1) $e^y \sin x \frac{dy}{dx} + (1+e^y)\cos x = Zero$

"Soul_"

* بالقسمة ع $(1+e^y)\sin x$

$$\frac{e^y}{(1+e^y)} \cdot \frac{dy}{dx} + \cot x = Zero$$

* فصل متغيرات.

$$\frac{e^y}{(1+e^y)} dy + \cot x \, dx = Zero$$

* بتكامل الطرفين

$$\int \frac{e^y}{(1+e^y)} dy + \int \cot x \, dx = \ln(c)$$

$$\ln|1+e^y| + \ln|\sin x| = \ln(c)$$

$$\ln (1+e^y)\sin x = c$$

$$\therefore (1+e^y)\sin x = c$$

$$e^y = \frac{c - \sin x}{\sin x}$$

$$2)\ (y^2 - x^2)\frac{dy}{dx} + 2xy = 0$$

((Soul))

$$\left(\frac{y^2}{x^2} - 1\right)\cdot\frac{dy}{dx} + \frac{2y}{x} = \text{Zero} \longrightarrow (1)$$ x^2 بالقسمة ع *

* معادلة متجانسة

* نفرض أن

$$y' = V + V'x \longleftarrow y = Vx$$

* بالتعويض عن (1)

$$\left(\frac{V^2\cancel{x^2}}{\cancel{x^2}} - 1\right)(V + V'x) + 2V = \text{Zero}$$

$$(V^2 - 1)(V + V'x) + 2V = \text{Zero}$$

$$(V^2 - 1)(V + V'x) = -2V$$

$$V + V'x = \frac{-2V}{(V^2-1)} \longrightarrow V'x = \frac{-2V}{(V^2-1)} - V$$

$$x\frac{dV}{dx} = V'x = \frac{-2V - V^3 + V}{(V^2-1)}$$

$$= \frac{-(V^3+V)}{(V^2-1)} = \frac{-V(V^2+1)}{(V^2-1)}$$

$$\frac{dx}{x} = \frac{(V^2-1)}{-(V^3+V)}dV = \left(\frac{1}{V} + \frac{-2V}{(V^2+1)}\right)dV$$

$$\frac{V^2-1}{-V(V^2+1)} = \frac{A}{-V} + \frac{BV+C}{V^2+1} = \frac{1}{V} + \frac{-2V}{V^2+1}$$

Let $V=0 \longrightarrow A=-1$

Let $V=1 \longrightarrow B+C=-2 \ (1)$

Let $V=-1 \longrightarrow -B+C=2 \ (2)$

$B=-2$ & $C=0$

* بتكامل الطرفين

$$\int \frac{dx}{x} = \int \frac{dV}{V} + \int \frac{-2V\,dV}{(V^2+1)}$$

$$\ln(x) + \ln(c) = \ln(V) - \ln(V^2+1)$$

$$\ln(xc) = \ln \frac{V}{(V^2+1)}$$

$$\therefore xc = \frac{V}{(V^2+1)}$$

$$\frac{1}{V}(V^2+1) = \frac{1}{xc}$$

$$V + \frac{1}{V} = \frac{1}{xc} \longrightarrow \frac{y}{x} + \frac{x}{y} = \frac{1}{xc}$$

* بالضرب * (xy)

$$y^2 + x^2 = \frac{y}{c}$$

$$\therefore c = \frac{y}{y^2+x^2}$$

3) $(x+1)\frac{dy}{dx} - 3y = (x+1)^5$

((sol —))

$(x+1)\,dy - 3y - (x+1)^5\,dx = \text{Zero} \longrightarrow ①$

$M = -3y - (x+1)^5 \qquad N = x+1$

$M_y = -3 \qquad \neq \qquad N_x = 1$

* معادلة غير تامة

$\frac{M_y - N_x}{N} = \frac{-4}{x+1} = f(x)$

$\therefore P = e^{\int f(x)\,dx} = e^{-4\int \frac{dx}{x+1}} = e^{-4\ln(x+1)}$

$= e^{\ln(x+1)^{-4}} = (x+1)^{-4}$

* بضرب المعادلة ① في P

$(x+1)^{-3}\,dy + (-3y)(x+1)^{-4} - (x+1)\,dx = \text{Zero}$

$M = -3y(x+1)^{-4} - (x+1) \qquad N = (x+1)^{-3}$

$M_y = -3(x+1)^{-4} \qquad N_x = -3(x+1)^{-4}$

∴ المعادلة تامة

$\int_x M\,dx = \int_x -3y(x+1)^{-4}\,dx - \int_x x\,dx - \int_x dx$

$$= y(x+1)^{-3} - \frac{1}{2}x^2 - x$$

$$\int_y N\,dy = \int_y (x+1)^{-3}\,dy$$

$$= y(x+1)^{-3}$$

$$\therefore\; y(x+1)^{-3} - \frac{1}{2}x^2 - x = C$$

4) $x^2 \frac{dy}{dx} + xy = y^2 \longrightarrow (*)$

((Soul-))

* بقسمة المعادلة (*) ع x^2

$$\therefore \frac{dy}{dx} + \frac{y}{x} = \frac{y^2}{x^2} \longrightarrow (1)$$

* معادلة برنولي

$n = 2$

نفرض أن

$$u = y^{-1} \longrightarrow u' = -y^{-2} y'$$

* بالتعويض في المعادلة (1)

$$\frac{-u'}{y^{-2}} + \frac{y}{x} = \frac{y^2}{x^2} \longrightarrow u' - \frac{u}{x} = \frac{-1}{x^2}$$

* معادلة خطية في (y)

$$S = e^{\int f(x)\,dx} = e^{-\int \frac{dx}{x}} = e^{-\ln(x)} = x^{-1}$$

$$Sy = \int S\,Q(x)\,dx = \int -x^{-1} \cdot x^{-2}\,dx$$

$$= \int x^{-3}\,dx = \frac{1}{2} x^{-2} + C$$

$$\therefore y = \frac{1}{2} x^{-1} + xC$$

5) $\frac{dy}{dx} = \frac{x+3y-5}{3x+y+1}$

((Sowl -))

$\because a_1 b_2 \neq a_2 b_1$

∴ الخطان متقاطعان

$x+3y-5 = Zero \longrightarrow ①$

$3x+y+1 = Zero \longrightarrow ②$

* بحل ① و ② لوجد النقطة التقاطع (α, β) هم $(-1, 2)$

* نفرض أن :- $x = X + \alpha$ و $y = Y + \beta$

$x = X - 1$ و $y = Y + 2$

$$\frac{dy}{dx} = \frac{dY}{dX}$$

* بالتعويض :

$$\frac{dY}{dX} = \frac{X-1+3Y+6-5}{3X-3+Y+2+1} = \frac{X+3Y}{3X+Y}$$

* المعادلة متجانسة

* نفرض أن :- $Y = VX \longrightarrow Y' = V+V'X$

$$V'X+V = \frac{X+3V\cancel{X}}{3X+VX} = \frac{1+3V}{3+V}$$

$$XV' = \frac{1+3V}{3+V} - V = \frac{1+\cancel{3V}-\cancel{3V}-V^2}{3+V} = \frac{1-V^2}{3+V}$$

$$X\frac{dV}{dX} = \frac{1-V^2}{3+V} \longrightarrow \frac{dX}{X} = \frac{3+V}{1-V^2}\,dV$$

* بتكامل الطرفين :-

$$\int \frac{dx}{x} = \int \frac{(3+V)}{(1-V^2)}\, dV = 3\int \frac{dV}{(1-V^2)} + \int \frac{V}{1-V^2}\, dV$$

$$\ln(x) + \ln(c) = \frac{-1}{2} \ln(1-V^2) + \frac{3}{2} \ln\left(\frac{V+1}{1-V}\right)$$

$$\frac{-1}{2} \ln(1-V^2) + \frac{3}{2} \ln(V+1)^3 - \frac{3}{2} \ln(1-V)$$

$$\ln(1-V^2)^{-1} + \ln(V+1)^3 - \ln(1-V)^3 = \ln(x^2c^2) = \ln(xc)$$

$$\ln \frac{(V+1)^3}{(1-V^2)(1-V)^3} = \ln(x^2c^2)$$

$$\therefore \frac{(V+1)^{\cancel{3}\,2}}{\cancel{(V+1)}(V-1)(1-V)^3} = x^2c^2$$

$$\therefore \frac{(V+1)^2}{(1-V)^4} = x^2c^2$$

$$\frac{\left(\frac{y}{x}+1\right)^2}{\left(1-\frac{y}{x}\right)^4} = x^2c^2$$

$$\frac{\left(\frac{y-2}{x+1}+1\right)^2}{\left(1-\frac{y-2}{x+1}\right)^4} = (x+1)^2c^2$$

$$6)\ \frac{dy}{dx} + y\,\ln(x) = e^{-x\ln(x)}$$

((Sowl__))

* معادلة خطية

$$f(x) = \ln(x) \qquad Q(x) = e^{-x\ln(x)}$$

$$S = e^{\int f(x)\,dx}$$

$$= e^{\int \ln(x)\,dx}$$

$$u = x \qquad dv = \frac{1}{x}\,dx$$
$$du = dx \qquad v = \ln(x)$$

$$\int \ln(x)\,dx = x\ln(x) - \int dx$$
$$= x\ln(x) - x$$
$$= x(\ln(x) - 1)$$

$$\therefore S = e^{x(\ln(x)-1)}$$

$$Sy = \int S\,Q(x)\,dx = \int e^{x(\ln x - 1)} \cdot e^{-x\ln(x)}\,dx$$

$$= \int e^{-x}\,dx = -e^{-x} + c$$

$$y = \frac{-e^{-x} + c}{e^{x(\ln x - 1)}}$$

7) $y^3 + xy' - y = \text{Zero}$

((Soul —))

* المعادلة ع صورة كليرو

$y = xy' + y'^3$

:- الحل العام هو

$\cancel{y} = cx + c^3 \rightarrow (1)$

* بالتفاضل بالنسبة لـ (c)

$0 = x + 3c^2 \quad \rightarrow c^2 = \frac{-x}{3}$

* بالتعويض في (1)

$\cancel{y} = c(x + c^2)$

$y^2 = c^2(x + c^2)^2 = \frac{-x}{3}\left(x - \frac{x}{3}\right)^2$

$= \frac{-4x^3}{27}$

$y^2 + \frac{4}{27}x^3 = \text{Zero} \longrightarrow \left(\frac{y}{2}\right)^2 + \left(\frac{x}{3}\right)^3 = \text{Zero}$

8) $xy'' - y' = x^2 e^x$

"Soul—"

Let $y' = P \longrightarrow y'' = P'$

* بالتعويض

$xP' - P = x^2 e^x \longrightarrow P' - \frac{1}{x} P = x e^x$

∴ المعادلة خطية في (P)

$f(x) = \frac{-1}{x}$, $Q(x) = x e^x$

$$S = e^{\int f(x)\,dx} = e^{-\int \frac{1}{x}\,dx} = e^{-\ln(x)} = x^{-1}$$

$$SP = \int S\, Q(x)\,dx = \int x^{-1} \cdot x e^x\,dx$$

$$= \int e^x\,dx = e^x + c_1$$

$$x^{-1} P = e^x + c_1 \longrightarrow P = x e^x + x c_1$$

$$\therefore \frac{dy}{dx} = P = x e^x + x c_1$$

$$\therefore dy = (x e^x + x c_1)\,dx$$

* بتكامل الطرفين

$$\int dy = \int x e^x\,dx + \int x c_1\,dx + c_2$$

$u = x \qquad dv = e^x\,dx$

$du = dx \qquad v = e^x$

$$\therefore y = x e^x - \int e^x\,dx + \frac{1}{2} x^2 c_1 + c_2$$

$$y = x e^x - e^x + \frac{1}{2} x^2 c_1 + c_2$$

$$y = e^x(x-1) + \frac{1}{2} x^2 c_1 + c_2$$

9) $x^2y'' - xy' + x = Zero$

"Soul__"

* معادلة متجانسة من نوع كوشي - أويلر

$y'' = M(M-1)x^{M-2} \leftarrow y' = Mx^{M-1} \leftarrow y = x^M$

* بالتعويض في المعادلة والحصول ع معادلة مساعدة

$M(M-1)x^M - Mx^M + x^M = Zero \div x^M$

$M^2 - 2m + 1 = Zero$

$(M-1)(M-1)$

$M_1 = M_2 = 1$

$y_1 = x^2 \qquad y_2 = x\ln(x)$

$y = x(C_1 + C_2\ln(x))$

10) $y''' - 2y'' + y' - 2y = 12\sin(2x) - 4x \longrightarrow (1)$

"Soul__"

* معادلة متجانسة

$y''' - 2y'' + y' - 2y = Zero$

Let $y = e^{mx}$

$m^3 - 2m^2 + m - 2 = Zero$

$m_1 = 2 \qquad m_{2,3} = \pm i$

$y_h = C_1e^{2x} + C_2\cos(x) + C_3\sin(x) \leftarrow$ * الحل المتجانس

Let $y_p = (A\sin(2x) + B\cos(2x) + Dx + C) \leftarrow$ * الحل الخاص

$$y'_p = 2A\cos(2x) - 2B\sin(2x) + D$$

$$y''_p = -4A\sin(2x) - 4B\cos(2x)$$

$$y'''_p = -8A\cos(2x) + 8B\sin(2x)$$

* بالتعويض في ①

$$-8A\cos(2x) + 8B\sin(2x) + 8A\sin(2x) + 8B\cos(2x) + 2A\cos(2x) - 2B\sin(2x) + D - 2A\sin(2x) - 2B\cos(2x) + Dx + C = 12\sin(2x) - 4x$$

* بالتجميع

$$6B + 6A = 12 \longrightarrow 12B = 12 \longrightarrow B = 1$$

$$-6A + 6B = \text{Zero} \longrightarrow 6A = 6B \longrightarrow A = B$$

$$\therefore A = B = 1$$

$$D = -4$$

$$D + C^2 = \text{Zero} \longrightarrow C = -4$$

$$y_p = \sin(2x) + \cos(2x) - 4x + 4$$

$$y = y_p + y_h = \sin(2x) + \cos(2x) - 4x + 4 + C_1 e^{2x} + C_2\cos(x) + C_3\sin(x)$$