use the digital filter specifications to determine a suitable normalized transfer function, $H(s)$.

(2) Determine the cutoff frequency (or passband edge frequency) of the digital filter and call this ω_p.

(3) Obtain an equivalent analogue filter cutoff frequency (ω'_p) using the relation (prewarped)

$$\omega'_p = k \tan\left(\frac{\omega_p T}{2}\right), \quad k = 1 \text{ or } \frac{T}{2}$$

(4) Denormalize the analogue filter by frequency scaling $H(s)$. This is achieved by replacing s with s/ω'_p.

(5) Apply the bilinear transformation to obtain the desired digital filter transfer function $H(z)$ by replacing s by $(z-1)/(z+1)$.

For computational efficiency we can combine step 4&5 as follows:

$$S = \frac{K(z-1)/(Z+1)}{K\tan(\frac{WpT}{2})} \longrightarrow \quad s = \cot\left(\frac{\omega_p T}{2}\right)\frac{z-1}{z+1}$$

Classical Analogue filters

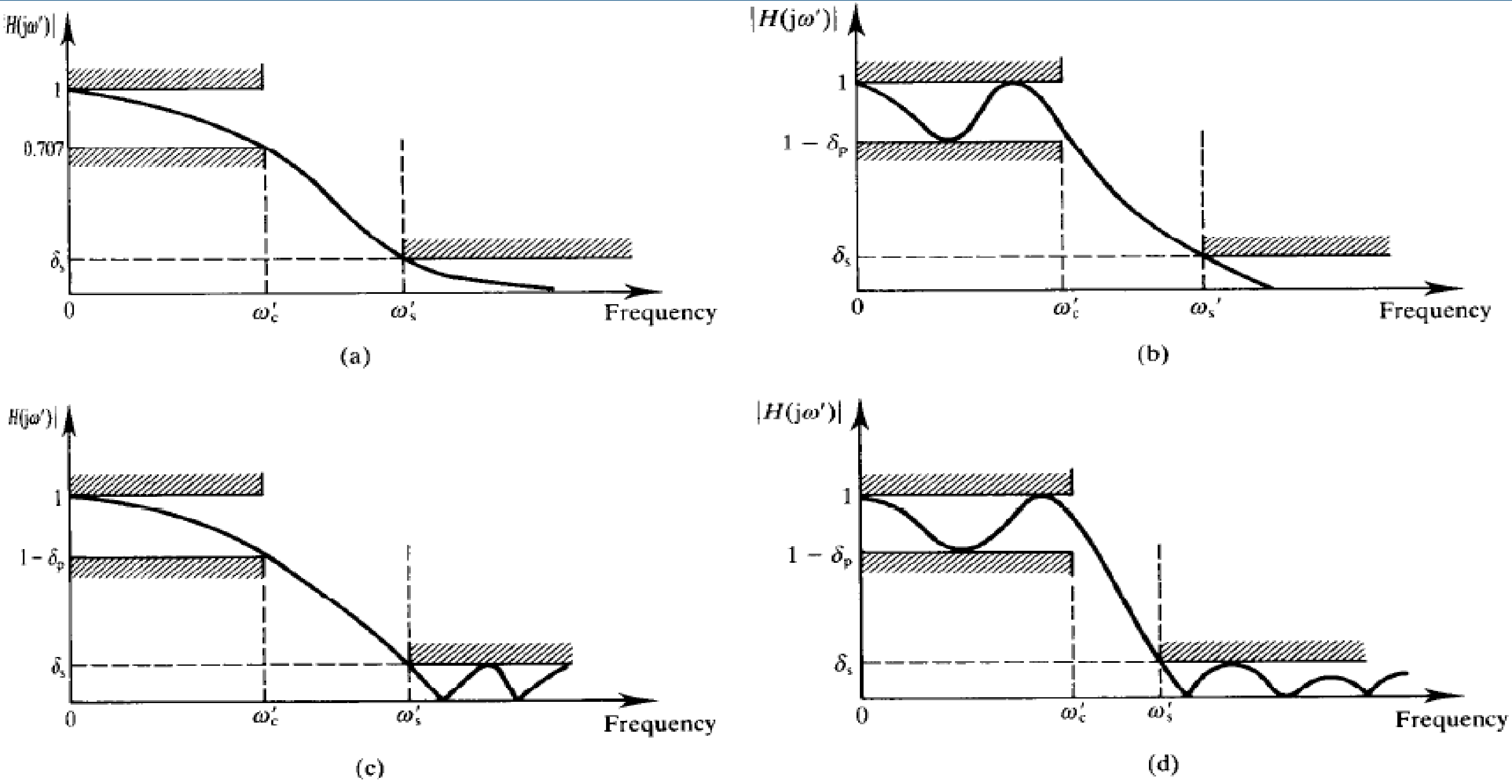

Figure 7.12 Sketches of frequency responses of some classical analogue filters: (a) Butterworth response; (b) Chebyshev type I; (c) Chebyshev type II; (d) elliptic.

Tables of the polynomials of $H(s)$ for the Butterworth, Chebyshev and elliptic characteristics are available in most analogue design books in normalized form

Butterworth filter

Law pass butterworth filter is characterized by:

$$|H(\omega')|^2 = \frac{1}{1 + (\omega'/\omega'_p)^{2N}}$$

where N is the order of the filter and ω'_p is the 3 dB cutoff frequency.

the filter order is

$$N \geqslant \frac{\log_{10}[(1/\delta_s) - 1]}{2\log_{10}(\omega'_s/\omega'_p)}$$

Butterworth filter transfer fn H(s)

n	Factors of Polynomial $B_n(s)$
1	$(s+1)$
2	$(s^2+1.4142s+1)$
3	$(s+1)(s^2+s+1)$
4	$(s^2+0.7654s+1)(s^2+1.8478s+1)$
5	$(s+1)(s^2+0.6180s+1)(s^2+1.6180s+1)$
6	$(s^2+0.5176s+1)(s^2+1.4142s+1)(s^2+1.9319s+1)$
7	$(s+1)(s^2+0.4450s+1)(s^2+1.2470s+1)(s^2+1.8019s+1)$
8	$(s^2+0.3902s+1)(s^2+1.1111s+1)(s^2+1.6629s+1)(s^2+1.9616s+1)$
9	$(s+1)(s^2+0.3473s+1)(s^2+s+1)(s^2+1.5321s+1)(s^2+1.879s+1)$
10	$(s^2+0.3129s+1)(s^2+0.9080s+1)(s^2+1.4142s+1)(s^2+1.7820s+1)(s^2+1.9754s+1)$

The normalized Butterworth polynomials can be used to determine the transfer function for any low-pass filter cut-off frequency ω_c, as follows

$$H(s) = \frac{G_0}{B_n(a)}, \text{ where } a = \frac{s}{\omega_c}.$$

G0 is the DC gain (gain at zero frequency)

Butterworth example 2

Obtain the transfer function of a lowpass digital filter meeting the following specifications:

passband	0–60 Hz
stopband	$>$ 85 Hz
stopband attenuation	$>$ 15 dB

Assume a sampling frequency of 256 Hz and a Butterworth characteristic.

Solution

(1) The critical frequencies for the digital filter are

$$W_P T = \frac{2\pi f_1}{F_s} = \frac{2\pi 60}{256} = 2\pi \times 0.2344$$

$$W_s T = \frac{2\pi f_2}{F_s} = \frac{2\pi 85}{256} = 2\pi \times 0.3320$$

(2) The prewarped equivalent analogue frequencies are:

$$W'_P = \tan\left(\frac{W_P T}{2}\right) = 0.906\,347 \quad W'_s = \tan\left(\frac{W_s T}{2}\right) = 1.715\,80$$

Butterworth example 2

(3) Next we need to obtain $H(s)$ with Butterworth characteristics, a 3 dB cutoff frequency of 0.906 347, and a response at 85 Hz that is down by 15 dB. For an attenuation of 15 dB, $\delta_s = 0.1778$ and so from Equation 7.16b $N = 2.468$. We use $N = 3$, since it must be an integer. A normalized third-order filter is given by

$$A_s = -20\log_{10}(\delta_s) = 15 \longrightarrow \delta_s = 0.1778$$

$$N \geqslant \frac{\log_{10}[(1/\delta_s) - 1]}{2\log_{10}(\omega'_s/\omega'_p)} \longrightarrow N = 2.468. \text{ We use } N = 3$$

$$H(s) = \frac{1}{(s+1)(s^2+s+1)} = \frac{1}{s+1}\,\frac{1}{s^2+s+1}$$

$$= H_1(s)\,H_2(s)$$

Then continue using bilinear z transform: we can combine step 4&5 in one equation using $s = \cot\left(\frac{\omega_p T}{2}\right)\frac{z-1}{z+1}$

Butterworth example 2

$$\cot\left(\frac{\omega_p T}{2}\right) = \cot\left(\frac{2\pi \times 0.2344}{2}\right) = 1.103\,155$$

Performing the transform in two stages, one for each of the factors of $H(s)$ above, we obtain

$$H_2(z) = H_2(s)|_s \quad = \cot(w_pT/2)[(z-1)/(z+1)]$$

$$= 0.3012 \frac{1 + 2z^{-1} + z^{-2}}{1 - 0.1307z^{-1} + 0.3355z^{-2}}$$

which we have arrived at after considerable manipulation. Similarly, we obtain $H_1(z)$ as

$$H_1(z) = 0.4754 \frac{1 + z^{-1}}{1 - 0.0490z^{-1}}$$

$H_1(z)$ and $H_2(z)$ may then be combined to give the desired transfer function, $H(z)$:

$$H(z) = H_1(z)H_2(z) = 0.1432 \frac{1 + 3z^{-1} + 3z^{-2} + z^{-3}}{1 - 0.1801z^{-1} + 0.3419z^{-2} - 0.0165z^{-3}}$$

Designing highpass, Bandpass and bandStop filters

The steps involved in the first method are outlined below.

(1) Use the digital filter specifications to determine a suitable normalized lowpass filter, $H(s)$.

(2) Determine and prewarp the critical frequencies of the digital filter. For lowpass or highpass filters there is just one critical frequency – the band edge or cutoff frequency, ω'_p; for bandpass or bandstop filters, we have the lower and upper band edge frequencies, ω'_1 and ω'_2.

(3) Replace s in the transfer function, $H(s)$, using one of the following transformations, depending on the type of filter required:

$$s = \frac{s}{\omega'_p} \qquad \text{lowpass to lowpass} \tag{7.22a}$$

$$s = \frac{\omega'_p}{s} \qquad \text{lowpass to highpass} \tag{7.22b}$$

$$s = \frac{s^2 + \omega_0^2}{Ws} \qquad \text{lowpass to bandpass} \tag{7.22c}$$

$$s = \frac{Ws}{s^2 + \omega_0^2} \qquad \text{lowpass to bandstop} \tag{7.22d}$$

where $\omega_0^2 = \omega'_1\omega'_2$, $W = \omega'_2 - \omega'_1$.

(4) Apply the BZT to the new $H(s)$:

$$s = \frac{z-1}{z+1}$$

Designing highpass Example 3:

Highpass filter design Convert the simple lowpass filter in Example 1 into an equivalent highpass discrete filter. The s-plane transfer function is given by

$$H(s) = \frac{1}{s+1}$$

Solution The critical frequency for the digital filter is $\omega_p = 2\pi \times 30$ rad.

$$\omega_p' = \tan(\omega_p T/2) = 0.7265$$

Using the LPF-to-HPF transformation of Equation 7.22b, the denormalized analogue transfer function is obtained as

$$H'(s) = H(s)|_{s = \omega_p'/s} = \frac{1}{\omega_p'/s + 1} = \frac{s}{s + 0.7265}$$

The z-plane transfer function is obtained by applying the BZT:

$$H(z) = H'(s)|_{s = (z-1)/(z+1)} = \frac{(z-1)/(z+1)}{(z-1)/(z+1) + 0.7265}$$

Designing highpass example

Simplifying, we have

$$H(z) = 0.5792 \frac{1 - z^{-1}}{1 + 0.1584 z^{-1}}$$

The coefficients of the digital filter are

$$a_0 = \quad 0.5792 \qquad\qquad b_1 = 0.1584$$

$$a_1 = -0.5792$$

Bandpass filter design Example 4

Bandpass filter design A discrete bandpass filter with Butterworth characteristics meeting the following specifications is required. Obtain the coefficients of its transfer function, $H(z)$.

passband	200–300 Hz
sampling frequency	2000 Hz
filter order	2

Solution

The prewarped passband edge frequencies are given by

$$\omega_1' = \tan\left(\frac{\omega_1 T}{2}\right) = \tan(200\pi/2000) = 0.3249$$

$$\omega_2' = \tan\left(\frac{\omega_2 T}{2}\right) = \tan(300\pi/2000) = 0.5095$$

Thus $\omega_0^2 = 0.1655$ and $W = \omega_2' - \omega_1' = 0.1846$. A first-order normalized analogue lowpass filter is required (half the order of the bandpass filter). Thus we have

$$H(s) = \frac{1}{s+1}$$

Using the lowpass-to-bandpass transformation (Equation 7.22c) we have

$$H'(s) = H(s)|_{s=(s^2+\omega_0^2)/Ws} = \frac{1}{(s^2+\omega_0^2)/Ws + 1}$$

$$= \frac{Ws}{s^2 + Ws + \omega_0^2}$$

Bandpass filter design Example 4

Applying the BZT to the analogue bandpass filter we have

$$H(z) = H'(s)\big|_{s=(z-1)/(z+1)} = \frac{W(z-1)/(z+1)}{[(z-1)/(z+1)]^2 + W(z-1)/(z+1) + \omega_0^2}$$

$$= \frac{W(z^2-1)/(1+W+\omega_0^2)}{z^2 + [2(\omega_0^2-1)/(1+W+\omega_0^2)]z + (1-W+\omega_0^2)/(1+W+\omega_0^2)}$$

Substituting the values of ω_0^2 and W and simplifying we have

$$H(z) = 0.1367\,\frac{1-z^2}{1-1.2362z^{-1}+0.7265z^{-2}}$$

Thanks for good listening

Best wishes

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