Problem Set #2

Back to Week 2



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0/1 points

1.

This question will give you further practice with the Master Method. Suppose the running time of an algorithm is governed by the recurrence $T(n)=7*T(n/3)+n^2$. What's the overall asymptotic running time (i.e., the value of T(n))?

Note: If you take this quiz multiple times, you may see different variations of this question.



 $\theta(n^{2.81})$

Incorrect Response

Make sure you're in the right case of the Master Method.

- $O \theta(n^2)$



1/1 points

2.

Consider the following pseudocode for calculating \boldsymbol{a}^b (where a and b are positive integers)

```
FastPower(a,b):
```

if b = 1

return a

else

c := a*a

ans := FastPower(c,[b/2])

if b is odd

return a*ans

else return ans

end

Here [x] denotes the floor function, that is, the largest integer less than or equal to x.

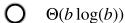
Now assuming that you use a calculator that supports multiplication and division (i.e., you can do multiplications and divisions in constant time), what would be the overall asymptotic running time of the above algorithm (as a function of b)?



 $\Theta(\log(b))$

Correct Response

Constant work per digit in the binary expansion of b.





$$oldsymbol{igsigma}_{\Theta(\sqrt{b})}$$

$$\bigcirc \Theta(b)$$

3.

Let $0<\alpha<.5$ be some constant (independent of the input array length n). Recall the Partition subroutine employed by the QuickSort algorithm, as explained in lecture. What is the probability that, with a randomly chosen pivot element, the Partition subroutine produces a split in which the size of the smaller of the two subarrays is $\geq \alpha$ times the size of the original array?

$$\bigcirc 1 - 2 * \alpha$$

Correct Response

That's correct!

$$\bigcirc$$
 2 - 2 * α

$$O 1 - \alpha$$

$$O$$
 α



1/1 points

4.

Now assume that you achieve the approximately balanced splits above in every recursive call --- that is, assume that whenever a recursive call is given an array of length k, then each of its two recursive calls is passed a subarray with length between αk and $(1-\alpha)k$ (where α is a fixed constant strictly between 0 and .5). How many recursive calls can occur before you hit the base case? Equivalently, which levels of the recursion tree can contain leaves? Express your answer as a range of possible numbers d, from the minimum to the maximum number of recursive calls that might be needed.

$$0 \le d \le -\frac{\log(n)}{\log(\alpha)}$$

$$O - \frac{\log(n)}{\log(1 - 2 * \alpha)} \le d \le - \frac{\log(n)}{\log(1 - \alpha)}$$

$$O - \frac{\log(n)}{\log(1-\alpha)} \le d \le - \frac{\log(n)}{\log(\alpha)}$$

Correct Response

That's correct!



1/1 points

5.

Define the recursion depth of QuickSort to be the maximum number of successive recursive calls before it hits the base case --- equivalently, the number of the last level of the corresponding recursion tree. Note that the recursion depth is a random variable, which depends on which pivots get chosen. What is the minimum-possible and maximum-possible recursion depth of QuickSort, respectively?



Minimum: $\Theta(\log(n))$; Maximum: $\Theta(n)$

Correct Response

The best case is when the algorithm always picks the median as a pivot, in which case the recursion is essentially identical to that in MergeSort. In the worst case the min or the max is always chosen as the pivot, resulting in linear depth.

- \bigcirc Minimum: $\Theta(\log(n))$; Maximum: $\Theta(n\log(n))$
- O Minimum: $\Theta(1)$; Maximum: $\Theta(n)$
- O Minimum: $\Theta(\sqrt{n})$; Maximum: $\Theta(n)$



