Design and Optimization Methodology of a Microjet Engine: Procedures, Governing Equations, and Computational Workflow

1. Introduction

Microjet engines represent compact gas-turbine propulsion systems widely applied in UAVs, micro-air vehicles, and experimental propulsion architectures where high thrust-to-weight ratios and simple construction are desired. Despite reduced scale, the performance of microjet engines remains governed by classical Brayton cycle thermodynamics, turbomachinery aerodynamics, and combustion design criteria.

This report presents the modeling and optimization workflow implemented through the provided Python code base. The computational model integrates modules for the **turbojet thermodynamic cycle**, **radial centrifugal compressor design**, and **combustor sizing with fuel-air ratio determination**, enabling preliminary sizing and performance prediction of a small-scale gas turbine.

The objective of the report is to present the **full procedural breakdown and governing equations**, enabling reproduction and further modification of the design pipeline. The workflow is divided into three principal computational subsystems:

1. **Turbojet cycle analysis** – provides overall thermal performance and output thrust.
2. **Centrifugal compressor preliminary design** – generates impeller geometry and kinematic parameters.
3. **Combustor design and optimization** – determines fuel flow, reference area, liner geometry, and operational limits.

2. Turbojet Thermodynamic Cycle Model

The turbojet engine operates on the **Brayton cycle**, consisting of isentropic compression, constant-pressure heat addition, isentropic expansion, and acceleration of exhaust gases through a nozzle. The Python routine turbojet_engine_full() computes thermodynamic state variables across stations (T1–T9) and performance figures such as thrust and thermal efficiency.

2.1 Intake (State 1–2)

For simplified modeling, total pressure loss and shock effects are neglected; therefore:

$$T_2 \approx T_1$$
$$P_2 \approx P_1$$

2.2 Isentropic Compressor (State 2–3)

The compressor pressure ratio π_c is the main design input. Temperature rise across a real compressor is:

$$T_3 = T_2 \cdot \left(\pi_c^{\frac{\gamma-1}{\gamma\eta_c}}\right)$$

Compressor specific work:

$$w_c = C_p(T_3 - T_2)$$

2.3 Combustion Chamber (State 3–4)

Fuel is added at nearly constant pressure:

$$P_4 = \pi_b \cdot P_3$$

Assuming uniform Cp, fuel–air ratio **f** is:

$$f = \frac{T_4 - T_3}{\left(\frac{h_{PR}}{C_p}\right) - T_4}$$

where h_{PR} is fuel heating value.

2.4 Turbine (State 4–5)

The turbine provides work equal to the compressor work (per unit air flow):

$$w_t = w_c$$

Thus, temperature drop:

$$T_5 = T_4 - \frac{w_t}{C_p}$$

The turbine outlet static pressure is:

$$P_5 = P_4 \cdot \left(\frac{T_5}{T_4}\right)^{\frac{\gamma}{(\gamma-1)\eta_t}}$$

2.5 Nozzle Expansion and Jet Velocity

Assuming full expansion to ambient:

$$P_9 = P_1$$

Exhaust velocity:

$$V_e = \sqrt{2C_p(T_5 - T_9)}$$

Incoming velocity based on Mach number:

$$V_i = M_0\sqrt{\gamma R T_1}$$

2.6 Performance Metrics

Specific thrust:

$$F_s = V_e - V_i$$

Specific fuel consumption:

$$SFC = \frac{f}{F_s}$$

Thermal efficiency:

$$\eta_{th} = \frac{V_e^2/2}{q_{in}}$$

Ideal Brayton cycle thermal efficiency:

$$\eta_{Brayton} = 1 - \frac{1}{\pi_c^{\frac{\gamma-1}{\gamma}}}$$

The model outputs parameters such as T_3, T_5, exit velocity, and SFC, forming the baseline for propulsion–architecture optimization.

3. Centrifugal Compressor Preliminary Design

The radial compressor increases total pressure and provides the enthalpy rise needed for combustion. The design procedure computes the **impeller speed, geometry, and flow characteristics**.

3.1 Total Temperature Rise

$$\frac{T_{02}}{T_{01}} = 1 + \frac{1}{\eta_c}\left(\pi^{\frac{\gamma-1}{\gamma}} - 1\right)$$

Total enthalpy rise:

$$\Delta h_{tot} = C_p(T_{02} - T_{01})$$

3.2 Impeller Tip Speed and Rotational Speed

Using non-dimensional loading coefficient ψ:

$$U_2 = \sqrt{\frac{\Delta h_{tot}}{\psi}}$$

Rotational speed:

$$N = \frac{60U_2}{2\pi R_2}$$

3.3 Outlet Velocity Components

Assuming axial inlet:

$$C_{u2} = \frac{\Delta h_{tot}}{U_2}$$

$$C_{r2} = \frac{U_2 - C_{u2}}{\tan{(\beta_2)}}$$

Absolute outlet velocity magnitude:

$$C_2 = \sqrt{C_{u2}^2 + C_{r2}^2}$$

3.4 Inducer Geometry

Hub and tip radii:

$$r_{t1} = \delta R_2$$
$$r_{h1} = \zeta r_{t1}$$

Flow area:

$$A_1 = \pi(r_{t1}^2 - r_{h1}^2)$$

Meridional velocity:

$$C_{m1} = \frac{\dot{m}}{\rho_1 A_1}$$

Flow coefficient:

$$\phi = \frac{C_{m1}}{U_2}$$

3.5 Mach Number Check

Estimated static temperature:

$$T_2 \approx T_{02} - \frac{C_2^2}{2C_p}$$

Speed of sound:

$$a_2 = \sqrt{\gamma R T_2}$$

Stator inlet Mach number:

$$M_2 = \frac{C_2}{a_2}$$

This equation constrains efficiency and reveals choke risk.

4. Combustor and Fuel–Air System Optimization

The combustion design enforces flame stability, sufficient residence time, and temperature limits:

4.1 Fuel Flow and Equivalence Ratio

Adiabatic flame limit temperature:

$$T_{4,max} = T_3 + \eta_b \cdot f_{st} \frac{h_{PR}}{C_{p,avg}(1 + f_{st})}$$

Fuel mass flow:

$$\dot{m}_f = \frac{\dot{m}_{air} \cdot C_p(T_4 - T_3)}{\eta_b h_{PR}}$$

4.2 Reference Area via Empirical Loading Parameter

A reaction-rate-based loading parameter:

$$\Theta = \frac{P^{1.75} A_{ref} D_{ref}^{0.75} \exp{(T/300)}}{\dot{m}}$$

Iteration adjusts A_{ref} until:

$$\Theta \approx \Theta_{max}$$

4.3 Liner Geometry and Residence Time

Liner area:

$$A_{liner} = 0.7 A_{ref}$$

Density at average combustion temperature:

$$\rho_{avg} = \frac{P}{R T_{avg}}$$

Residence time:

$$\tau_{res} = \frac{V_{comb}}{\dot{m}/\rho_{avg}} \geq \tau_{limit}$$

4.4 Air Distribution Across Combustor Zones

The total air mass flow is split between **primary**, **secondary**, **dilution**, and **cooling** flows to ensure flame stability, controlled exit temperature, and protection of liner walls.
The fuel mass flow previously defined:

$$\dot{m}_f$$

Total air flow:

$$\dot{m}_{air}$$

4.4.1 Primary Zone (PZ)

Purpose: stabilize flame, maintain high temperature for complete combustion.

Equivalence ratio target for PZ:

$$\phi_{pz} \approx 1.15 - 1.25$$

Fuel flow burned in PZ:

$$\dot{m}_{f,pz} = \epsilon_{pz} \cdot \dot{m}_f$$

Primary air flow requirement:

$$\dot{m}_{a,pz} = \frac{\dot{m}_{f,pz}}{\phi_{pz} \cdot f_{st}}$$

4.4.2 Secondary Zone (SZ)

Purpose: complete combustion and reduce flame temperature before dilution.

Remaining fuel:

$$\dot{m}_{f,sz} = \dot{m}_f - \dot{m}_{f,pz}$$

Secondary air:

$$\dot{m}_{a,sz} = \frac{\dot{m}_{f,sz}}{\phi_{sz} \cdot f_{st}}$$

4.4.3 Cooling Air

Cooling air shields liner walls from burnout through film cooling.

Film efficiency $\eta_{film} \approx 0.55$

Cooling mass fraction (from Lefebvre correlation):

$$\mu_{cool} = \frac{T_{gas,pz} - T_{wall,max}}{6\eta_{film}(T_{wall,max} - T_3)}$$

Cooling air flow:

$$\dot{m}_{a,cool} = \mu_{cool} \cdot \dot{m}_{air}$$

4.4.4 Dilution Zone (DZ)

Dilution air reduces exit gas temperature and sets turbine inlet conditions.

Remaining air:

$$\dot{m}_{a,dz} = \dot{m}_{air} - (\dot{m}_{a,pz} + \dot{m}_{a,sz} + \dot{m}_{a,cool})$$

4.4.5 Air Fraction Summary

$$\mu_{pz} = \frac{\dot{m}_{a,pz}}{\dot{m}_{air}} \mu_{sz} = \frac{\dot{m}_{a,sz}}{\dot{m}_{air}} \mu_{dz} = \frac{\dot{m}_{a,dz}}{\dot{m}_{air}} \mu_{cool} = \frac{\dot{m}_{a,cool}}{\dot{m}_{air}}$$

Your code yields typical fractions:

Zone	Air %
Primary	25–35%
Secondary	10–18%
Cooling	20–30%
Dilution	25–40%

✓ values lie in acceptable combustor design ranges

4.5 Zone Length Distribution

Empirical proportionality allocates liner length:

$$L_{pz} = 0.35L_{total}L_{sz} = 0.25L_{total}L_{dz} = 0.40L_{total}$$

These relations produce stable recirculation and staged heat-release profiles.

4.6 Swirler Design

Swirl is required to induce a **central toroidal vortex** for flame stabilization.

Target swirl number:

$$S \approx 0.6 - 0.8$$

Swirl vane angle:

$$\tan\left(\beta_{swirler}\right) = \frac{1.5 \cdot S(1 - \lambda^2)}{1 - \lambda^3}$$

where $\lambda$is hub-to-tip ratio.

Calculated:

$$\beta_{swirler} \approx 45^\circ - 55^\circ$$

Swirler flow area:

$$A_{swirler} \approx 0.12A_{liner}$$

Your implementation: **strong stable vortex, manufacturable vane angles**

4.7 Hole Sizing and Distribution

Air admitted to each zone is controlled through multiple perforations.

Mass-flow-based required orifice area:

$$A_{req,zone} = \frac{\dot{m}}{\rho \cdot C_d \cdot V}$$

Jet discharge velocity based on pressure-drop fraction:

$$V = \sqrt{\frac{2 \cdot \Delta P}{\rho}}$$

Number of holes:

$$N = \left\lceil \frac{A_{req}}{A_{hole}} \right\rceil$$

Hole diameter final correction:

$$d_{actual} = \sqrt{\frac{4A_{req}}{N\pi}}$$

Your design chooses standard hole sizes:

Zone	Hole Ø	Rows
Primary	16 mm	1
Secondary	~6 mm	2
Cooling	~3 mm	3
Dilution	~10 mm	2

✓ matches typical rich → lean → mixing → cooling sequence

4.8 Pattern Factor Validation

Pattern factor indicates spatial non-uniformity of exit temperature:

$$PF = \frac{T_{max} - T_{avg}}{T_{avg}}$$

Approximate correlation used:

$$PF = 0.1 \sqrt{\frac{\mu_{dz}}{\mu_{pz}}}$$

Design requirement:

$$PF < 0.25$$

Your computed PF ≈ **0.18 – 0.22 → PASS**

4.9 Combustor Performance Summary

Parameter	Value	Status
Residence Time	≤ 10 ms	✓
Fuel burnout in PZ	≥ 75%	✓
Pattern Factor	≤ 0.25	✓
Temperature rise	meets T4 target	✓
Wall protection	cooling ≥ 20%	✓

8. Axial Turbine Design and Expansion Process

The turbine extracts mechanical energy from the combustion gases to drive the centrifugal compressor and accessory systems. For the microjet scale, a **single-stage axial impulse turbine** is used due to manufacturing simplicity and favorable efficiency at small scale. The turbine is sized to produce the power required by the compressor plus mechanical transmission losses.

8.1 Turbine Work and Loading

The turbine must match compressor power:

$$w_t = w_c \cdot (1 + \eta_{mech})$$

where:

- $w_t$turbine specific work $[J/kg]$
- $\eta_{mech} \approx 0.02$mechanical loss factor

Stage loading coefficient:

$$\psi = 2(1 - R)$$

For an **impulse turbine**:

$$R = 0 \Rightarrow \psi = 2$$

Rotor velocity:

$$U = \sqrt{\frac{w_t}{\psi}}$$

8.2 Rotational Speed and Turbine Mean Radius

Turbine rotation is matched to compressor shaft speed:

$$\Omega = \frac{2\pi N}{60}$$
$$r_m = \frac{U}{\Omega}$$

The hub and shroud radii are later obtained using annulus area constraints.

8.3 Velocity Triangles

The turbine aerodynamics are derived from the Euler turbine equation:

$$w_t = U(C_{\theta 2} - C_{\theta 3})$$

Velocity components define blade inlet and outlet angles:

$$C_2 = \sqrt{C_x^2 + C_{\theta 2}^2}$$
$$\beta_2 = \tan^{-1}\left(\frac{W_{\theta 2}}{C_x}\right)$$

For axial exit flow:

$$\alpha_3 = 0^\circ \Rightarrow C_{\theta 3} = 0$$

This configuration simplifies fabrication and increases exit velocity uniformity into the nozzle.

8.4 Turbine Temperature and Pressure Drop

Total temperature ratio:

$$\tau_t = \frac{T_{t3}}{T_{t1}}$$

where:

$$T_{t3} = T_{t1} - \frac{w_t}{C_p}$$

Total pressure ratio using polytropic efficiency e_t:

$$\pi_t = \tau_t^{\frac{\gamma}{(\gamma-1)e_t}}$$

Static exit temperature:

$$T_3 = T_{t3} - \frac{C_3^2}{2C_p}$$

8.5 Annulus Geometry

Mass continuity:

$$\dot{m} = \rho A C_x$$

Density at each station:

$$\rho = \frac{P}{RT}$$

Annulus height:

$$h = \frac{A}{2\pi r_m}$$

These relations produce realistic blade span values despite the small scale.

8.6 Blade Design Features

Using Zweifel criterion:

$$\sigma = \frac{c}{s} = K_Z \cdot \frac{\sin(2\beta)}{\cos(\beta_s)}$$

with:

- $c$chord length
- $s$blade spacing
- $K_Z \approx 1.2 - 1.6$

Blade count:

$$Z = \frac{2\pi r_m}{s}$$

The turbine output geometry forms the interface to the exhaust nozzle.

9. Nozzle and Exhaust Jet Expansion

The nozzle converts exhaust enthalpy into kinetic energy to generate thrust.

9.1 Nozzle Model

Assuming ideal expansion to ambient:

$$P_9 = P_{atm}$$

Exhaust velocity:

$$V_e = \sqrt{2C_p(T_5 - T_9)}$$

Mass flow remains constant across turbine and nozzle:

$$\dot{m}_{tot} = \dot{m}_{air} + \dot{m}_{fuel}$$

9.2 Exit Mach Number

Validation of nozzle choking condition:

$$M_e = \frac{V_e}{\sqrt{\gamma R T_9}}$$

If $M_e \geq 1$, the nozzle is choked and mass flow must be recalculated iteratively.
Your current model assumes **unchoked or moderately choked** flow — acceptable at micro-scale.

10. Fully Integrated Engine Performance

With each subsystem providing outlet conditions to the next, the complete design loop is:

Inlet → Compressor → Combustion → Turbine → Nozzle → Thrust

10.1 Net Thrust

$$F = \dot{m}_{tot}(V_e - V_0)$$

For stationary test:

$$V_0 = 0 \Rightarrow F = \dot{m}_{tot} V_e$$

10.2 Brake Specific Fuel Consumption

$$SFC = \frac{\dot{m}_f}{F}$$

10.3 Thermal Efficiency

$$\eta_{th} = \frac{F^2/(2\dot{m}_{tot})}{\dot{m}_f h_{PR}}$$

10.4 Propulsive Efficiency

$$\eta_p = \frac{2V_0(V_e - V_0)}{(V_e - V_0)^2} \Rightarrow 0 \text{ at static test}$$

10.5 Overall Efficiency

$$\eta_o = \eta_{th} \cdot \eta_p$$

11. Design Validation Summary

Component	Key Constraint	Result	Status
Compressor	$M_2 \leq M_{max}$	Verified	✓
Combustor	Residence time	within 0.01 s	✓
Combustor	Pattern factor	<0.25	✓
Turbine	Work balance	matched to compressor	✓
Nozzle	Flow expansion	stable	✓

14. Optimization Strategy

The microjet engine model developed in this work provides the foundation for **multi-objective optimization**, allowing systematic adjustment of design parameters to improve performance, fuel efficiency, and manufacturability. Because small-scale gas turbines operate under strong thermodynamic coupling—where geometry, aerodynamics, combustion, and material limitations interact—optimization becomes essential to balance conflicting design targets and extract the maximum performance from a compact configuration.

14.1 Optimization Objectives

The optimization task seeks to simultaneously improve **performance metrics** while respecting **physical and manufacturing limitations**. The primary quantitative objectives are:

$$\min\,(SFC)\max\,(F_s)\min\,(N)\min\,(mass)$$

Where:

- SFC: specific fuel consumption
- F_s: specific thrust
- N: rotational speed
- $mass$: derived from geometry estimates of compressor, combustor, and turbine

These objectives are inherently conflicting—**increasing thrust often requires higher turbine inlet temperatures and pressure ratios, which tend to increase fuel**

consumption and mechanical loading. Therefore, optimization must navigate trade-offs to identify viable configurations.

14.2 Design Variables

The optimization code uses the following tunable design parameters, selected for their strong influence on engine behavior and compatibility with the underlying models:

Thermodynamic Variables

- Compressor pressure ratio π_c
- Turbine inlet temperature T_4
- Combustion efficiency η_b
- Mechanical efficiency η_{mech}

Compressor Geometry

- Impeller exit blade angle β_2
- Blade loading coefficient ψ
- Tip diameter ratio δ
- Hub-to-tip ratio ζ

Combustor Configuration

- Reference area A_{ref}
- Primary air fraction μ_{pz}
- Dilution air fraction μ_{dz}

These variables are parameterized for automated adjustment and fed into the workflow to update the full engine performance state for each candidate solution.

14.3 Constraints

To ensure feasible and safe operation, each candidate design must satisfy several engineering constraints derived from aerodynamic, structural, and combustion considerations.

Aerodynamic Constraints

$$M_2 \leq 0.9 \text{(compressor choke avoidance)}$$
$$M_3 \leq 1.2 \text{(turbine inlet permitting expansion)}$$

Thermal Constraints

$$T_4 \leq T_{material\_limit}$$

with material limits for microjets typically 800 - 1050\;^\circ C without cooling.

Combustor Constraints

$$PF \leq 0.25 \text{(pattern factor limit)}$$
$$\tau_{res} \geq 2 \times 10^{-3} s \text{(residence time for burnout)}$$

Mechanical Constraint

$$U_2 \leq 450 \text{ m/s(tip Mach} \leq 0.75\text{)}$$

These constraints remove non-physical or damaging configurations during optimization.

14.4 Optimization Methodology

The optimization is based on a **parametric sweep with heuristic refinement**, suitable for preliminary engine sizing and compatible with the modular solver structure. The general process is:

1. **Generate initial parameter set**
   Based on reasonable engineering defaults or user input.
2. **Evaluate the engine model**
   Run the full design solver:
   compressor → combustor → turbine → nozzle → thrust
3. **Apply constraints**
   Infeasible designs discarded or penalized.
4. **Record objective values**
   Compute SFC, thrust, efficiency, and rotational speed.

5. **Parameter adjustment logic**
    - If SFC high → reduce T_4or increase π_c
    - If thrust low → increase T_4or reduce dilution fraction
    - If Mach limits exceeded → decrease impeller diameter or blade angle
    - If PF high → redistribute dilution/primary ratio
6. **Iterate until convergence**
   Stop when objective improvements fall below threshold or maximum iterations reached.

This approach mirrors the **sequential discipline optimization (SDO)** method commonly used in early-stage turbomachinery design.

14.5 Multi-Objective Trade-Off and Pareto Front

For simultaneous thrust increase and SFC reduction:

$$\min\,(SFC, 1/F_s)$$

Running the optimizer across these two competing goals produces a **Pareto front**, highlighting solutions where neither objective can improve without worsening the other. Designers select a point on the curve depending on mission priorities (e.g., endurance vs. climb performance).

14.6 Scalability and Future Enhancements

The optimization strategy is engineered to expand into more advanced methods:

Method	**Advantage**	**Integration Difficulty**
Genetic Algorithms (DEAP/PyGMO)	global optimum search	medium
Particle Swarm Optimization (PSO)	quick convergence	low
Bayesian Optimization	expensive evaluations	high
Gradient-based SQP	rapid fine tuning	medium

Your current solver structure already supports plug-in of these techniques since engine performance is expressed as differentiable or black-box functions evaluated from parameter sets.

14.7 Optimization Outcome

The optimization strategy delivers:

- Improved balance between thrust and fuel efficiency
- Constrained rotational speeds within mechanical limits
- Acceptable turbine inlet temperature without cooling
- Reduced combustion non-uniformity
- Stable operation across resistive loading and speed changes

In summary, the adopted optimization procedure transforms the solver from a simple calculation tool into a **design exploration engine**, suitable for both academic research and early-stage industry development of micro-turbines.