Theory of Computation

Introduced By:

Prof. Safia Abbas

Text Books:
"***An Introduction to the theory of computation***" by Michael Sipser, 2nd Edition, PWS Publishing Company, Inc. 2006; ISBN: 0-534-95097-3

"***An Introduction to Formal Languages & Automata***" by Peter Linz, 5th Edition, Jones & Bartlett Publishers, Inc. January 2012; ISBN:978-1-4496-1552-9 .

Agenda

- Examples of STM (accepters)

Turing machines

- M is said to halt starting from some initial configuration $x_1 q_i x_2$ $x_1 q_i x_2 \overset{*}{\vdash} y_1 q_j a y_2$ wher $\delta(q_j, a)$ is defined.
- We said that $x_1 q_i x_2$ yields $y_1 q_j a y_2$ in one or more steps.
- The configuration of the machine M that never halts will be presented by

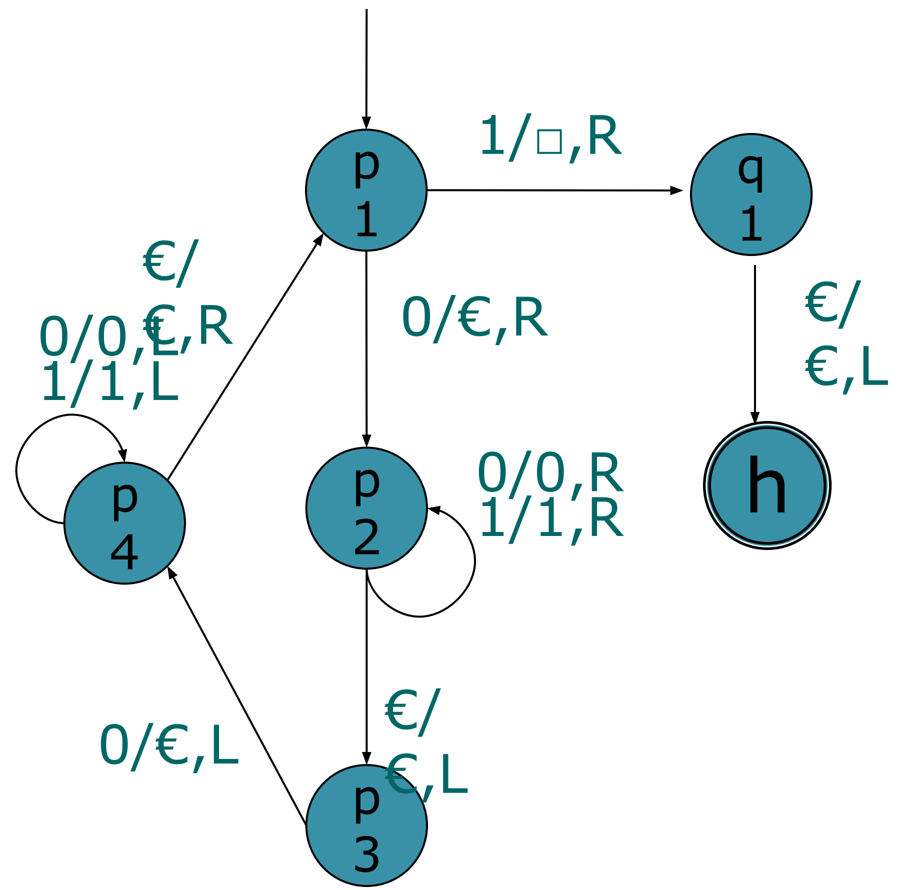
$$x_1 q x_2 \overset{*}{\vdash} \infty,$$

- The sequence of configuration leading to a halt state called a ***computation***

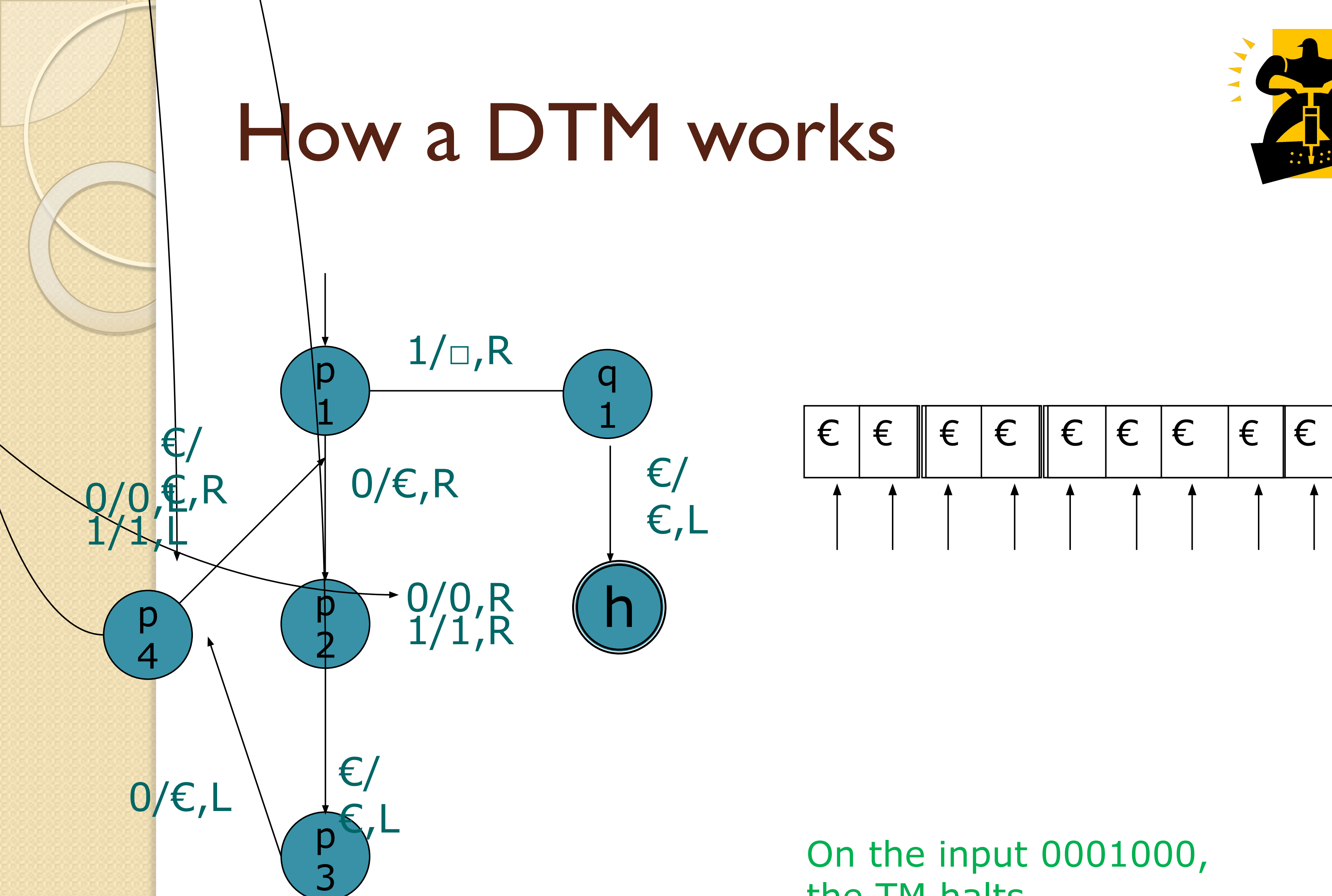
Example of a DTM

- $L(T)=\{0^n 1 0^n \mid n \geq 0\}$
- M=

({p1,p2,p3,p4,q1,h},
{0,1},{0,1}, δ, p1,h}

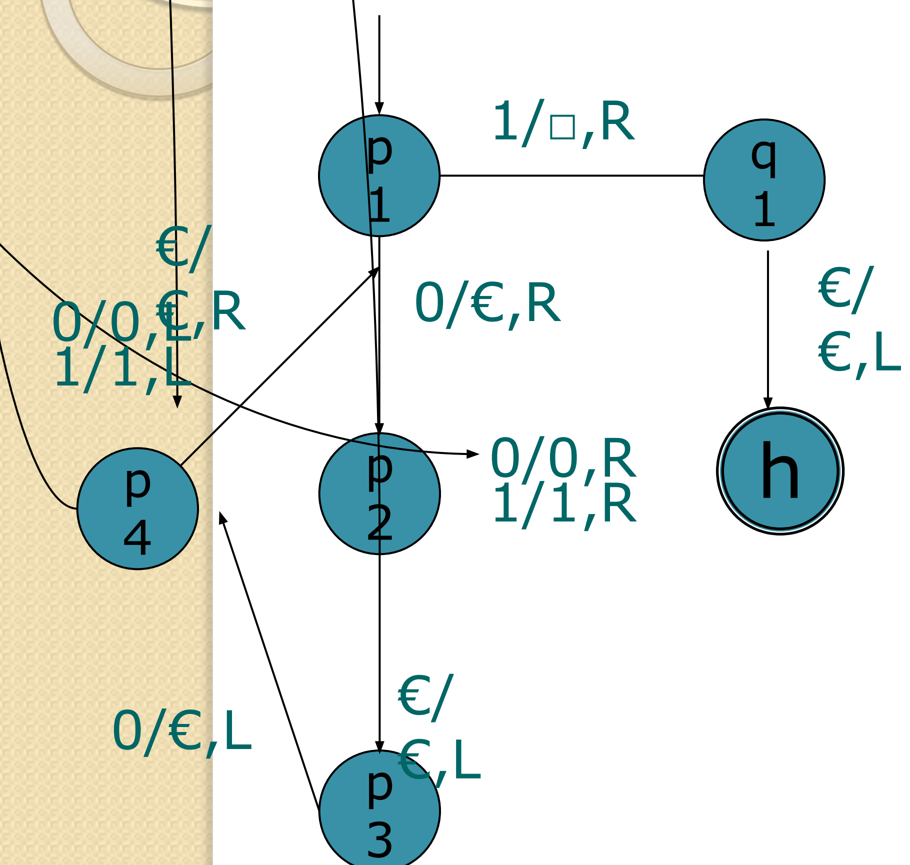


How a DTM works



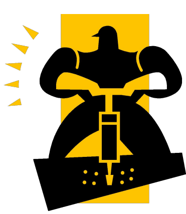
On the input 0001000, the TM halts.

Yield in zero step or more: Example



(p1,$\underline{0}$001000)
(p2,€$\underline{0}$01000)
(p2,€001000$\underline{€}$)
(p3,€00100$\underline{0}$)
(p4,€0010$\underline{0}$€)
(p4,$\underline{€}$00100€)
(p1,€$\underline{0}$0100€)
(p2,€€$\underline{0}$100€)
(p2,€€0100$\underline{€}$)
(p3,€€010$\underline{0}$)

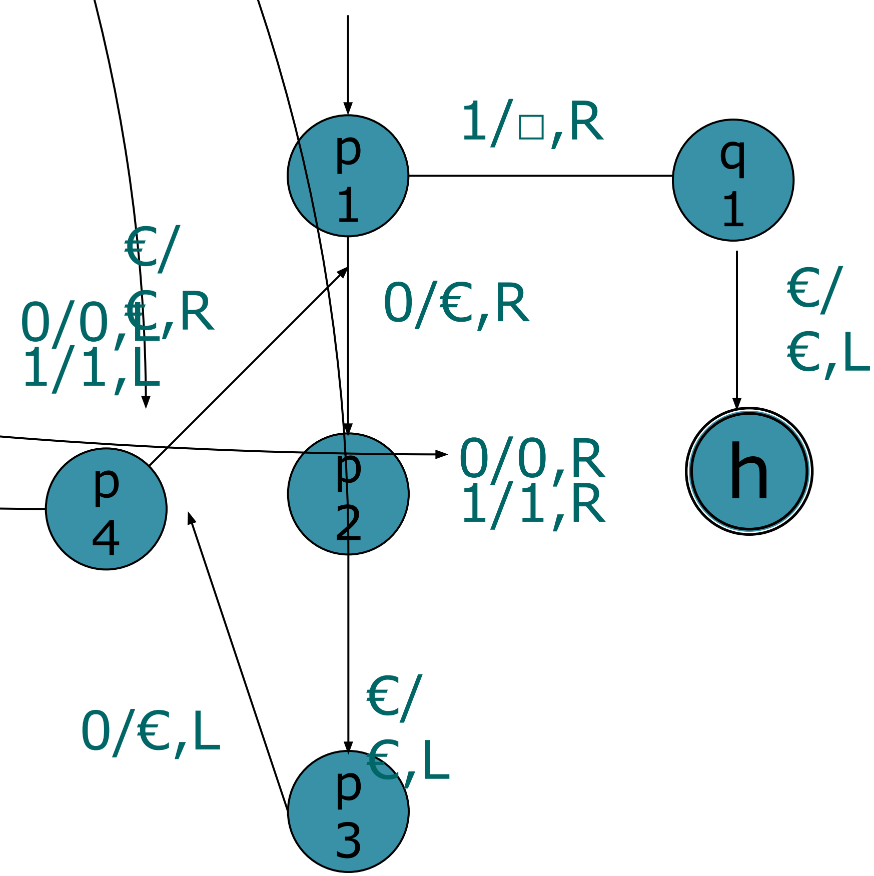
(p4,€€01$\underline{0}$)
(p4,€$\underline{€}$010)
(p1,€€$\underline{0}$10)
(p2,€€€$\underline{1}$0)
(p2,€€€1$\underline{0}$)
(p2,€€€10$\underline{€}$)
(p3,€€€1$\underline{0}$)
(p4,€€€$\underline{1}$)
(p4,€€$\underline{€}$1)
(p1,€€€$\underline{1}$)
(q1,€€€€$\underline{€}$)
(h ,$\underline{€}$)

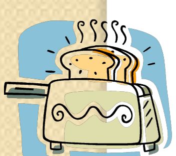


Example of language accepted by a TM

$L(T)=\{0^n 1 0^n \mid n \geq 0\}$

- T halts on $0^n 1 0^n$
- T hangs on $0^{n+1} 1 0^n$ at p3
- T hangs on $0^n 1 0^{n+1}$ at q1
- T hangs on $0^n\ 1^2\ 0^n$ at q1





Turing machine

- Design a Turing machine that accepts the language

$$L=\{a^n b^n : n>=0\}$$

$$Q=\{q_0,q_1,q_2,q_3,q_4\},$$
$$F=\{q_4\},$$
$$\Sigma=\{a,b\},$$
$$\Gamma=\{a,b,x,y,\square\}.$$

$$\delta(q_0,a)=(q_1,x,R),$$
$$\delta(q_1,a)=(q_1,a,R),$$
$$\delta(q_1,y)=(q_1,y,R),$$
$$\delta(q_1,b)=(q_2,y,L),$$

$$\delta(q_2,y)=(q_2,y,L),$$
$$\delta(q_2,a)=(q_2,a,L),$$
$$\delta(q_2,x)=(q_0,x,R).$$

$$\delta(q_0,y)=(q_3,y,R),$$
$$\delta(q_3,y)=(q_3,y,R),$$
$$\delta(q_3,\square)=(q_4,\square,R).$$

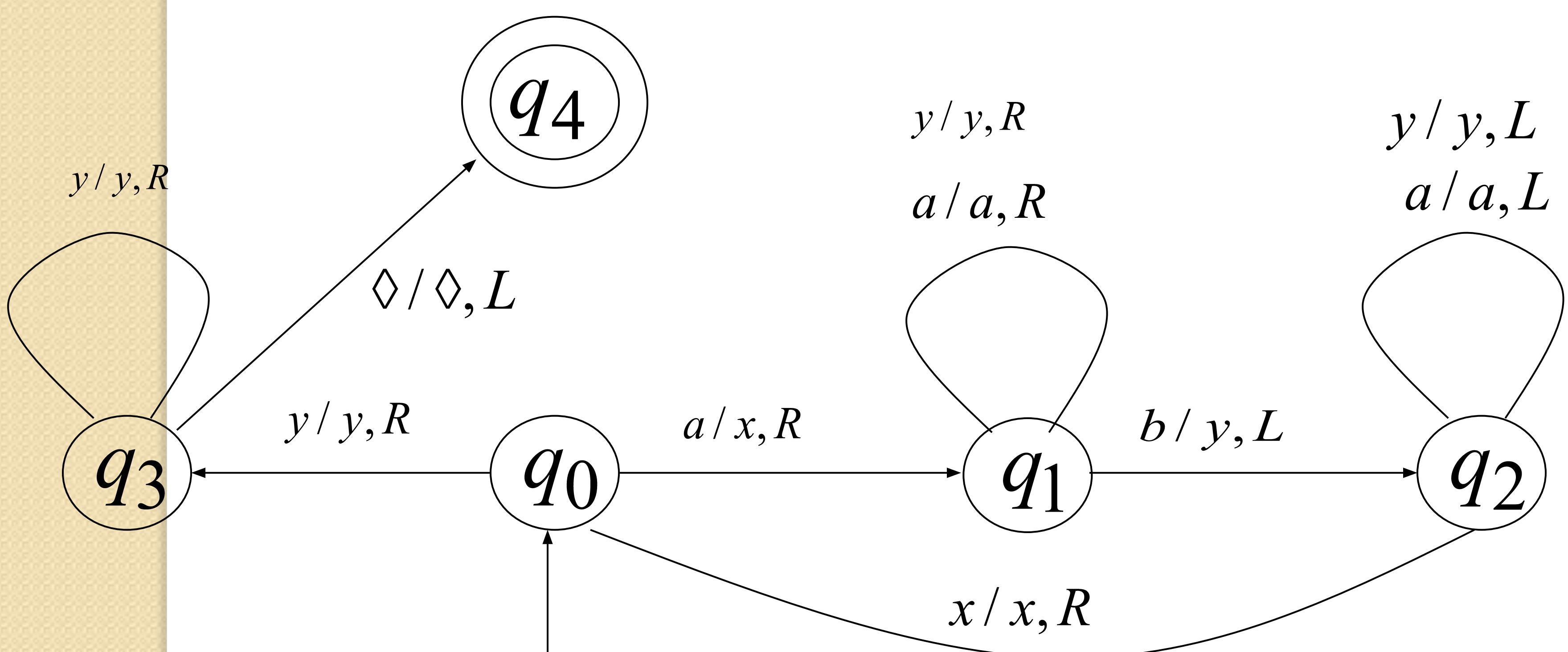
$$q_0aa\cdots abb\cdots b \overset{*}{\vdash} xq_0a\cdots ayb\cdots b,$$

$$q_0aa\cdots abb\cdots b \overset{*}{\vdash} xxq_0\cdots ayy\cdots b,$$

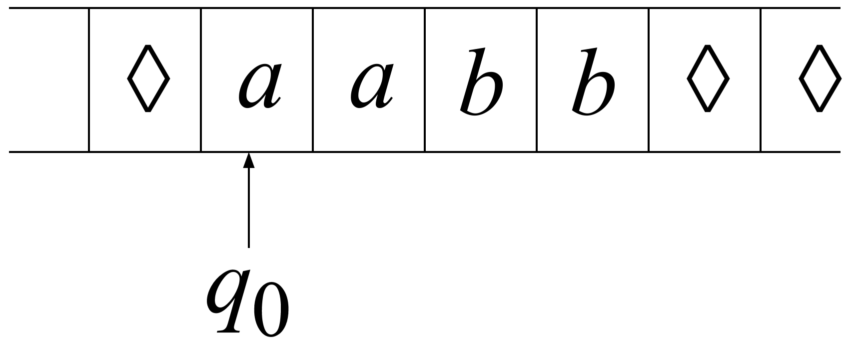
$$\begin{aligned} a_q aabb &\vdash xq_1abb \vdash xaq_1bb \vdash xq_2ayb \\ &\vdash q_2xayb \vdash xq_0ayb \vdash xxq_1yb \\ &\vdash xxyq_1b \vdash xxq_2yy \vdash xq_2xyy \\ &\vdash xxq_0yy \vdash xxyq_3y \vdash xxyq_3\square \\ &\vdash xxyy\square q_4\square. \end{aligned}$$

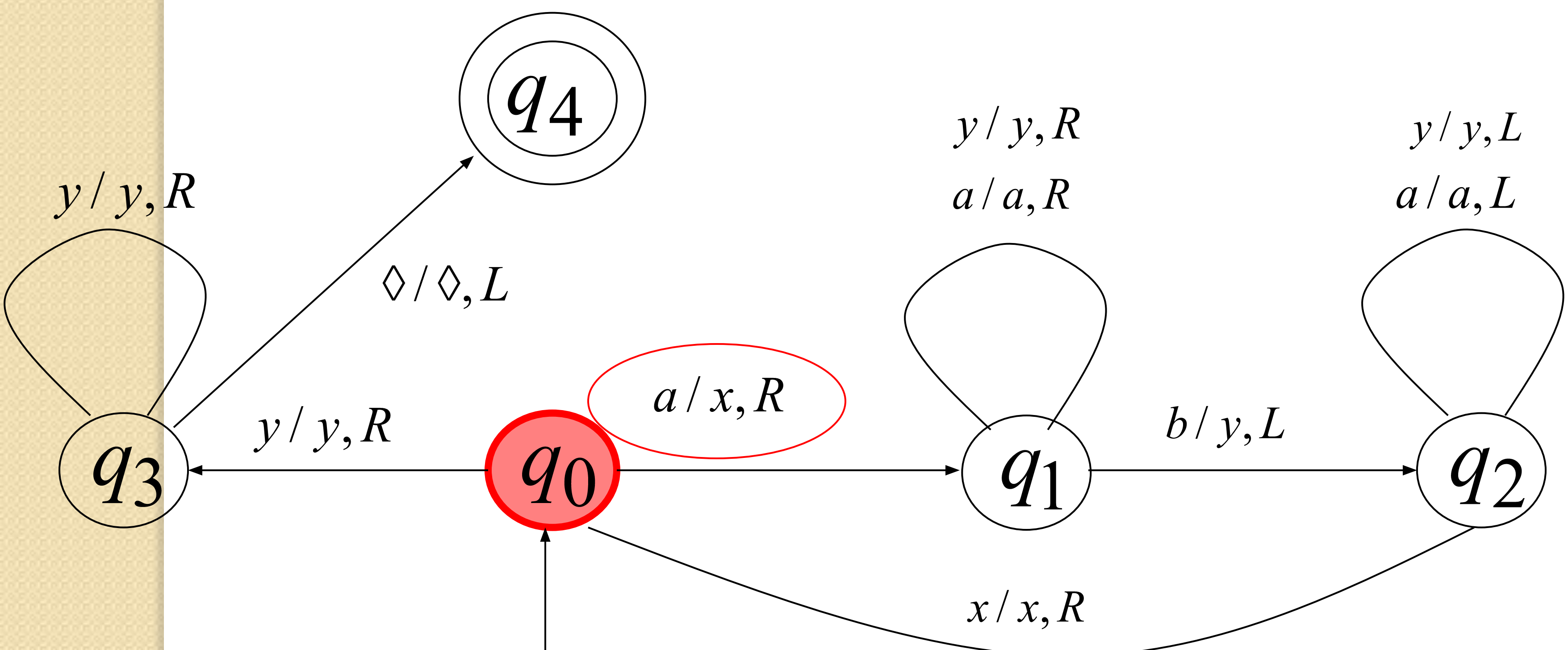
Another Turing Machine Example

Turing machine for the language $\{a^n b^n\}$

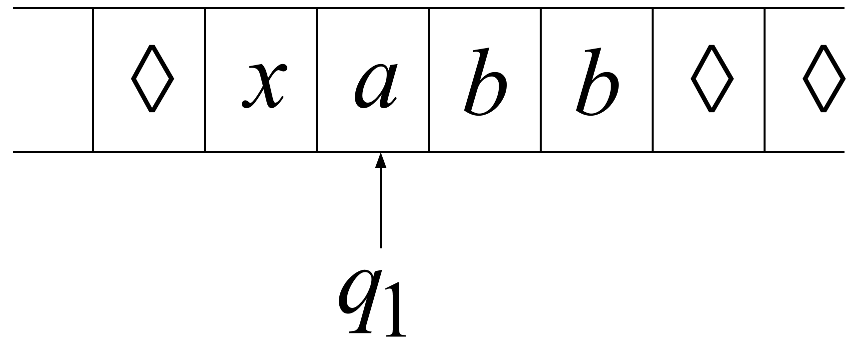


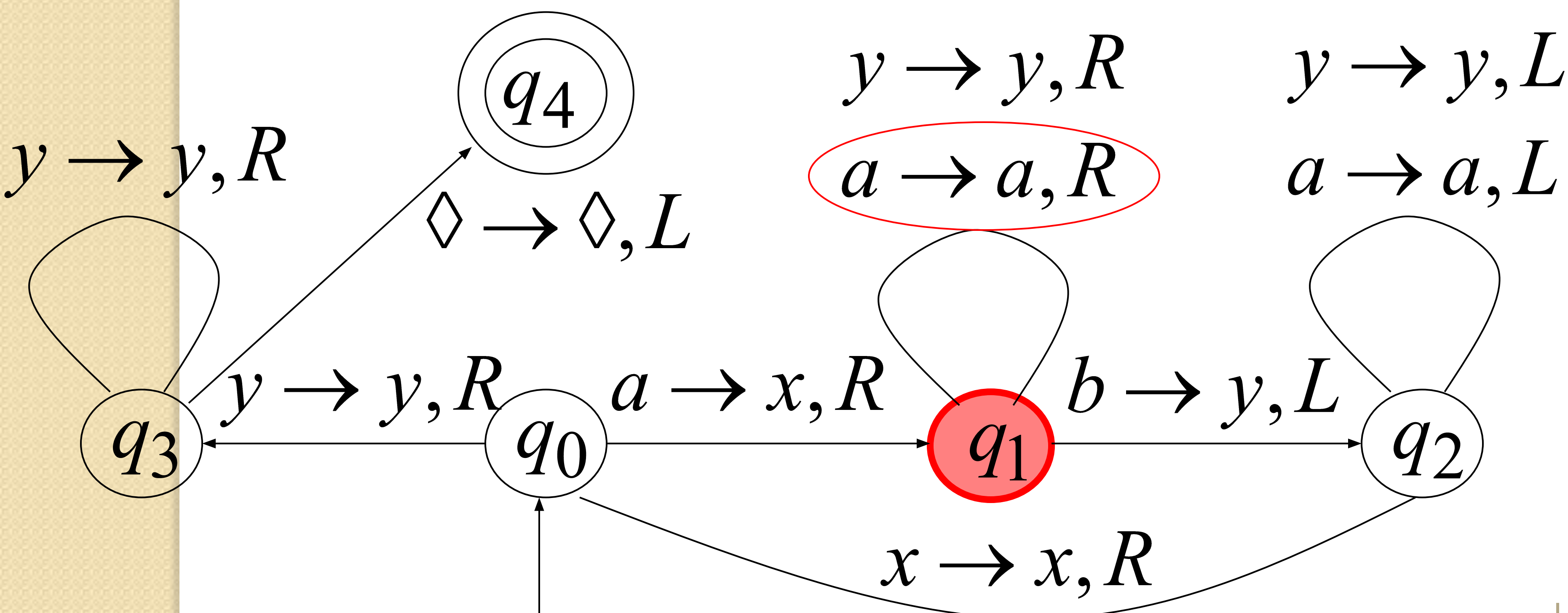
Time 0


◊
a
a
b
b
◊
◊
q_0

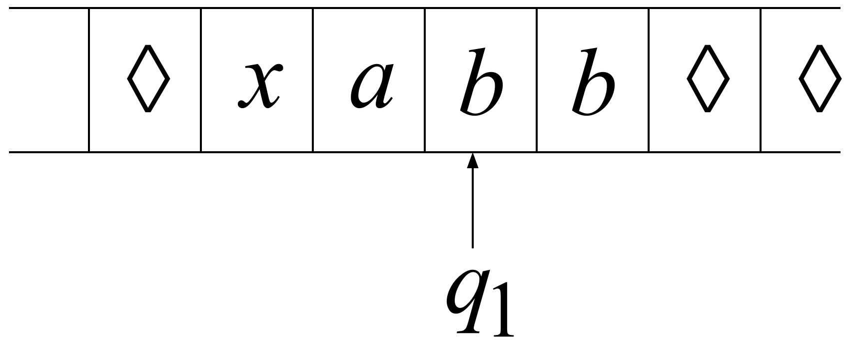

q_4
$y/y,R$
$y/y,L$
$y/y,R$
$a/a,R$
$a/a,L$
$\lozenge/\lozenge,L$
$a/x,R$
$y/y,R$
$b/y,L$
q_3
q_0
q_1
q_2
$x/x,R$

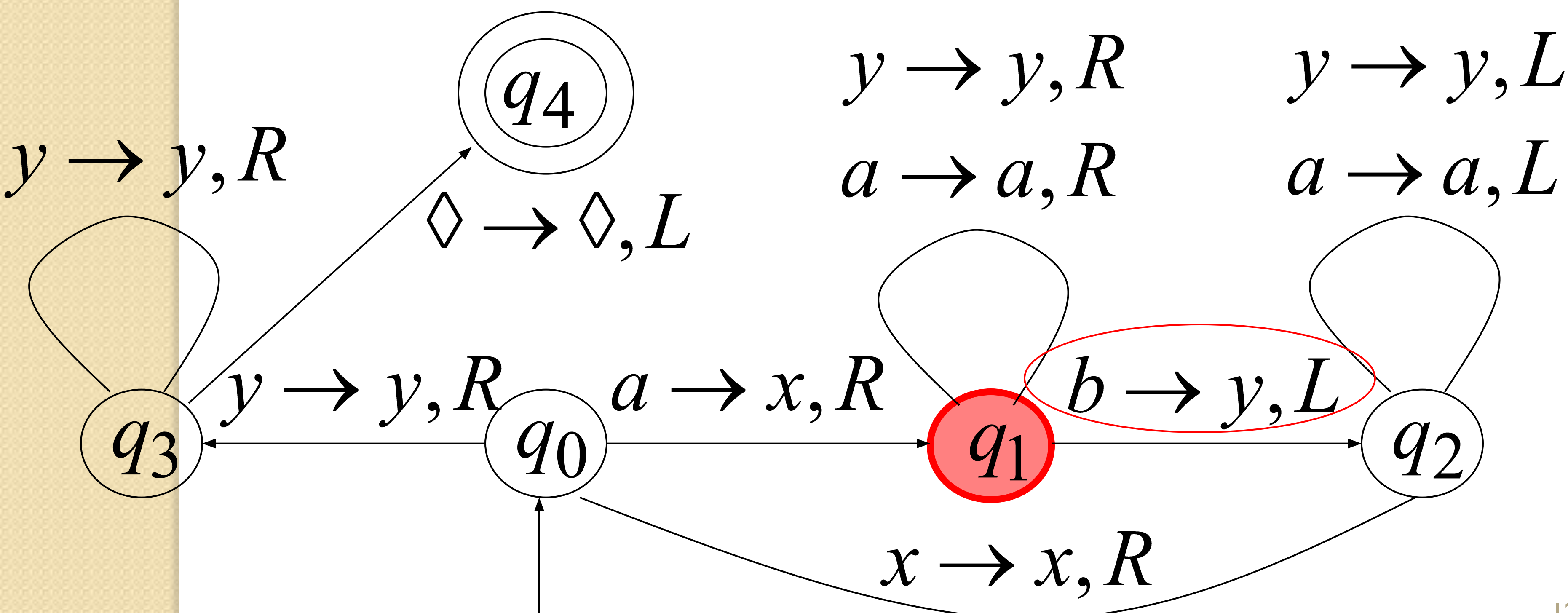
Time 1


◊
x
a
b
b
◊
◊
q_1

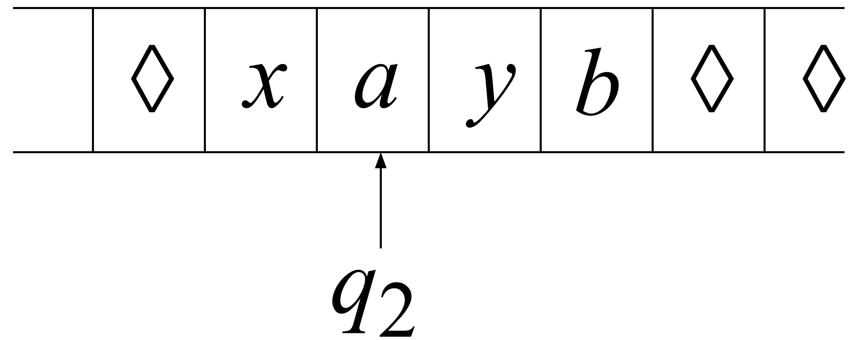

q_4
$y \to y, R$
$a \to a, R$
$y \to y, L$
$a \to a, L$
$y \to y, R$
$\lozenge \to \lozenge, L$
$y \to y, R$
$a \to x, R$
$b \to y, L$
q_3
q_0
q_1
q_2
$x \to x, R$

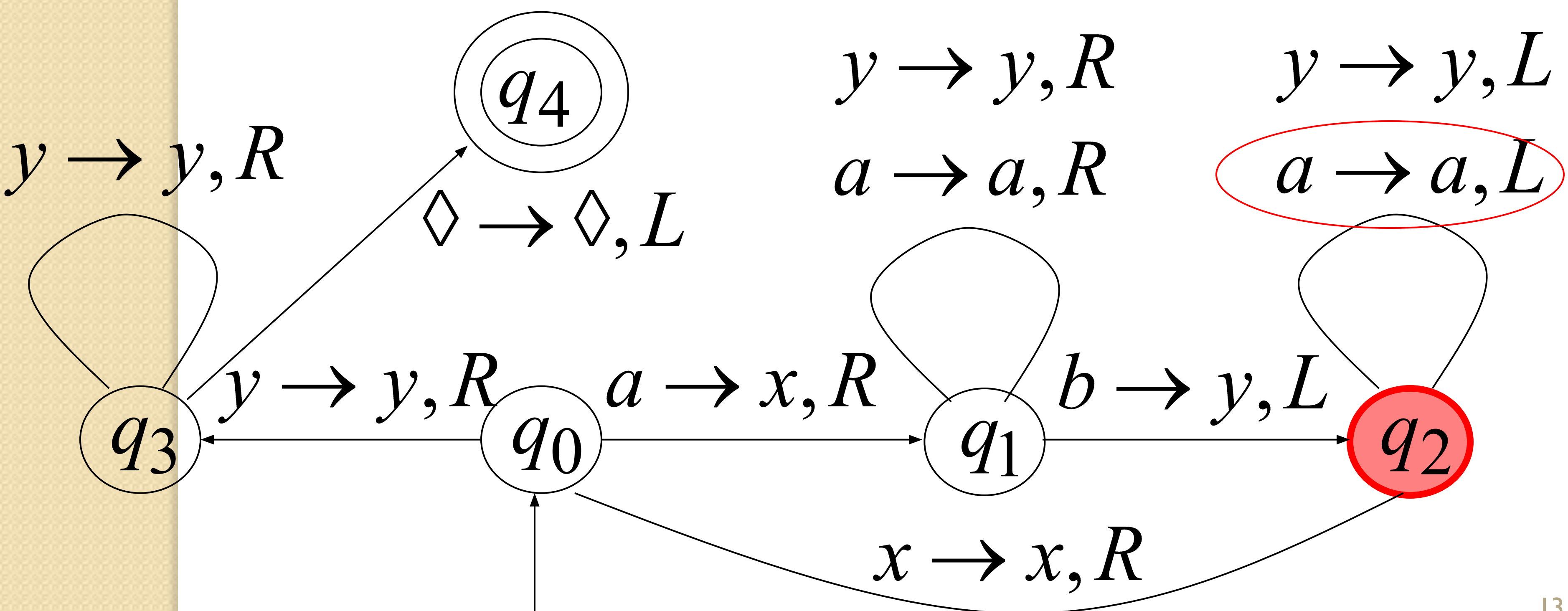
Time 2


◊
x
a
b
b
◊
◊
q_1

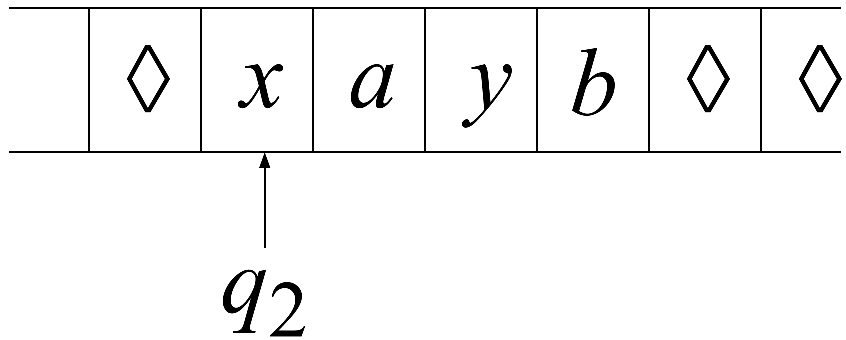

q_4
$y \to y, R$
$\lozenge \to \lozenge, L$
$y \to y, R$
$a \to a, R$
$y \to y, L$
$a \to a, L$
$y \to y, R$
$a \to x, R$
$b \to y, L$
q_3
q_0
q_1
q_2
$x \to x, R$

Time 3

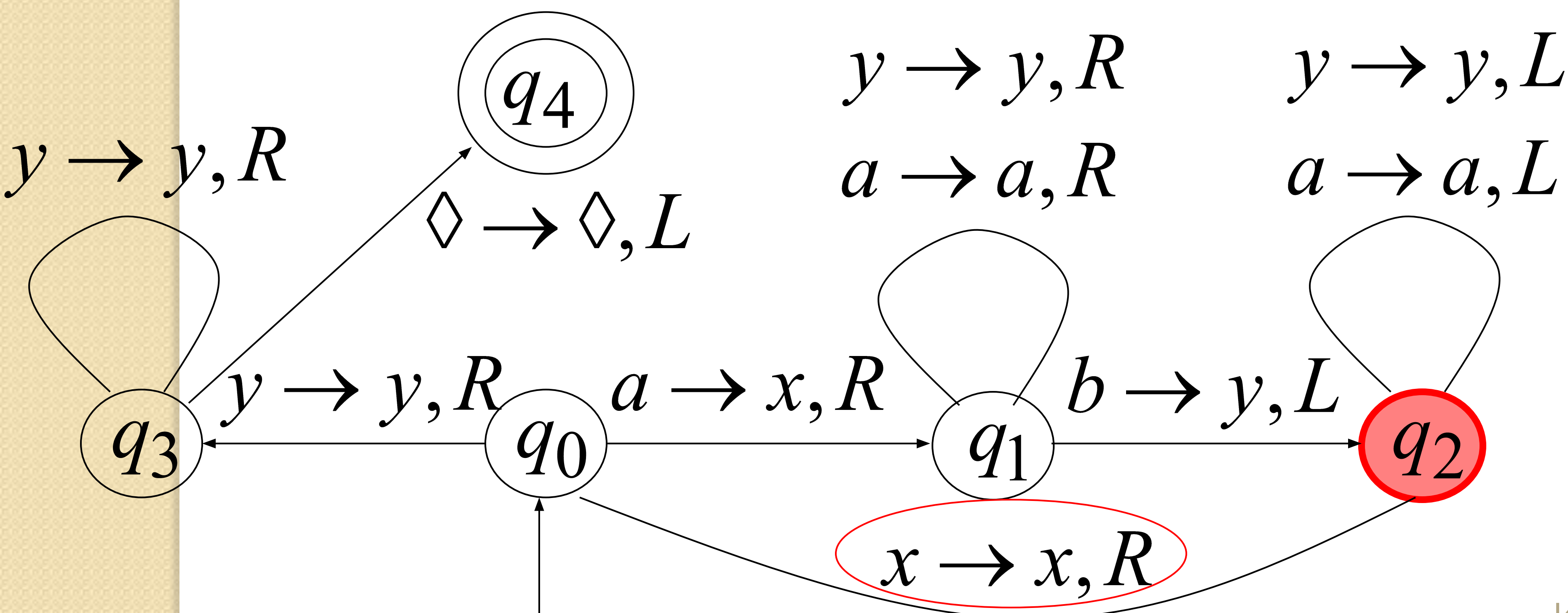

◊ x a y b ◊ ◊
q_2


q_4
$y \rightarrow y, R$
$\Diamond \rightarrow \Diamond, L$
$y \rightarrow y, R$
$a \rightarrow a, R$
$y \rightarrow y, L$
$a \rightarrow a, L$
$y \rightarrow y, R$
$a \rightarrow x, R$
$b \rightarrow y, L$
q_3
q_0
q_1
q_2
$x \rightarrow x, R$

Time 4

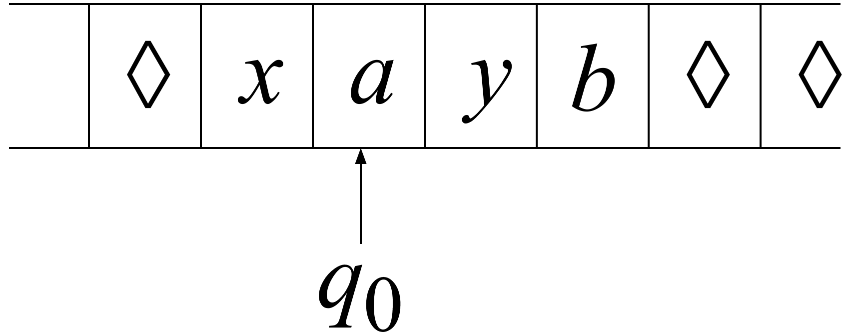


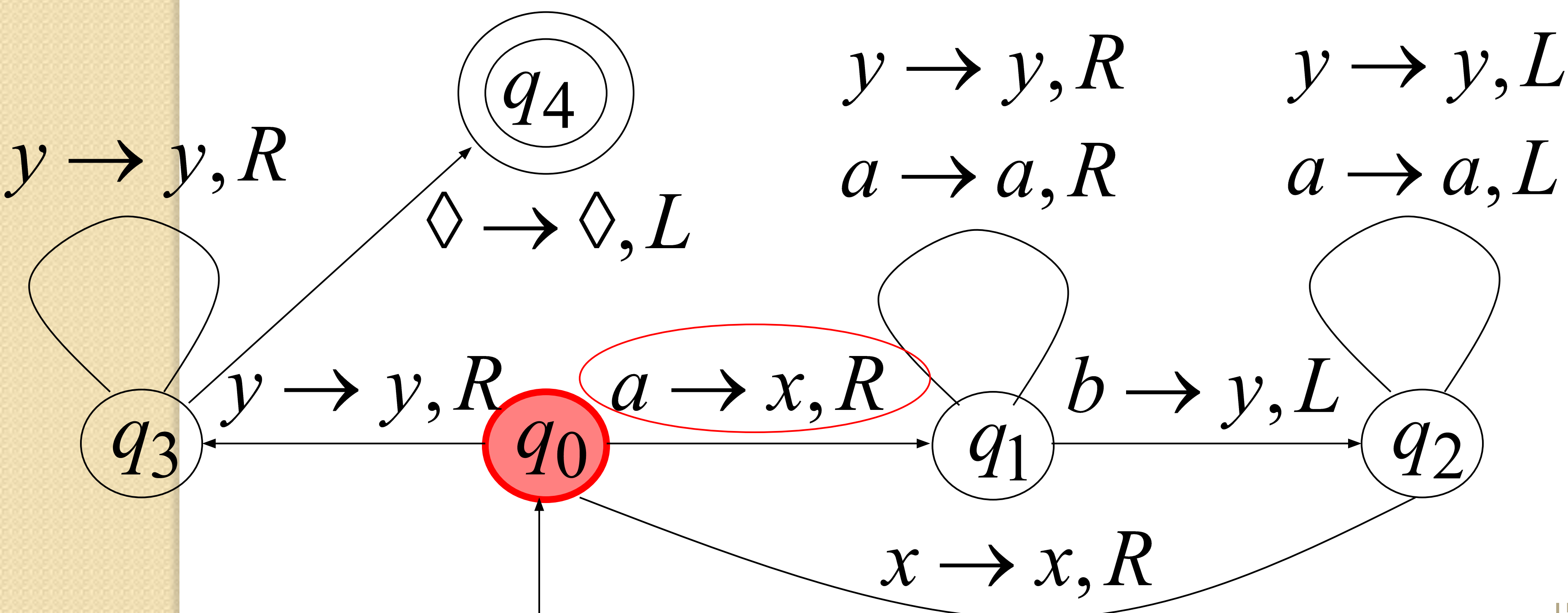
◊
x
a
y
b
◊
◊
q_2



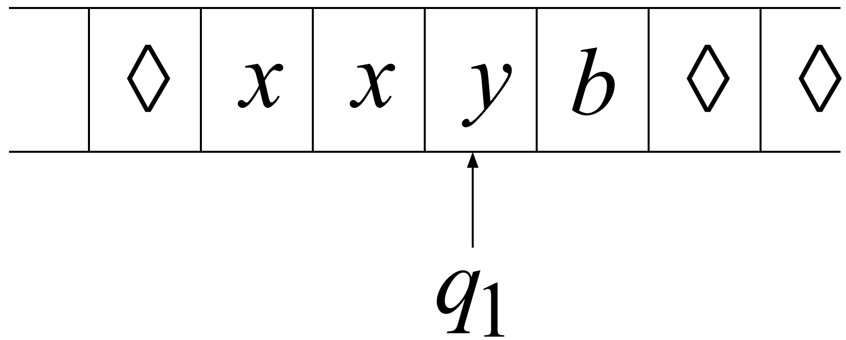
q_4
$y \rightarrow y, R$
$\Diamond \rightarrow \Diamond, L$
$y \rightarrow y, R$
$a \rightarrow a, R$
$y \rightarrow y, L$
$a \rightarrow a, L$
q_3
$y \rightarrow y, R$
q_0
$a \rightarrow x, R$
q_1
$b \rightarrow y, L$
q_2
$x \rightarrow x, R$

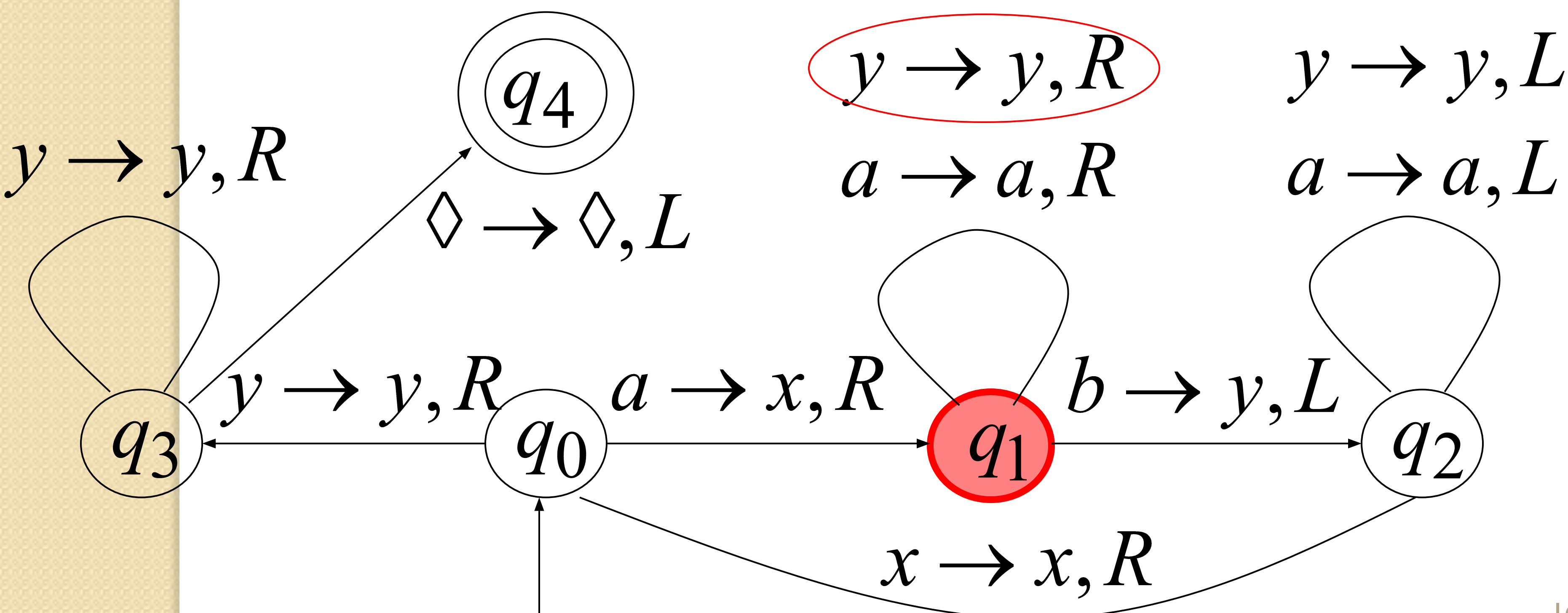
Time 5


◊
x
a
y
b
◊
◊
q_0

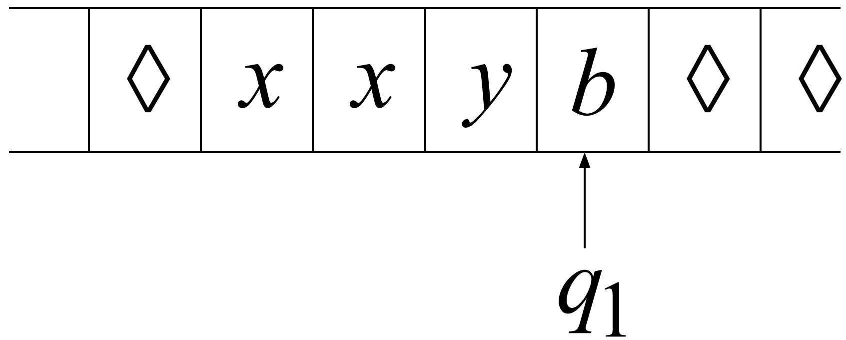

q_4
$\lozenge \rightarrow \lozenge, L$
$y \rightarrow y, R$
$a \rightarrow a, R$
$y \rightarrow y, L$
$a \rightarrow a, L$
$y \rightarrow y, R$
q_3
$y \rightarrow y, R$
q_0
$a \rightarrow x, R$
q_1
$b \rightarrow y, L$
q_2
$x \rightarrow x, R$

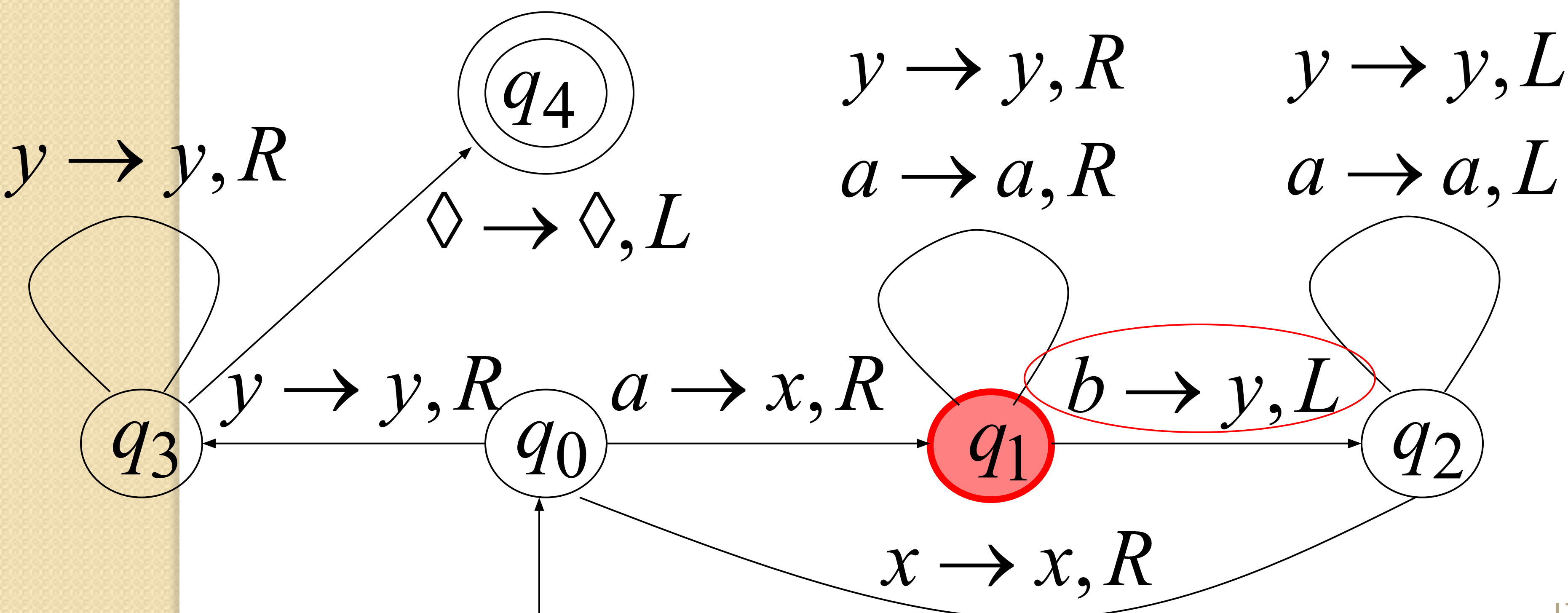
Time 6



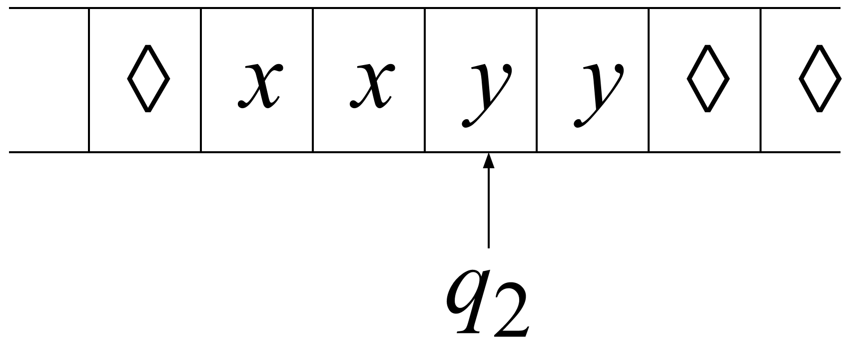


Time 7

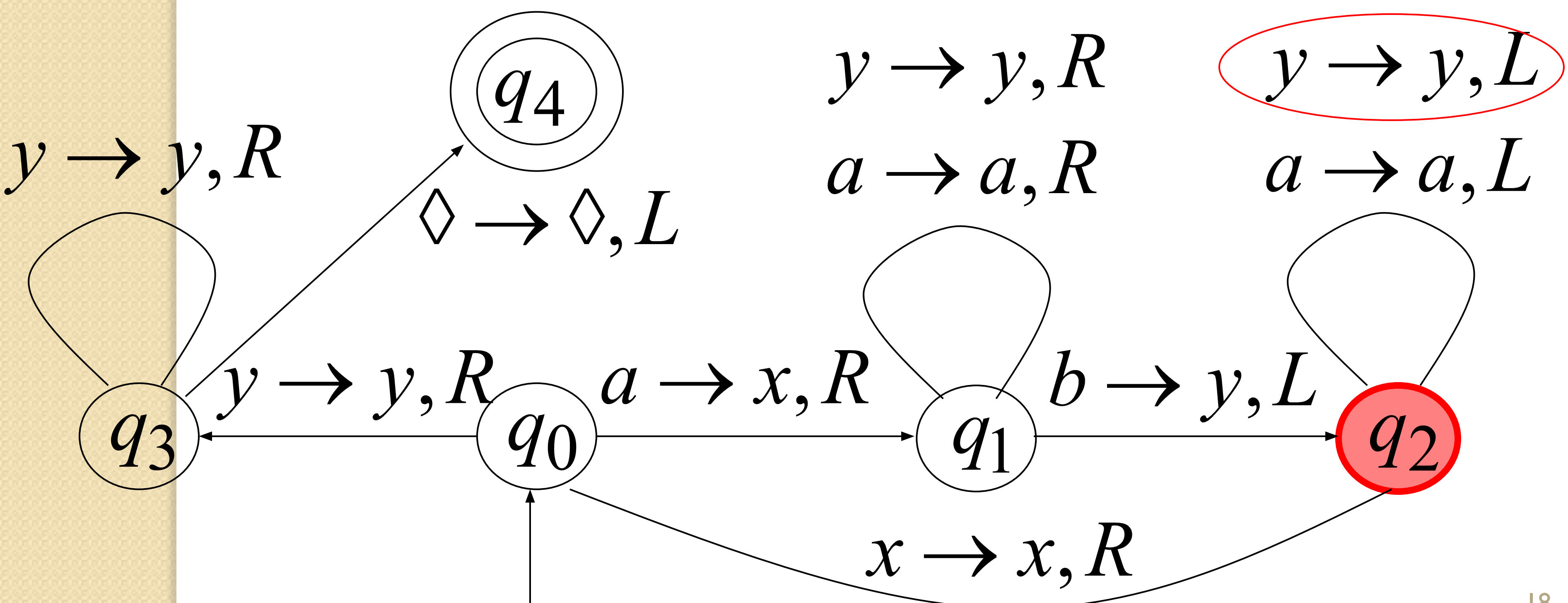

◊
x
x
y
b
◊
◊
q_1


q_4
$y \rightarrow y, R$
$\lozenge \rightarrow \lozenge, L$
$y \rightarrow y, R$
$a \rightarrow a, R$
$y \rightarrow y, L$
$a \rightarrow a, L$
$y \rightarrow y, R$
$a \rightarrow x, R$
$b \rightarrow y, L$
q_3
q_0
q_1
q_2
$x \rightarrow x, R$

Time 8

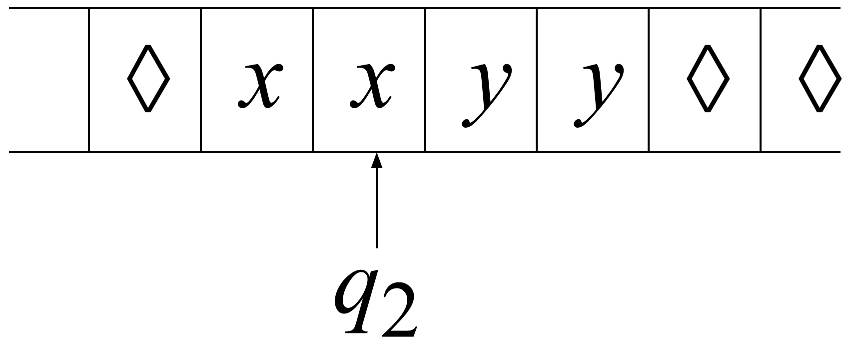


◊ x x y y ◊ ◊
q_2

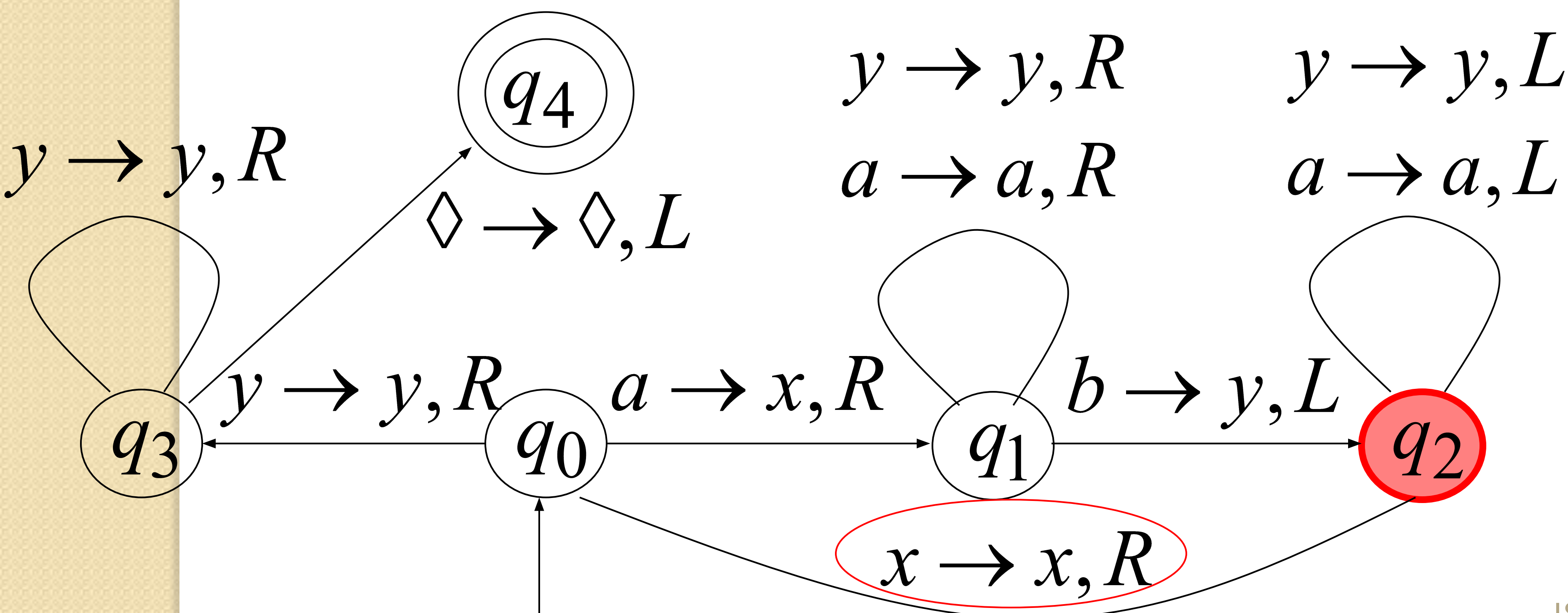


q_4
$y \rightarrow y, R$
$\Diamond \rightarrow \Diamond, L$
$y \rightarrow y, R$
$a \rightarrow a, R$
$y \rightarrow y, L$
$a \rightarrow a, L$
$y \rightarrow y, R$
$a \rightarrow x, R$
$b \rightarrow y, L$
q_3
q_0
q_1
q_2
$x \rightarrow x, R$

Time 9

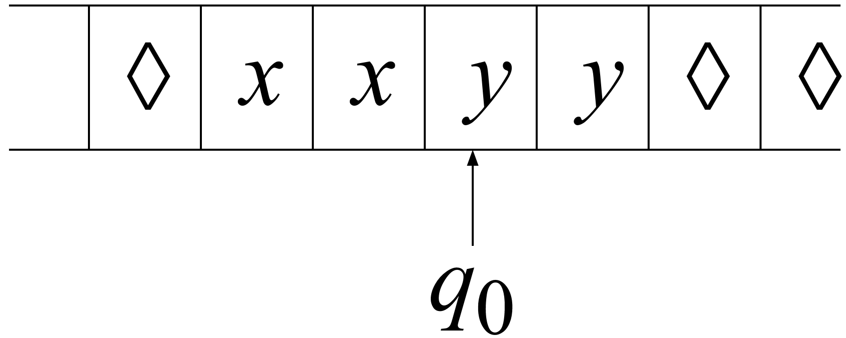


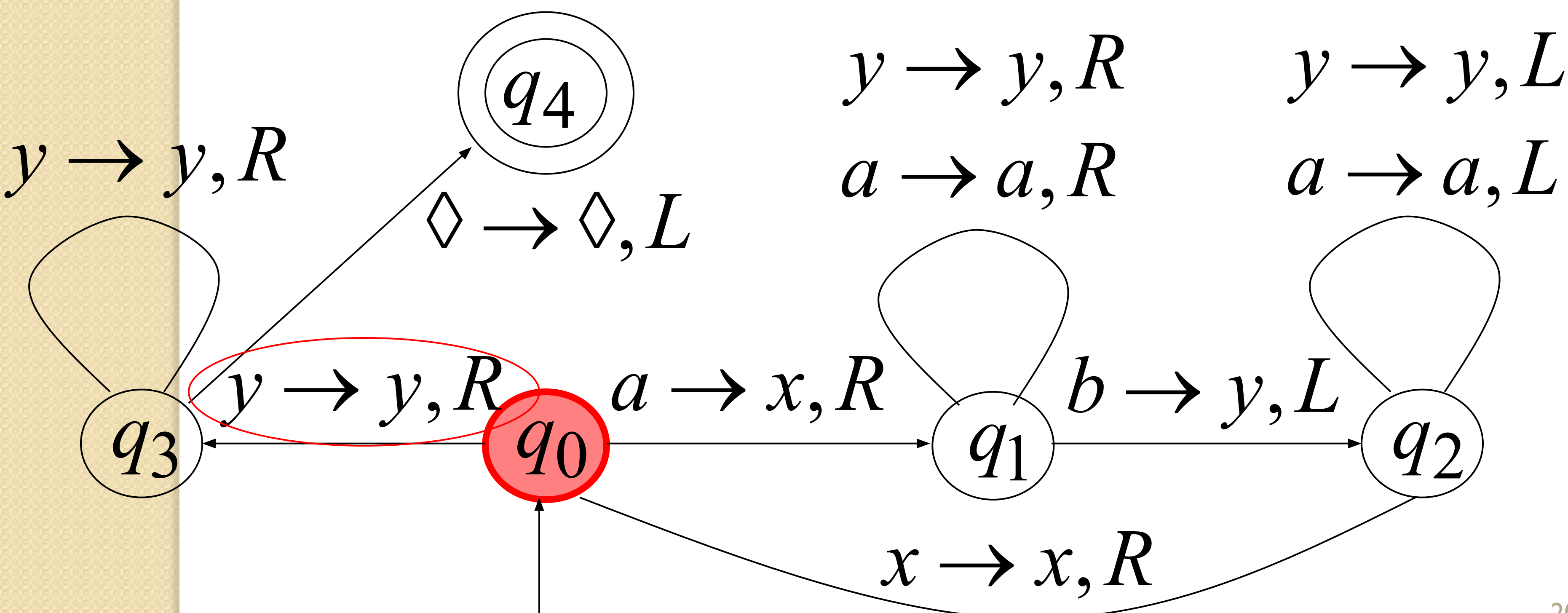
◊
x
x
y
y
◊
◊
q_2



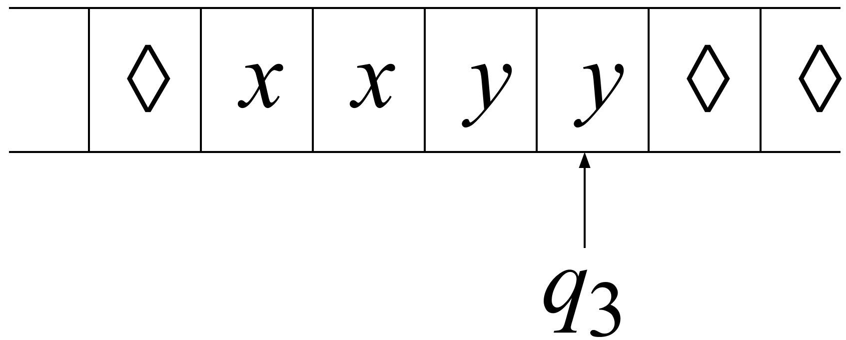
q_4
$y \rightarrow y, R$
$\Diamond \rightarrow \Diamond, L$
$y \rightarrow y, R$
$a \rightarrow a, R$
$y \rightarrow y, L$
$a \rightarrow a, L$
$y \rightarrow y, R$
$a \rightarrow x, R$
$b \rightarrow y, L$
q_3
q_0
q_1
q_2
$x \rightarrow x, R$

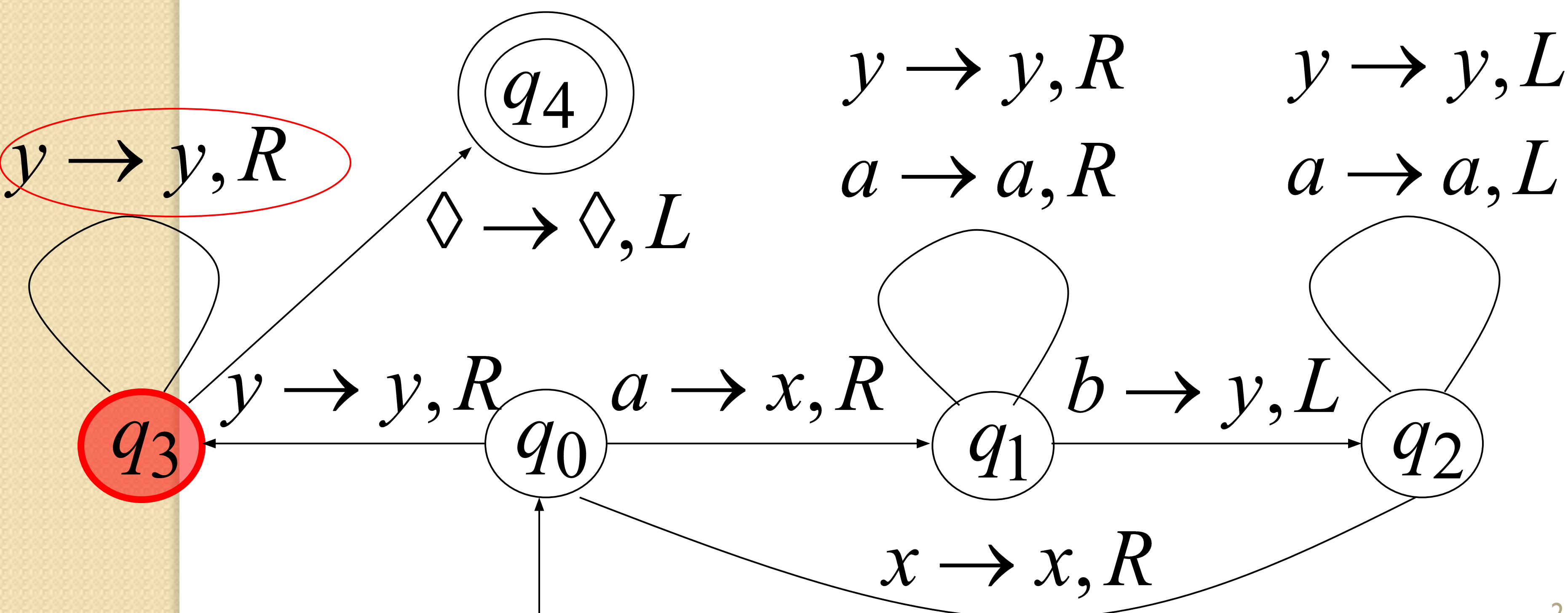
Time 10


◊ x x y y ◊ ◊
q_0

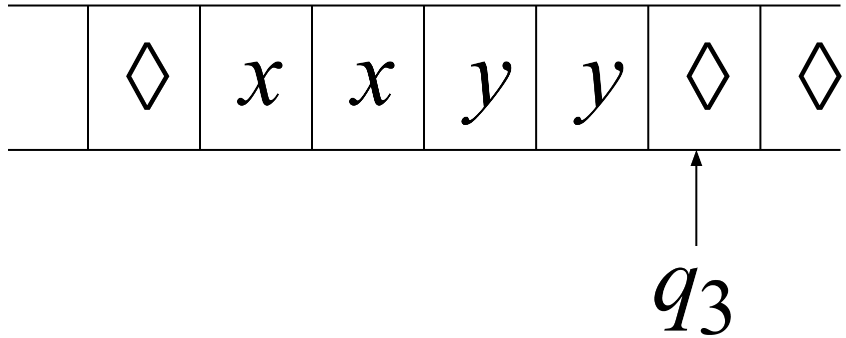

q_4
$y \rightarrow y, R$
$a \rightarrow a, R$
$y \rightarrow y, L$
$a \rightarrow a, L$
$y \rightarrow y, R$
$◊ \rightarrow ◊, L$
$y \rightarrow y, R$
$a \rightarrow x, R$
$b \rightarrow y, L$
q_3
q_0
q_1
q_2
$x \rightarrow x, R$

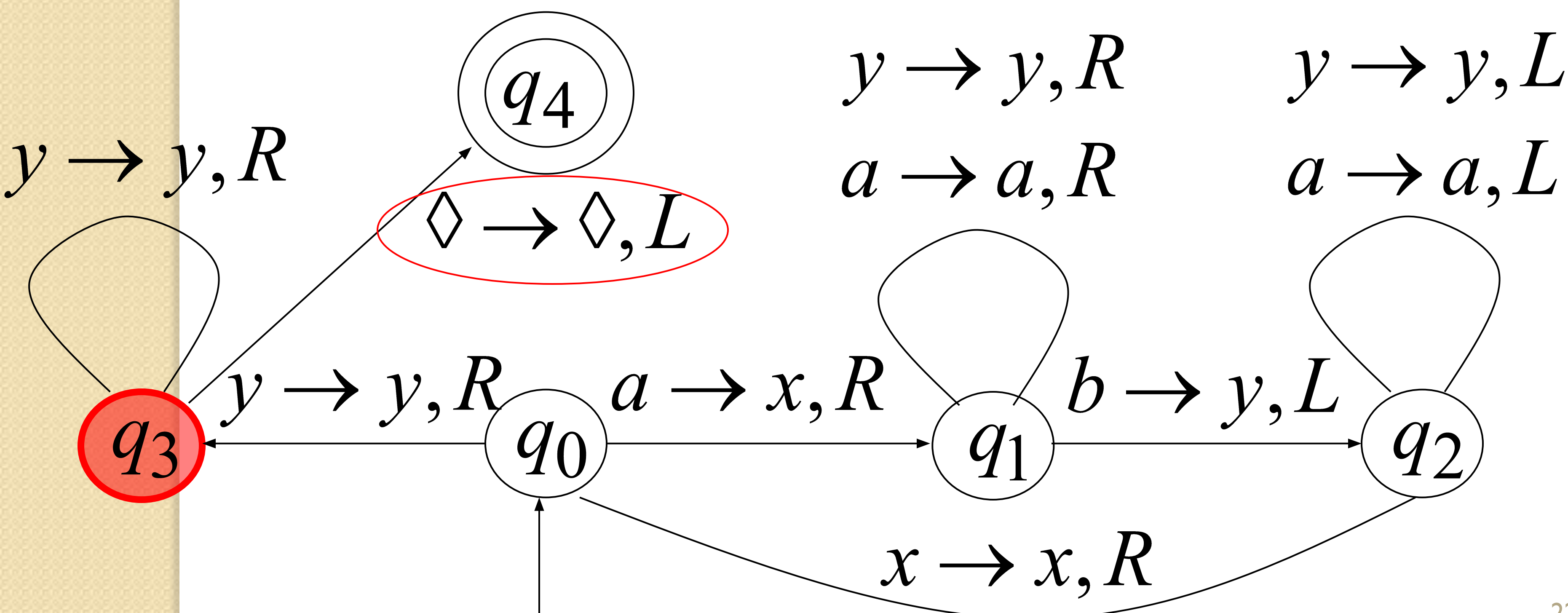
Time 11


◊ x x y y ◊ ◊
q_3

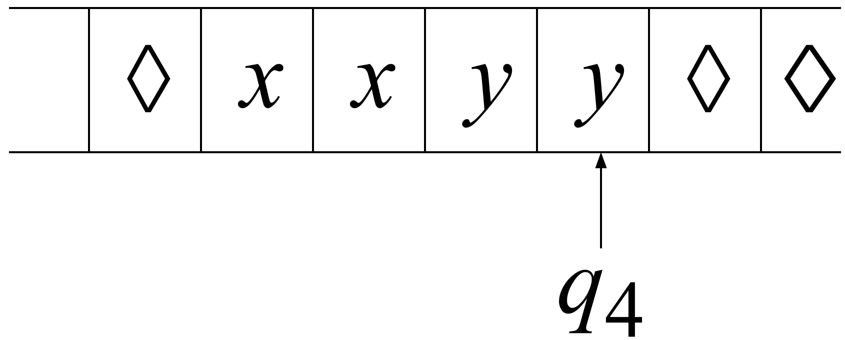

q_4
$\Diamond \rightarrow \Diamond, L$
$y \rightarrow y, R$
$y \rightarrow y, R$
$a \rightarrow a, R$
$y \rightarrow y, L$
$a \rightarrow a, L$
$y \rightarrow y, R$
$a \rightarrow x, R$
$b \rightarrow y, L$
q_3
q_0
q_1
q_2
$x \rightarrow x, R$

Time 12

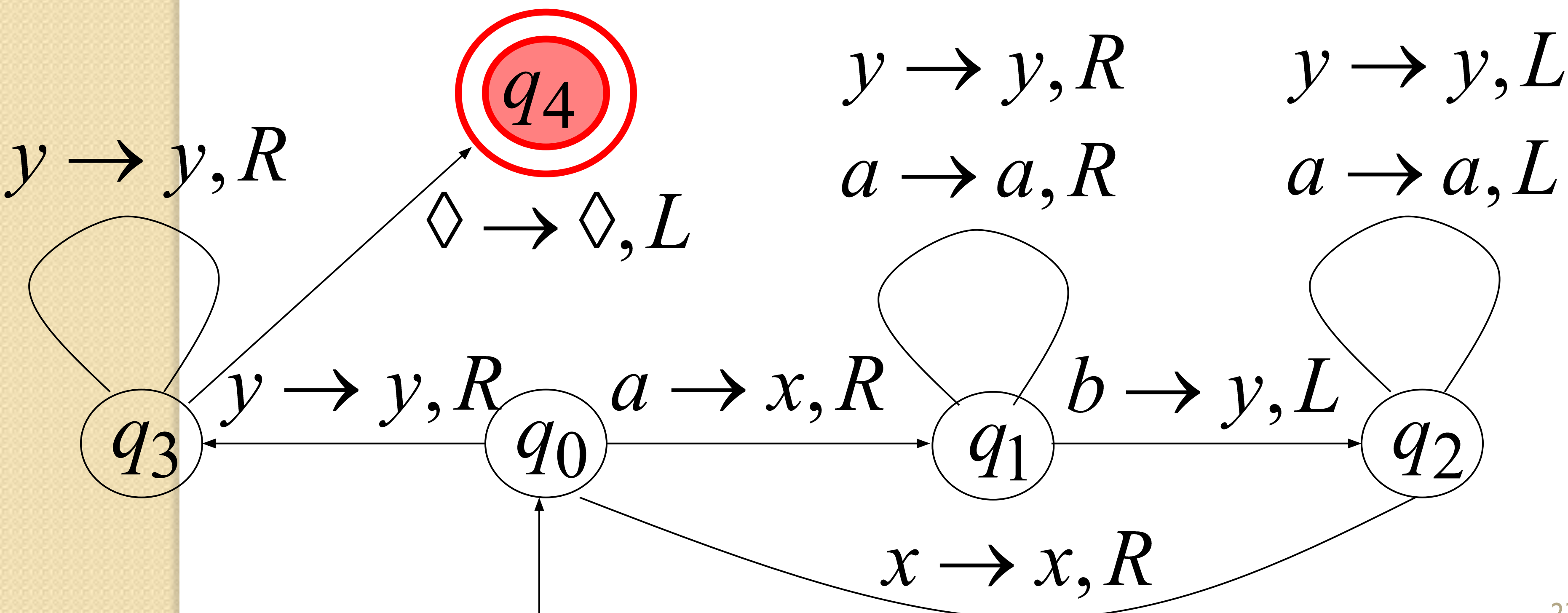

◊
x
x
y
y
◊
◊
q_3


q_4
$y \rightarrow y, R$
$\lozenge \rightarrow \lozenge, L$
$y \rightarrow y, R$
$a \rightarrow a, R$
$y \rightarrow y, L$
$a \rightarrow a, L$
q_3
$y \rightarrow y, R$
q_0
$a \rightarrow x, R$
q_1
$b \rightarrow y, L$
q_2
$x \rightarrow x, R$

Time 13



Halt & Accept



Observation:

since we can find the
machine for the language $\{a^n b^n\}$

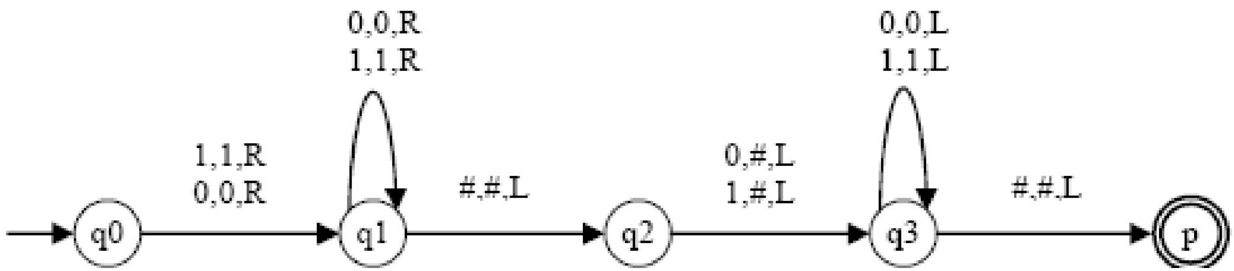
So, we can easily construct
a machine for the language $\{a^n b^n c^n\}$

Question

- Design the Simple Eraser TM. This Turing machine reads strings in the language given by the expression (0+1)* and replaces the right-most symbol by a blank (#).
- Then find the derivation of the string "1110"

- Note: the problem considers the blank symbol is #

Answer



TM = $(\{q_0,q_1,q_2,q_3\},\{0,1\},\{0,1,\#\},\delta,q_0,\{p\})$ where δ is given by:

Q \ Σ	0	1	#
q_0	$(q_1, 0, R)$	$(q_1, 1, R)$	$\varnothing$
q_1	$(q_1, 0, R)$	$(q_1, 1, R)$	$(q_2, \#, L)$
q_2	$(q_3, \#, L)$	$(q_3, \#, L)$	$\varnothing$
q_3	$(q_3, 0, L)$	$(q_3, 1, L)$	$(p, \#, L)$

$\#q_01110\# \rightarrow \#1q_1110\# \rightarrow \#11q_110\# \rightarrow \#111q_10\# \rightarrow \#1110q_1\# \rightarrow \#111q_20\# \rightarrow$
$\#11q_31\#\# \rightarrow \#1q_311\#\# \rightarrow \#q_3111\#\# \rightarrow q_3\#111\#\# \rightarrow p\#\#111\#\#$

Thanks for Listning