Theory section5 - General

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→ language L is not Regular → if there is No FA that accepts L

→ is L Regular? $L = \{a^n b^n : n \geq 0\}$ ×

* Proof language is not Regular using Pumping lemma:

① language (L) and an integer m.

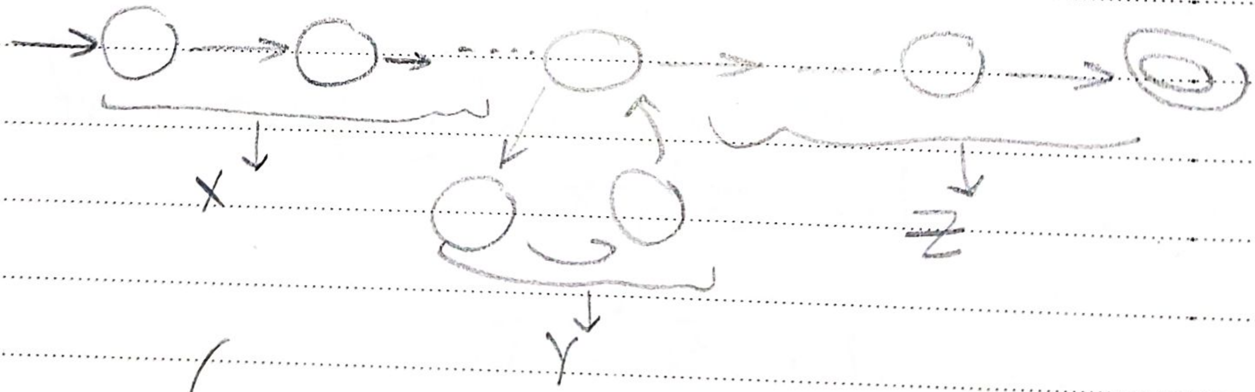
② for any string $w \in L$ with length $w \geq \underset{\#states}{m}$

we can write:

$$w = xyz$$

with $xy \leq m$, $y \geq 1$

such that $xy^iz \in L$ $i = 0, 1, 2, \dots$



string XZ Accept

" XYZ "

" XYYZ "

" XYYYZ "

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① Proof that $L = \{a^n b^n : n \geq 0\}$ is not Regular?

- let m be the integer in the Pumping lemma
  - w string : $w \in L$
  - length $w \geq m$
  - Assume $w = a^m b^m$

$$a^m b^m = xyz$$

- length $xy \leq m$, $y \geq 1$

$$a^m b^m = \overbrace{\underbrace{aaaa\ldots}_{x}\underbrace{\ldots aa}_{y}}^{m}\overbrace{\underbrace{bb\ldots bbb}_{z}}^{m}$$

- $a^m b^m = xyz$ $\quad y = a^k \quad k \geq 1$

- from pumping lemma:

$$xy^i z \in L \quad i = 0, 1, 2$$

$$xy^2 z \in L \checkmark$$

$$xyyz = a^m a^k b^m$$
$$= a^{m+k} b \notin L$$

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Proof that $L = \{ww^R | k \in \{a,b\}\}$ is not aregular language?

→ let m be integer

– $w \in L$ → Assume $w = a^m b^m b^m a^m$
length $w \geq m$

– $a^m b^m b^m a^m = XYZ$

$$\overbrace{\underbrace{aaa}_{x}\,\underbrace{\_\_aa}_{Y}}^{m}\,\underbrace{\overbrace{bb\_\,b}^{m}\,\overbrace{b\ldots bb}^{m}\,\overbrace{\ldots aa\ldots a}^{m}}_{Z}$$

– $Y = a^k \quad k \geq 1$

– from pumping lemma:

$$XY^iZ \in L \,, \quad i = 0, 1, 2, \ldots$$

$$XY^2Z = XYYZ = a^m a^k b^m b^m a^m$$

$$= a^{m+k} b^m b^m a^m \notin L$$

② proof that $L = \{a^n b^l c^{n+l} : n,l \geq 0\}$
is not Regular language?

- let m be integer
- $w \in L \rightarrow$ Assume $w = a^m b^m c^{2m}$
  length $w \geq m$
- $a^m b^m c^{2m} = xyz$
- $\overbrace{\underbrace{aaa}_{x}\ldots\underbrace{aa}_{y}}^{m}\underbrace{\overbrace{bb\ldots b}^{m}\overbrace{c\ldots cc}^{c}}_{z}$ $\quad xy \leq m$
  $y \geq 1$
- $y = a^k \quad k \geq 1$
- from pumping lemma:
  $xy^iz \in L \;,\; i = 0,1,2\ldots$

$xy^2z = xyyz = a^m a^k b^m c^{2m}$
$= a^{m+k} b^m c^{2m} \notin L$

③ proof that $L = \{a^n b^{2n} : n \geq 0\}$ is Not Regular language?

- let m is an integer
- $w \in L$, length of $w \geq m$

  Assume : $w = a^m b^{2m}$

- $a^m b^{2m} = xyz$

$$\overbrace{\underbrace{aaa}_{x}\ \underbrace{\dots aa}_{y}}^{m}\ \overbrace{\underbrace{bbb\dots bb}_{z}}^{2m} \qquad \begin{array}{l} xy \leq m \\ y \geq 1 \end{array}$$

- $y = a^k$
- from pumping lemma:

$$xy^iz \in L \qquad i = 0,1,2,\dots$$

$$xy^2z = xyyz = a^m a^k b^{2m}$$

$$= a^{m+k} b^{2m} \notin L$$

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-write the Context free grammer of the following languages.

① $L = \{ww^R : w \{a,b\}^*\}$

$S \rightarrow aSa \mid bSb \mid \lambda \mid a \mid b$

② $L = \{a^n b^m : n \neq m\}$

$S \rightarrow AB$

$A \rightarrow aA \mid \lambda$

$B \rightarrow aBb \mid \lambda$

③ $L = \{a^n b^{n+1} : n \geq 0\}$

$S \rightarrow Ab$

$A \rightarrow aAb \mid \lambda$

④ $L = \{a^n b^n : n \geq 0\}$

$S \rightarrow aSb \mid \lambda$

⑤ $L = \{ab^n c : n \geq 0\}$

$S \rightarrow aBc$

$B \rightarrow bB \mid \lambda$

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⑥ $L = \{ab^{2n+1}c : n \geq 0\}$

$S \rightarrow aBc$
$B \rightarrow bbB \mid b$

⑦ $L = \{(ab)^n : n \geq 0\}$

$S \rightarrow abS \mid \lambda$

⑧ $L = \{a^n b^m abb^j : m \geq 1, n, j \geq 0\}$

$S \rightarrow AbB\, abB$
$A \rightarrow aA \mid \lambda$
$B \rightarrow bB \mid \lambda$

⑨ $L = \{a^n b^m a^{2n} \mid n, m \geq 0\}$

$S \rightarrow aSaa \mid B$
$B \rightarrow bB \mid \lambda$

⑩ $L = \{0^i 1^j 2^n : i, j, n \geq 1, i+j=n\}$

$S \rightarrow 0A2$
$A \rightarrow 1A2 \mid 12$