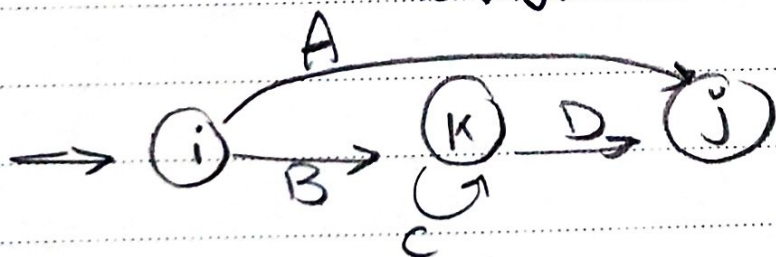
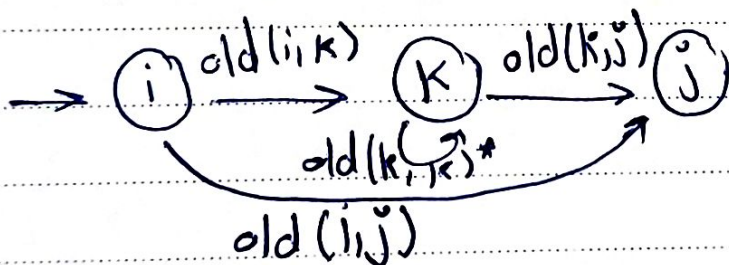


Theory sec (4) - General

→ Convert NFA to Regular Expression:

- ① Connect a new start state (S) to the start states, and connect each final to the FA to a new final state (F).
- ② if needed, Combine all multiple edges between same node into one edge with label sum of the labels.
- ③ eliminate each state K of the FA by constructing a new edge (i, j)

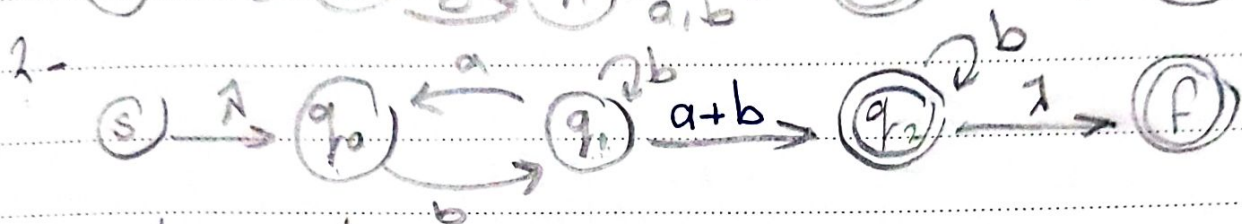
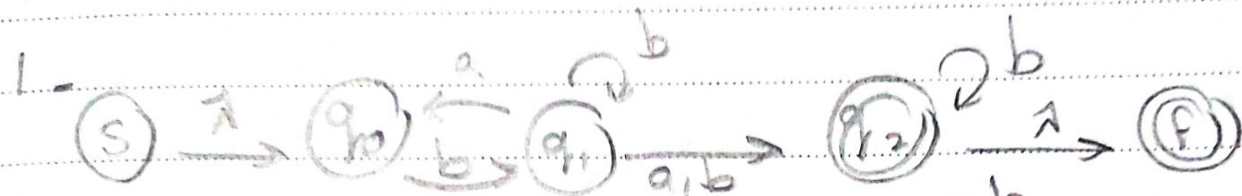
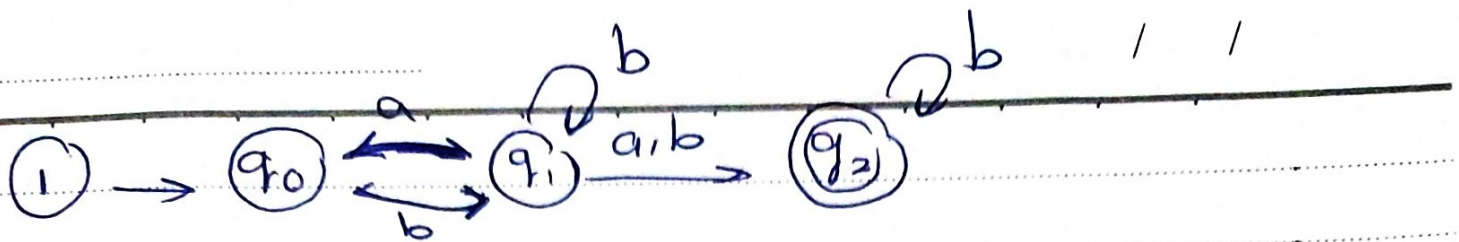
$$\text{new}(i, j) = \text{old}(i, j) + \text{old}(i, k) \text{old}(k, k)^* \text{old}(k, j)$$



$$\Rightarrow \text{new}(i, j) = \text{old}(i, j) + \text{old}(i, k) \text{old}(k, k)^* \text{old}(k, j)$$

$$= A + B C^* D$$

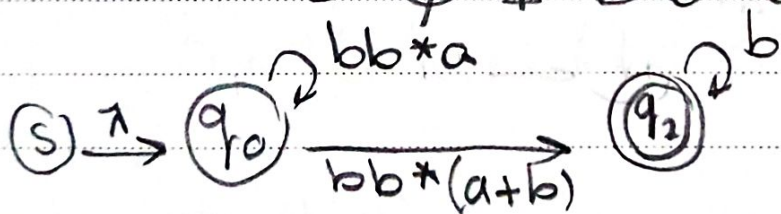




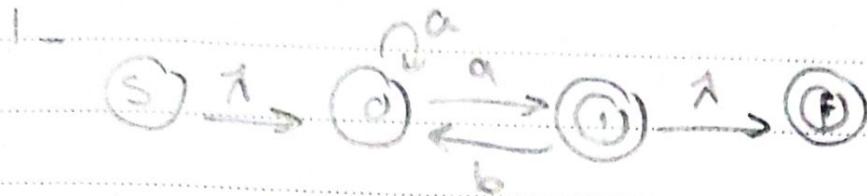
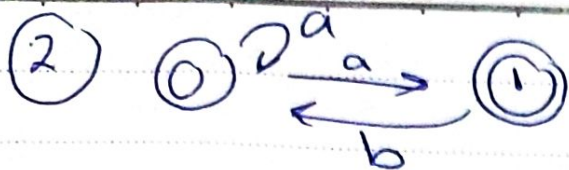
3 - eliminate (q_1)

$$\begin{aligned} \text{new}(q_0, q_2) &= \text{old}(q_0, q_2) + \text{old}(q_0, q_1) \text{old}(q_1, q_1)^* \text{old}(q_1, q_2) \\ &= \emptyset + b \cdot b^* (a+b) \end{aligned}$$

$$\begin{aligned} \text{new}(q_0, q_0) &= \text{old}(q_0, q_0) + \text{old}(q_0, q_1) \text{old}(q_1, q_1)^* \text{old}(q_1, q_0) \\ &= \emptyset + b b^* a \end{aligned}$$



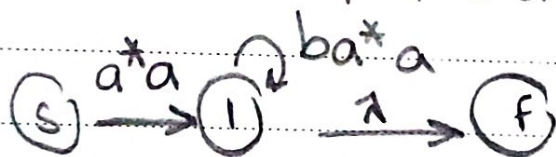
$$\Rightarrow RE = (bb^*a)^* bb^*(a+b) b^*$$



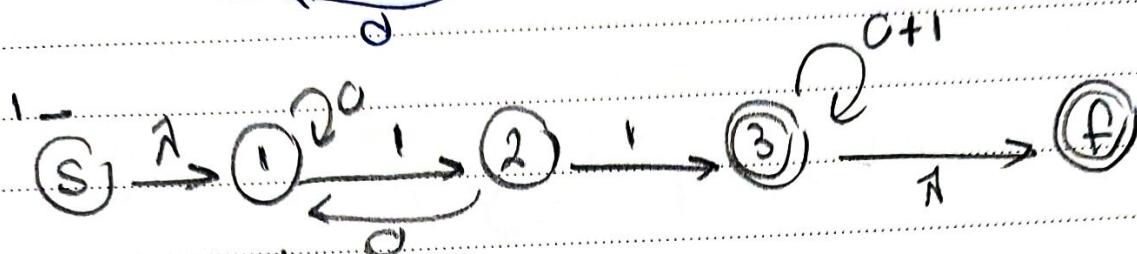
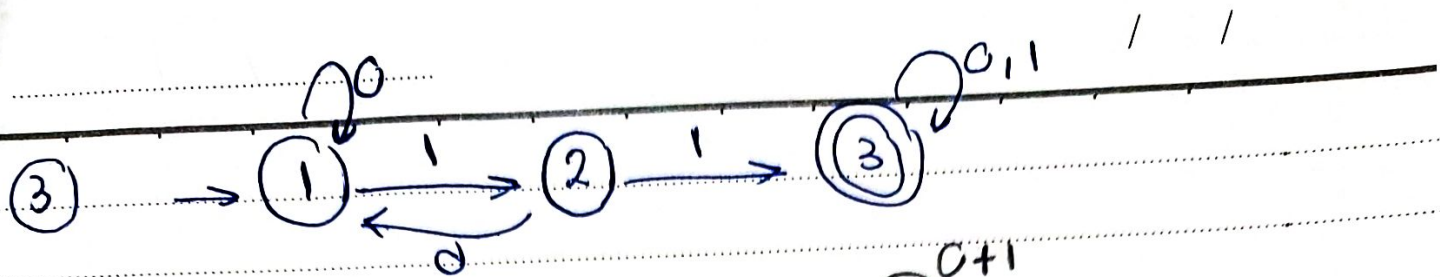
2. eliminate (0)

$$\begin{aligned} \text{new}(S,1) &= \text{old}(S,1) + \text{old}(S,0) \text{old}(0,0)^* \text{old}(0,1) \\ &= \phi + 1 a^* a = a^* a \end{aligned}$$

$$\begin{aligned} \text{new}(1,1) &= \text{old}(1,1) + \text{old}(1,0) \text{old}(0,0)^* \text{old}(0,1) \\ &= \phi + b a^* a \end{aligned}$$



$$RE = (a^*a) \cdot (ba^*a)^*$$



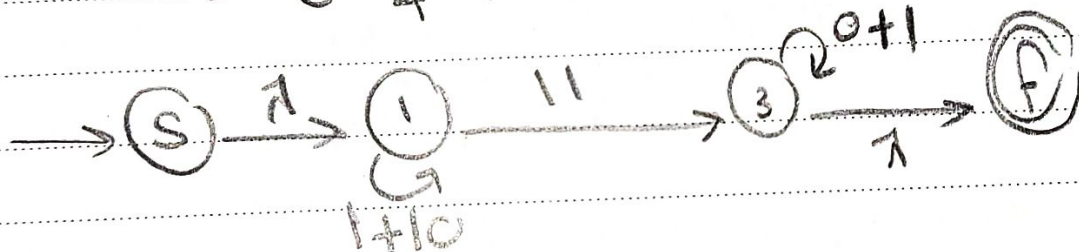
2. Eliminate 2:

$$N(1, 3) = o(1, 3) + o(1, 2) o(2, 2)^* o(2, 3)$$

$$= \phi + 1 \cdot 1 = 1$$

$$N(1, 1) = o(1, 1) + o(1, 2) o(2, 2)^* o(2, 1)$$

$$= 0 + 1 \cdot 1 \cdot 0 = 1 + 0$$



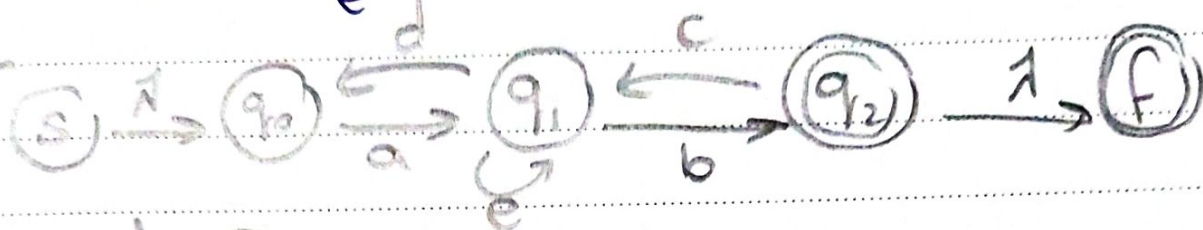
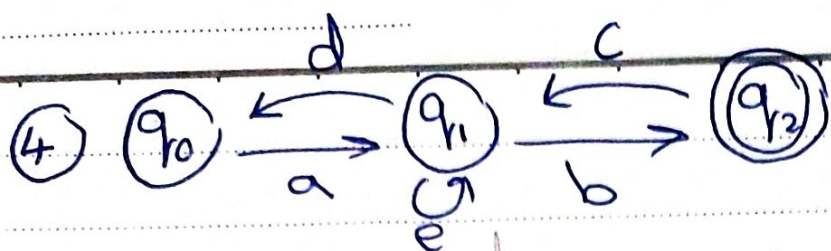
Eliminate 1:

$$N(S, 3) = o(S, 3) + o(S, 1) o(1, 1)^* o(1, 3)$$

$$= \phi + 1 (1+0)^* 1$$



$$RE = (1+0)^* 1 (0+1)^*$$



Eliminate q_1 :

$$N(q_0, q_2) = o(q_0, q_2) + o(q_0, q_1) o(q_1, q_1)^* o(q_1, q_2)$$

$$= \phi + a e^* b = a e^* b$$

$$N(q_0, q_0) = o(q_0, q_0) + o(q_0, q_1) o(q_1, q_1)^* o(q_1, q_0)$$

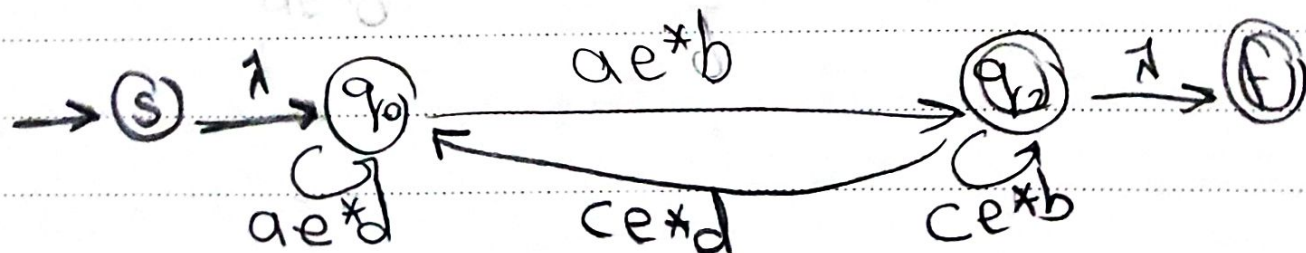
$$= \phi + a e^* d = a e^* d$$

$$N(q_2, q_2) = o(q_2, q_2) + o(q_2, q_1) o(q_1, q_1)^* o(q_1, q_2)$$

$$= \phi + c e^* b = c e^* b$$

$$N(q_2, q_0) = o(q_2, q_0) + o(q_2, q_1) o(q_1, q_1)^* o(q_1, q_0)$$

$$= \phi + c e^* d = c e^* d$$



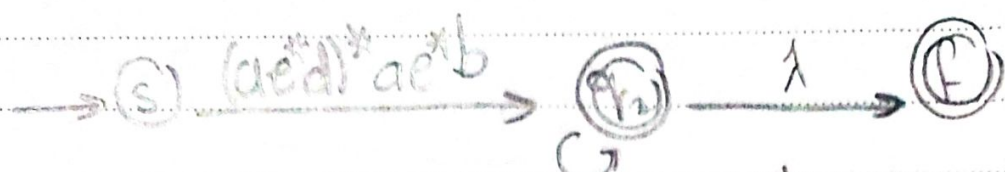
Eliminate q_0 :

$$N(S, q_2) = o(S, q_2) + o(S, q_0) o(q_0, q_0)^* o(q_0, q_2)$$

$$= \phi + \lambda (a e^* d)^* a e^* b$$

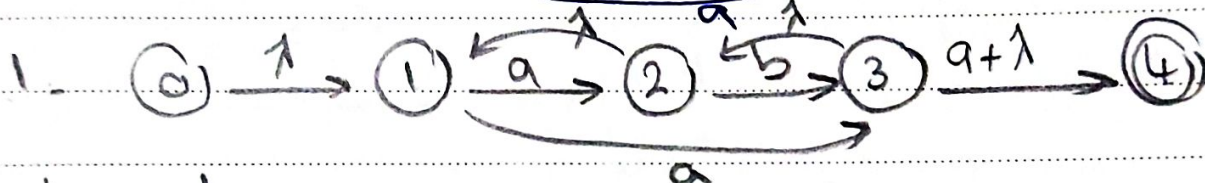
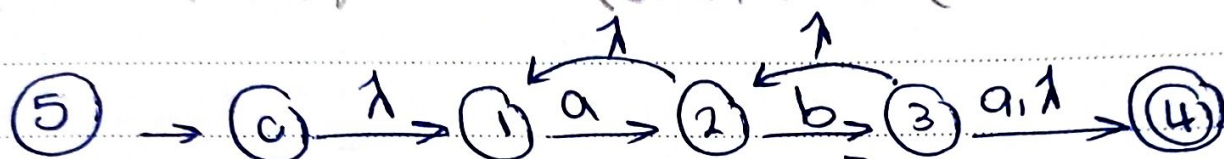
$$N(q_2, q_2) = o(q_2, q_2) + o(q_2, q_0) o(q_0, q_0)^* o(q_0, q_2)$$

$$= c e^* b + c e^* d \cdot a e^* d \cdot a e^* b$$



$$ce^*b + ce^*d, (ae^*d)^*, (ae^*b)$$

$$RE = (ae^*d)^*ae^*b \cdot (ce^*b + ce^*d \cdot (ae^*d)^* \cdot (ae^*b))^*$$



Eliminate 2,

$$N(1,3) = o(1,3) + o(1,2) o(2,2)^* o(2,3)$$

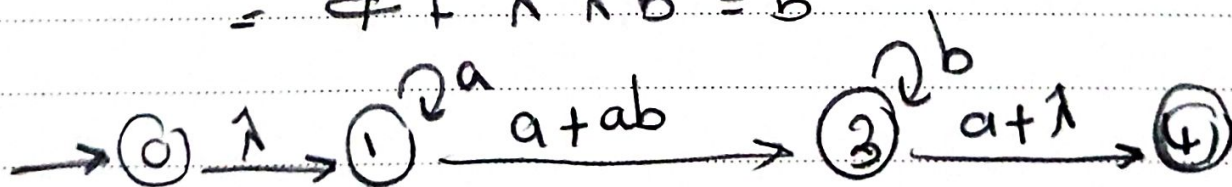
$$= a + a \lambda b = a + ab$$

$$N(1,1) = o(1,1) + o(1,2) o(2,2)^* o(2,1)$$

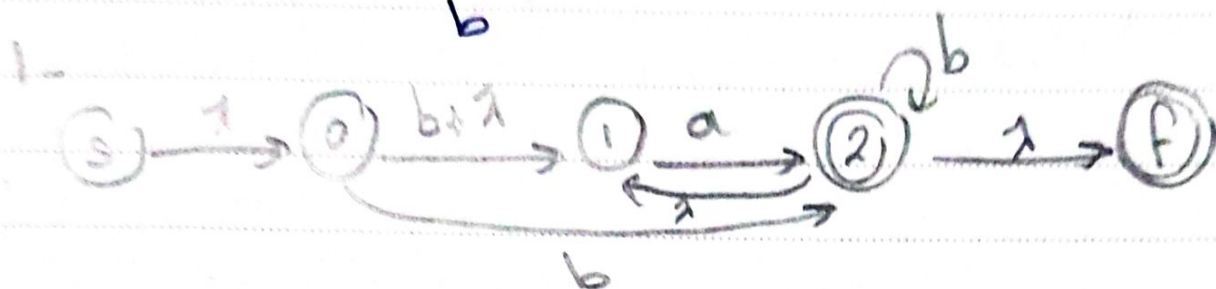
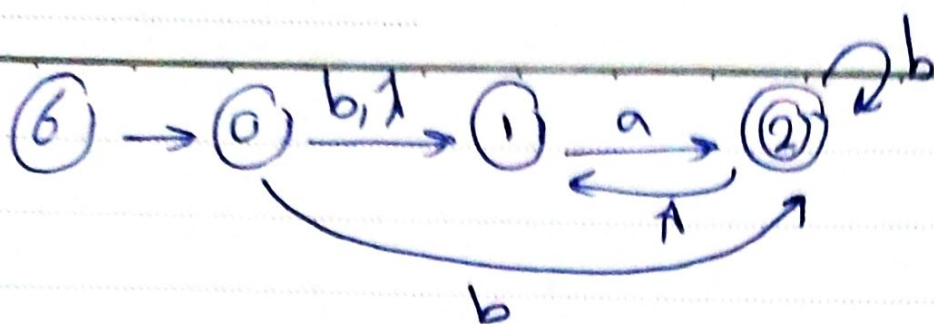
$$= \phi + a \lambda \lambda = a$$

$$N(3,3) = o(3,3) + o(3,2) o(2,2)^* o(2,3)$$

$$= \phi + \lambda \lambda b = b$$



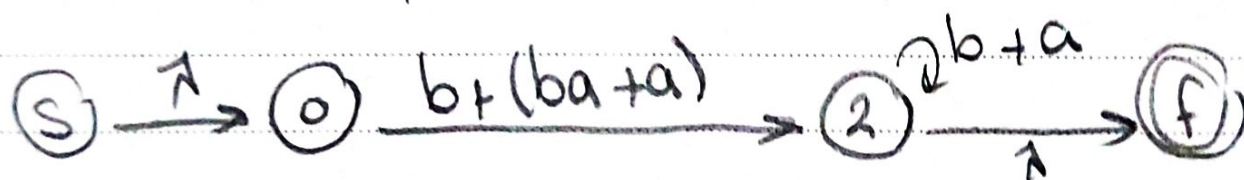
$$RE = a^*(a+ab) b^*(a+\lambda)$$



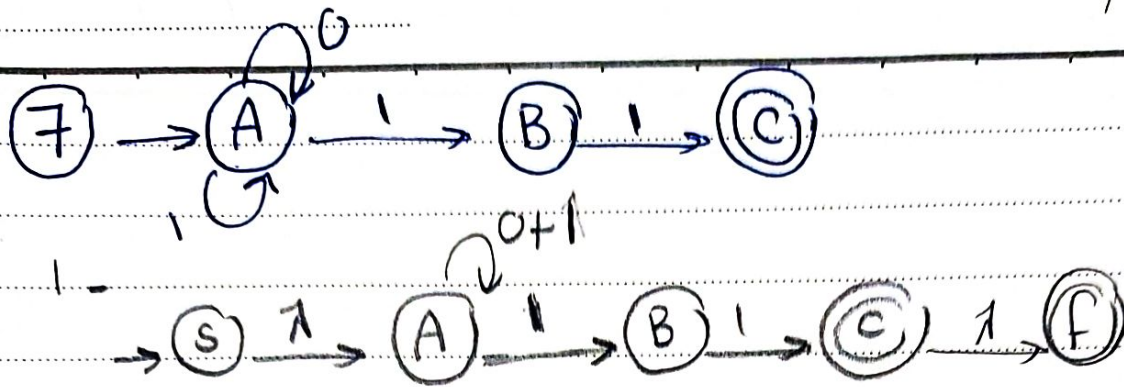
Eliminate 1:

$$\begin{aligned}
 N(0,2) &= o(0,2) + o(0,1) o(1,1)^* o(1,2) \\
 &= b + (b+1) 1 a = b + (b+1)a \\
 &= b + (ba + a)
 \end{aligned}$$

$$\begin{aligned}
 N(2,2) &= o(2,2) + o(2,1) o(1,1)^* o(1,2) \\
 &= b + 1 \phi^* a = b + a
 \end{aligned}$$



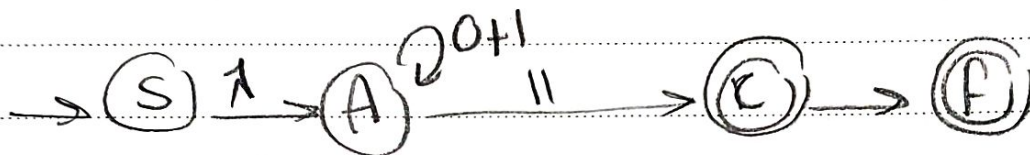
$$RE = (b + (ba + a)) (b + a)^*$$



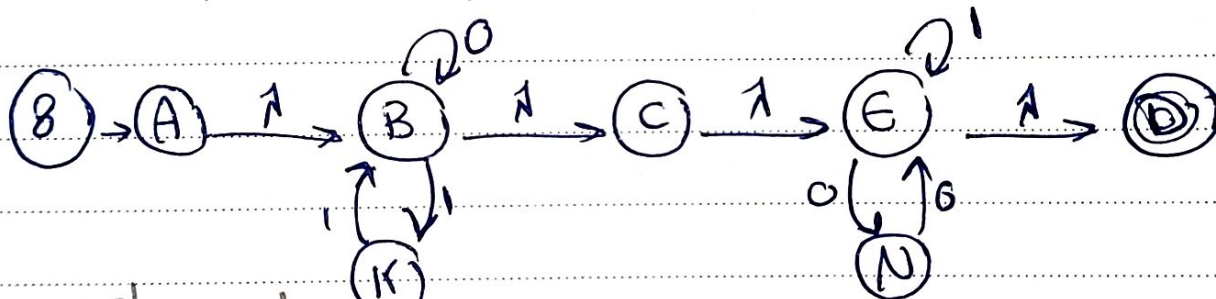
Eliminate B:

$$N(A, C) = o(A, C) + o(A, B) o(B, B)^* o(B, C)$$

$$= \emptyset + 1 \emptyset^* 1 = 11$$



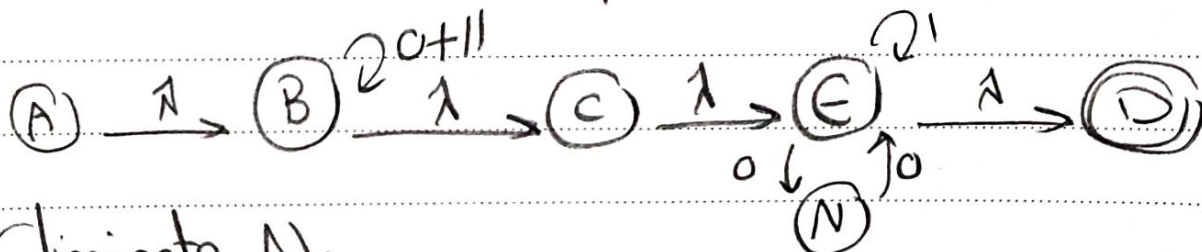
$$RE = (0+1)^* 11$$



Eliminate K:

$$N(B, B) = o(B, B) + o(B, K) o(K, K)^* o(K, B)$$

$$= 0 + 1 \emptyset^* 1 = 0 + 11$$



Eliminate N:

$$N(E, E) = o(E, E) + o(E, N) o(N, N)^* o(N, E)$$

$$= 1 + 00$$

$$RE = (0+11)^* (1+00)^*$$