Percents

The word percent means "hundredth". The symbol "%" is used to express the word percent. For example, "17 percent" means "17 hundredths" and can be written with a % symbol, as a fraction, or as a decimal:

$$17\% = \frac{17}{100} = 0.17.$$

To convert a percent to a decimal, or a percent to a fraction, follow these rules:

1. To convert a percent to a decimal, drop the % symbol and move the decimal point two places to the left, adding 0's if necessary. (Remember: it is assumed that there is a decimal point to the right of any whole number).

2. To convert a percent to a fraction, drop the % symbol, write the number over 100, and reduce.

$$25\% = 0.25 = \frac{25}{100} = \frac{1}{4} \qquad 100\% = 1.00 = \frac{100}{100}$$

$$12.5\% = 0.125 = \frac{12.5}{100} = \frac{125}{1000} = \frac{1}{8}$$

$$1\% = 0.01 = \frac{1}{100} \qquad \frac{1}{2}\% = 0.5\% = 0.005 = \frac{0.5}{100} = \frac{1}{200}$$

$$250\% = 2.50 = \frac{250}{100} = \frac{5}{2}$$

To convert a decimal to a percent, or a fraction to a percent, follow these rules:

1. To convert a decimal to a percent, move the decimal point two places to the right, adding 0's if necessary, and add the % symbol.

2. To convert a fraction to a percent, first convert the fraction to a decimal, then do step 1.

$0.375 = 37.5\%$ $0.3 = 30\%$ $1.25 = 125\%$ $10 = 1000\%$

$\frac{3}{4} = 0.75 = 75\%$ $\frac{1}{3} = 0.333333\ldots = 33.333\ldots\% = 33\frac{1}{3}\%$

$\frac{1}{5} = 0.2 = 20\%$

Solving Percent problems

Now, consider these three questions:
(i) What is 45% of 200?
(ii) 90 is 45% of what number?
(iii) 90 is what percent of 200?
In each case, there is one unknown; call it x. Now, just translate each sentence, replacing "is" by "=" and the unknown by x.

(1) $x=\frac{45}{100}\times 200=90$

(2) $90=\frac{45}{100}\times x$

$90=0.45x$

$x=\frac{90}{0.45}=200$

(3) $90=\frac{x}{100}\times 200$

$90=2x$

$x=\frac{90}{2}=45$

Many students have been taught to answer questions such as these by writing the proportion $\frac{\text{is}}{\text{of}}=\frac{\text{percent}}{100}$.
To use this method, think of is, of, and percent as variables. In each percent problem you are given two variables and asked to find the third, which you label x. Of course, you then solve the equation by cross-multiplying.

For example, the three problems solved above could be handled as follows:
(i) What is 45% of 200? (Let x = the is number).
$\frac{x}{200}=\frac{45}{100}\Rightarrow 100x=45(200)=9000\Rightarrow x=90.$
(ii) 90 is 45% of what number? (Let x = the of number).
$\frac{90}{x}=\frac{45}{100}\Rightarrow 9000=45x\Rightarrow x=200.$
(iii) 90 is what % of 200 ? (Let x = the *percent*).
$\frac{90}{200}=\frac{x}{100}\Rightarrow 200x=9000\Rightarrow x=45.$

Example:
Brian gave 20% of his baseball cards to Scott and 15% to Adam. If he still had 520 cards. How many did he have originally ?
Solution:
Originally, Brian had 100% of the cards (all of them). After he gave away 35% of them, he had 100% – 35% = 65% of them left. Then 520 is 65% of what number?
$520=0.65x \Rightarrow x=520\div 0.65=800$
Example:
After Michael gave 110 baseball cards to Sally and 75 to Heidi, he had 315 left. What percent of his cards did Michael give away?
Solution:
Michael gave away a total of 185 cards and had 315 left. Therefore, he started with 185 + 315 = 500 cards. Then 185 is what percent of 500?
$185=\frac{x}{100}(500) \Rightarrow 5x=185 \Rightarrow x=185\div 5=37$
Michael gave away 37% of his cards.

° Since percent means "hundredth", the easiest number to use in any percent problem is 100

$a\%\ of\ 100 = \frac{a}{100}(100) = a$

In any problem involving percents, use the number 100.

Example:

In 1970 the populations of town A and town B were the same. From 1970 to 1980, however, the population of town A increased by 60% while the population of town B decreased by 60%. In 1980, population of town B was what percent of the population of town A ?

(A) 25% (B) 36% (C) 40% (D) 60% (E) 120%

Solution:

In your math class, you would let x be the population of town A in 1970 and then proceed to set up an algebra problem. *Don't do that on the SAT.* Assume that the populations of both towns were 100 in 1970. Then, since 60% of 100 is 60, in 1980 the populations were 100 + 60 = 160 (town A) and 100 - 60 = 40 (town B). Then, in 1980, town B's population was $\frac{40}{160} = \frac{1}{4} = 25\%$ of town A's. Choice A is correct.

Percent Increase and Decrease

The percent increase of a quantity is

$$\frac{actual\ \ increase}{original\ amount} \times 100\%$$

The *percent decrease* of a quantity is

$$\frac{actual\ \ decrease}{original\ amount} \times 100\%$$

For example:

° If the price of a DVD player rises from $80 to $100, the actual increase is $20, and the percent increase is

$$\frac{20}{80} \times 100\% = \frac{1}{4} \times 100\% = 25\%$$

° If a $100 DVD player is on sale for $80, the actual decrease in price is $20, and the percent decrease is

$$\frac{20}{100} \times 100\% = 20\%$$

° Note that the percent increase in going from 80 to 100 is not the same as the percent decrease in going from 100 to 80.

° To increase a number by k%, multiply it by (1+ k%); to decrease a number by k%, multiply it by (1- k%).

For example:

° The value of a $1600 investment after a 25% increase is $1600 (1 + 25%) = $1600 (1.25) = $2000.

° If the investment then loses 25% of its value, it is worth $2000(1-25%) = $2000 (0.75) = $1500.

Note that, after a 25% increase followed by a 25% decrease, the value is $1500, $100 less than the original amount.

° A decrease of a% followed by a decrease of b% always results in a smaller decrease than a single decrease of (a + b)%. Similarly, an increase of a% followed by an increase of b% always results in a larger increase than a single increase of (a + b)%. In particular, an increase (or decrease) of a% followed by another increase (or decrease) of a% is *never* the same as a single increase (or decrease) of 2 a%.

Example:
Bill and George were each hired in January at the same salary. Bill got two raises of 10%, one in May and the other in October. George received only one raise, in November. If, after George received his raise, he and Bill had identical salaries, by what percent was George's salary raised?
Solution:
Since this is a percent problem, assume both starting salaries were $100. First, in May, Bill's salary rose 10% to $110. Later, in October, his salary again rose 10%. Since 10% of $110 is $11, his final salary was 121. George's one raise brought his salary from $100 to $121, the same as Bill's salary. George had an actual increase of $21 and a percent increase of $\overline{BC}$.
Example:
In January, the value of a stock increased by 25%; and in February, it decreased by 20%. How did the value of the stock at the end of February compare with its value at the beginning of January?

(a)- It was less.

(b)- It was the same.

(c)- It was 5% greater.

(d)- It was more than 5% greater.

(e)- It depends on the value of the stock.

Solution:
assume that at the beginning of January the stock was worth $100. Then at the end of January it was worth $125. Since 20% of 125 is 25, during February its value decreased from $125 to $100. The value of the stock was the same at the end of February as at the beginning of January (B).
Example:
From 2003 to 2004, the number of applicants to a college increased 15% to 5060. How many applicants were there in 2003?
(A) 459 (B) 4301 (C) 4400 (D) 5819 (E) 5953
Solution:
The number of applicants in 2003 was 5060 ÷ 1.15 = 4400 (C).

Finding an original amount after a percent change

If you know the number that results after a given number is increased by P%, you can find the original number by dividing the new amount by 1+P%.

$$\text{Original amount} = \frac{\text{New amount after an increase of P\%}}{1+\frac{P}{100}}$$

Similarly, if you know the number that results after a given number is decreased by P%, you can find the original number by dividing the new amount by 1- P%.

$$\text{Original amount} = \frac{\text{New amount after an increase of P\%}}{1-\frac{P}{100}}$$

The Quadratic Formula

If you need to find the solutions to quadratic equation, you may find it easier to use the quadratic formula

if $ax^2 + bx + c = 0$, then by:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}, \text{ where } a \neq 0$$

Example

What are the solutions to $y^2 + 6y + 7 = 0$?

Solution

Since the quadratic trinomial $y^2 + 6y + 7$ is not factorable, solve by either completing the square or by using the quadratic formula:

METHOD 1: Rewrite the equation as $y^2 + 6y = -7$ and solve it by completing the square.

- Add $\left(\frac{6}{2}\right)^2 = 9$ to both sides of the equation:

$$y^2 + 6y + 9 = -7 + 9 \text{ so } (y+3)^2 = 2$$

- Take the square root of both sides of the equation:

$$y + 3 = \pm\sqrt{2}$$

- Solve for y:

$$y = -3 + \sqrt{2} \quad \text{or} \quad y = -3 - \sqrt{2}$$

METHOD 2: Solve using the quadratic formula where $a = 1$, $b = 6$ and $c = 7$:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$
$$= \frac{-6 \pm \sqrt{6^2 - 4(1)(7)}}{2(1)}$$
$$= \frac{-6 \pm \sqrt{36 - 28}}{2}$$
$$= \frac{-6 \pm \sqrt{8}}{2}$$
$$= \frac{-6 \pm 2\sqrt{2}}{2}$$
$$= \frac{2\left(-3 \pm \sqrt{2}\right)}{2}$$
$$= -3 \pm \sqrt{2}$$

1 College Road

Which of the following is equal to $\frac{1}{10}\%$?

(A) 10
(B) 1
(C) 0.1
(D) 0.001

2 College Road

20 percent of 5 is 50 percent of what number?

(A) 40
(B) 20
(C) 10
(D) 2

3 College Road

If 0.03 percent of n is 3, what is 3 percent of n?

(A) 900
(B) 600
(C) 300
(D) 0.006

4 College Road

If 30 percent of m is 40, what is 15 percent of m?

(A) 15
(B) 20
(C) 25
(D) 30

5 College Road

If 20 percent of 100 is equal to 500 percent of n, then n is equal to what number?

7 College Road

Which of the following must be equal to 30 percent of x?

(A) $\frac{3x}{1{,}000}$

(B) $\frac{3x}{100}$

(C) $\frac{3x}{10}$

(D) 3x

6 College Road

If y% of 50 is 32, then what is 200% of y?

(A) 16

(B) 32

(C) 64

(D) 128

8 College Road

If x% of 30 is 12, what is 4x% of 15?

(A) 6

(B) 12

(C) 18

(D) 24

9 College Road

If n is positive and $\frac{50}{n}$ equals n percent of 50, what is the value of n?

(A) 1
(B) 5
(C) 10
(D) 50

10 College Road

If 7.5 is x percent of 75, what is x percent of 10 ?

(A) 10
(B) 1
(C) 0.75
(D) 0.1

11 College Road

Which expression is equivalent to $\frac{2}{5}\%$?

(A) 0.40
(B) 0.04
(C) 0.004
(D) 0.0004

12 College Road

If $x > 0$, then 2 percent of 5 percent of $3x$ equals what percent of x ?

(A) 0.03%
(B) 0.3%
(C) 0.6%
(D) 3%

13 College Road

The sum of 25% of 32 and 40% of 15 is what percent of 35?

(A) 20%
(B) 25%
(C) 30%
(D) 40%

14 College Road

If 25% of x is 12.5, what is 12.5% of 2x?

(A) 6.25
(B) 12.5
(C) 25
(D) 37.5

15 College Road

If 75 percent of m is equal to k percent of 25, where $k > 0$, what is the value of $\frac{m}{k}$?

(A) $\frac{3}{16}$
(B) $\frac{1}{3}$
(C) $\frac{3}{4}$
(D) 3

16 College Road

If $\frac{1}{8}$ of a number is 9, what is 75% of the same number?

(A) 36
(B) 48
(C) 54
(D) 72

17 College Road

What percent of the integers from 3 to 12, inclusive, are neither primes nor multiples of 4 ?

(A) 20%
(B) 30%
(C) 40%
(D) 60%

19 College Road

If 30 percent of m is 40, what is 15 percent of m ?

(A) 15
(B) 20
(C) 25
(D) 30

18 College Road

Three percent of 4,200 is equal to 6 percent of what number?

(A) 8,400
(B) 2,100
(C) 1,260
(D) 252

20 College Road

Hao and Mike shared a box of popcorn. If they ate all the popcorn in the box and Mike ate three times as much popcorn as Hao, what percent of the popcorn did Hao eat?

(A) 20%

(B) 25%

(C) 30%

(D) $33\frac{1}{3}\%$

1 College Road

If 48 of the 60 seats on a bus were occupied, what percent of the seats were not occupied?

(A) 12%
(B) 20%
(C) 25%
(D) 60%

2 College Road

Bob took 20 math tests last year. If he failed 6 of them, what percent of the math tests did he pass?

(A) $37\frac{1}{2}\%$
(B) 50%
(C) $62\frac{1}{2}\%$
(D) 70%

3 College Road

In a certain box of gloves, 12 pairs are size 7 and 24 pairs are size 6. If all the gloves in the box are either size 6 or size 7, what percent of the gloves in the box are size 6?

(A) $33\frac{1}{3}\%$
(B) 50%
(C) $66\frac{2}{3}\%$
(D) 75%

4 College Road

A closet contains 24 pairs of shoes. If 25 percent of those pairs of shoes are black, how many pairs are NOT black?

(A) 4
(B) 6
(C) 12
(D) 18

5 College Road

A word-processing operator typed 44 words per minute. After practice, the operator's speed increased to 55 words per minute. By what percent did the operator's speed increase?

(A) 10%
(B) 11%
(C) 20%
(D) 25%

6 College Road

What is the percent discount on a jacket marked down from \$120 to \$100?

(A) $16\frac{2}{3}\%$
(B) 20%
(C) 30%
(D) $33\frac{1}{3}\%$

7 College Road

In a certain class, if Edie's average rose from 72 to 84, by what percent did her average increase?

(A) 12%
(B) $14\frac{2}{7}\%$
(C) $16\frac{2}{3}\%$
(D) $66\frac{2}{3}\%$

8 College Road

The price of a newspaper rises from 5 cents to 15 cents. What is the percent increase in price?

(A) 50%
(B) 75%
(C) 100%
(D) 200%

9 College Road

If the value of a certain building increased from \$1.6 million to \$2.0 million, what was the percent increase in the value?

(A) 4%
(B) 8%
(C) 16%
(D) 25%

10 College Road

Which of the following is equivalent to $\frac{1}{2}$ of 23 percent of 618 ?

(A) 23% of 309

(B) 23% of $\frac{309}{2}$

(C) $22\frac{1}{2}$% of 618

(D) $\frac{23}{2}$% of 309

11 College Road

A big-screen TV is on sale at 15% off the regular price. If the regular price of the TV is \$420, what is the sale price?

(A) \$63
(B) \$126
(C) \$357
(D) \$405

12 College Road

In a class of 720 students, 35% are boys. How many girls are in the class?

13 College Road

If 25 students took an exam and 4 of them failed, what percent of them passed?

14 College Road

A company produced 300 appliances in the first week of the month. Because it received additional machinery, its production increased 30 percent from the first week to the second week. How many appliances did the company produce the second week?

15 College Road

In a movie theater, 480 of the 500 seats were occupied. What percent of the seats were NOT occupied?

(A) 0.4%
(B) 2%
(C) 4%
(D) 20%

16 College Road

If 20% of a number is 8, what is 25% of the number?

(A) 2
(B) 10
(C) 12
(D) 14

17 College Road

If 65 percent of x is 195, what is 75 percent of x ?

(A) 215
(B) 225
(C) 235
(D) 250

18 College Road

75 percent of 88 is the same as 60 percent of what number?

(A) 100
(B) 103
(C) 105
(D) 110

19 College Road

If $n > 0$, which of the following is equivalent to $\frac{n}{5}$?

(A) 5% of n
(B) 20% of n
(C) 25% of n
(D) 50% of n

20 College Road

What percent of 50 is 6 ?

(A) 3%
(B) $8\frac{1}{3}\%$
(C) 12%
(D) 30%

1 College Road

36 percent of 18 is 18 percent of what number?

(A) 9
(B) 24
(C) 36
(D) 40

2 College Road

$\frac{4}{5}$ is what percent of $\frac{1}{5}$?

(A) 20%
(B) 25%
(C) 80%
(D) 400%

3 College Road

What percent of 4 is 5 ?

(A) 75%
(B) 80%
(C) 125%
(D) 150%

4 College Road

If $x > 0$, what is 30 percent of $15x$?

(A) $0.5x$
(B) $2x$
(C) $4.5x$
(D) $5x$

5 College Road

30% of 150 equals 4.5% of

(A) 10
(B) 100
(C) 250
(D) 1000

6 College Road

If x is a positive number, then 50 percent of $10x$ equals

(A) $2x$
(B) $4x$
(C) $5x$
(D) $20x$

7 College Road

What percent of 50 is b?

(A) $\frac{b}{50}$ (B) $\frac{b}{2}$ (C) $\frac{50}{b}$ (D) $\frac{2}{b}$

8 College Road

What is 10 percent of 20 percent of 30 ?

9 College Road

9 is $\frac{1}{3}$% of what number?

(A) 0.03 (B) .27 (C) 3 (D) 2700

11 College Road

What is 10% of 20% of 30%?

(A) 0.006% (B) 0.6% (C) 6% (D) 60%

10 College Road

$\frac{1}{2}$ is what percent of $\frac{1}{5}$?

(A) 250%
(B) 210%
(C) 140%
(D) 40%

12 College Road

What percent of 800 is 5?

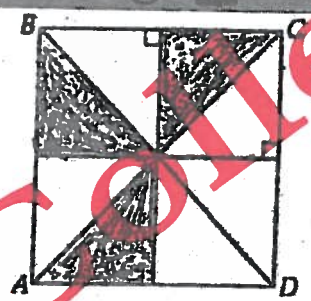
(A) $\frac{1}{160}$%
(B) $\frac{5}{8}$%
(C) 1.6%
(D) 62.5%

13 College Road

What percent of 5 is 8?

(A) 50%
(B) 63%
(C) 125%
(D) 160%

14 College Road



In the figure above, $ABCD$ is a square. What percent of the square is shaded?

(A) 25%

(B) $33\frac{1}{3}\%$

(C) $37\frac{1}{2}\%$

(D) 40%

15 College Road

20 percent of 5 is 50 percent of what number?

(A) 40
(B) 20
(C) 10
(D) 2

16 College Road

If a is a positive number, 400% of a is what percent of $400a$?

(A) 0.01 (B) 0.1 (C) 1 (D) 10

17 College Road

A box contains 2,900 solid-colored marbles that are either orange, blue, or green. If 29 percent of the marbles are orange and 29 percent of the marbles are blue, what percent are green?

(A) 29%
(B) 42%
(C) 52%
(D) 58%

18 College Road

A box of staples contains 4,600 staples that are either silver, black, or red. If 46 percent of the staples are silver and 46 percent are black, what percent are red?

(A) 8%
(B) 18%
(C) 46%
(D) 54%

19 College Road

Which of the following is equal in value to 1 plus (100 percent of 1)?

(A) 100 percent of 1
(B) 101 percent of 1
(C) 110 percent of 1
(D) 200 percent of 1

20 College Road

Which of the following is equivalent to $\frac{1}{2}$ of 23 percent of 618?

(A) 23% of 309

(B) 23% of $\frac{309}{2}$

(C) $22\frac{1}{2}$% of 618

(D) $\frac{23}{2}$% of 309

1 College Road

Minimum Age Requirement (years)	Number of States
14	7
15	12
16	28
17	2
18	2

The table above shows the minimum age requirement for obtaining a driver's license. In what percent of the states can a person obtain a driver's license before the age of 16?

(A) 94%
(B) 47%
(C) 38%
(D) 19%

2 College Road

At the beginning of 1999, the population of Rockville was 204,000 and the population of Springfield was 216,000. If the population of each city increased by exactly 20% in 1999, how many more people lived in Springfield than in Rockville at the end of 1999?

(A) 9,600
(B) 10,000
(C) 12,000
(D) 14,400

3 College Road

Of the students at South High, $\frac{1}{3}$ are seniors. Of the seniors, $\frac{3}{4}$ will go to college next year. What percent of the students at South High will go to college next year?

(A) 20
(B) 25
(C) $33\frac{1}{3}$
(D) 50

4 College Road

Hao and Mike shared a box of popcorn. If they ate all the popcorn in the box and Mike ate three times as much popcorn as Hao, what percent of the popcorn did Hao eat?

(A) 20%
(B) 25%
(C) 30%
(D) $33\frac{1}{3}\%$

5 College Road

A salesman's monthly gross pay consists of $1,200 plus 20 percent of the dollar amount of his sales. If his gross pay for one month was $2,500, what was the dollar amount of his sales for that month? (Disregard the $ sign when gridding your answer.)

6 College Road

A survey found that 80 percent of the apartments in City G have smoke alarms installed. Of these, 20 percent have smoke alarms that are not working. What percent of the apartments in City G were found to have working smoke alarms?

(A) 60%

(B) 64%

(C) $66\frac{2}{3}\%$

(D) 70%

7 College Road

In 1997 it was predicted that in the year 2020 the total school-age population of Country X will be approximately 42 million. This represents a 20 to 25 percent increase from the 1997 school-age population. Which of the following could have been the 1997 school-age population of Country X?

(A) 20 million
(B) 24 million
(C) 28 million
(D) 34 million

8 College Road

The price of a telephone was first increased by 10 percent and then the new price was decreased by 25 percent. The final price was what percent of the initial price?

(A) 78%
(B) 80%
(C) 82.5%
(D) 85%

9 College Road

At a sale, Lisa bought a sweater that was 25% off the original price. Not including sales tax, she paid $18 for the sweater. What was the original price of the sweater?

(A) $36
(B) $52
(C) $56
(D) $64

10 College Road

A class of 30 girls and 40 boys organized a hayride. If 60 percent of the girls and 25 percent of the boys went on the ride, what percent of the class went on the ride?

(A) 30%
(B) 35%
(C) 40%
(D) 50%

11 College Road

In a certain election, 60 percent of those who voted were females. If 8,000 males voted, what was the total number of people who voted in the election?

(A) 12,800
(B) 14,000
(C) 18,600
(D) 20,000

12 College Road

What number exceeds 50 percent of itself by 10 ?

(A) 5
(B) 10
(C) 15
(D) 20

13 College Road

A salesperson's commission is k percent of the selling price of a car. Which of the following represents the commission, in dollars, on 2 cars that sold for $14,000 each?

(A) $280k$

(B) $7{,}000k$

(C) $28{,}000k$

(D) $\frac{14{,}000}{100 + 2k}$

14 College Road

State University plans on accepting a total of 1,000 students for next year's class. Of the 800 students accepted so far, 60 percent are female and 40 percent are male. How many of the remaining students to be accepted must be male in order for half of the total number of students accepted to be male?

(A) 100
(B) 120
(C) 160
(D) 180

15 College Road

A shipment of coal is loaded into 5 railroad cars, each having the same weight capacity. The first 4 cars are loaded to their weight capacity and the fifth car is loaded to 60 percent of its weight capacity. If the total weight of the shipment is 368 tons, how many tons of coal are loaded into the fifth car?

16 College Road

On a certain day, 85 men and 155 women each made purchases at a department store. If 40 percent of these people purchased clothing and if 42 of those who purchased clothing were men, how many women purchased clothing?

(A) 42
(B) 54
(C) 62
(D) 82