**Summary:**

Black and Scholes (1973) proposed a valuation model for a European option, a contract that allows the holder of the option to exercise the right to buy or sell stocks at the expiration date. Unlike European options, where the payoff is determined by the price of the underlying asset at the exercise date, another primary type of options called American options give the holder the right to early exercise the options at any time before the expiration date. The American option valuation problem can be viewed as a free-boundary problem, that is, there exists an unknown boundary dependent on time that sets the line between early exercising and holding the option. Since an American option offers the holder greater rights than a European option, it usually has a larger value. The explicit formula for European options no longer works for American options. In this project I was asked to value American and European option on dividend paying underlying asset using Monte Carlo Euler’s scheme for European options, Longstaff and Schwartz regression method for American options and finite difference methods for both American and European options. To solve system of equation obtained from Finite difference method and Crank Nicolson I implemented Thomas, Successive over relaxation, Brennan-Schwartz and projected SOR algorithm.

**Analysis of Numerical Results**: The results in Table1 are obtained through implementing the explicit and implicit finite difference methods for European options and Table 2 shows European Call and Put option price obtained through explicit Euler method. Table1 shows that the three finite difference methods provide us with similar values and the values almost matches with closed form solution, but Euler’s Call option price and Put option price doesn’t converge to the closed form values, the reason is as for measurement uncertainty, Euler’s approach has an error of O(dt) although better approximation exist , e.g. the Milstein method which has an error of O(dt^2) so time step size plays an important role, in our case time step(delta\_t) is too small 0.00125, but if I increase my time step size from 0.00125 to 0.01 I am pretty closed to closed form values. Euler's method uses the line tangent to the function at the beginning of the interval as an estimate of the slope of the function over the interval, assuming that if the step size is small, the error will be small. However, even when extremely small step sizes are used, over a large number of steps the error starts to accumulate and the estimate diverges from the actual functional value.

On the other hand the explicit scheme is stable only when the ratio of the time step to the square of the space step is not greater than 0.5, which imposes restrictions on the number of necessary time steps. Though the explicit method is relatively easy to implement, the fully-implicit and the Crank-Nicolson methods have better stability properties.

Table 1 European Options Algorithmic parameters S0=100, K=100,r=2%,delta=1%volatility=60%,T=1

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
|  | Black Sholes | Explicit FDM | Implicit FDM and Thomas | Implicit FDM and SOR | Crank Nicolson Thomas | Crank Nicolson SOR |
| CALL | 23.727169 | 23.687471 | 23.678058 | 23.678137 | 23.697955 | 23.697974 |
| PUT | 22.742053 | 22.710984 | 22.694432 | 22.694557 | 22.714305 | 22.714461 |

Table 2 Euler's European N=500, time\_step\_size=0.00125, time domain [0,1]

|  |  |
| --- | --- |
|  | Explicit Euler’s Scheme |
| Call | 27.159944 |
| Put | 22.018020 |

Below results are obtained through implementing the explicit and implicit finite difference for American options and Explicit Euler’s Regression technique (Table 3 shows those results). Table3 shows that the three finite difference methods provide us similar values, but the regression method values differ from finite difference methods the reason is when extremely small step sizes are used, over a large number of steps the error starts to accumulate and the estimate diverges from the actual functional value so regression method tends to underperform when time steps are extremely small due to accumulating variance so when I reduce time step size in regression method say from 0.00125 to 0.01 keeping N paths constant my result matches the FDM results.

Table 3 American Options

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
|  | Regression  Method | Explicit FDM | Implicit FDM and Brennan-Scwartz | Implicit FDM and PSOR | Crank Nicolson Thomas | Crank Nicolson FDM and PSOR |
| CALL | 25.118006 | 23.686620 | 23.698196 | 23.698226 | 23.715737 | 23.715772 |
| PUT | 23.435479 | 22.701866 | 22.822386 | 22.824411 | 22.844502 | 22.845661 |

**Photocopy of my console Output:**