

1. For a vector  $\mathbf{x} = \begin{bmatrix} 6 \\ 0 \\ 0 \end{bmatrix}$  and the subspace  $U$  spanned by the basis vectors  $\mathbf{b}_1 = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$  and  $\mathbf{b}_2 = \begin{bmatrix} 0 \\ 1 \\ 2 \end{bmatrix}$ , which of the following statements are true?

4 / 4 points

You can use the formula slide that comes with the corresponding lecture.

- The coordinates of the projected point with respect to  $\mathbf{b}_1, \mathbf{b}_2$  are  $\begin{bmatrix} 0 \\ 0 \end{bmatrix}$ .
- The projection of  $\mathbf{x}$  onto  $U$  is  $\begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$
- The coordinates of the projected point with respect to  $\mathbf{b}_1, \mathbf{b}_2$  are  $\begin{bmatrix} 5 \\ -3 \end{bmatrix}$ .

✓ **Correct**

Excellent job!

- The projection matrix is  $\begin{bmatrix} 0 & 0 & 0 \\ 0 & 1 & 2 \\ 0 & 2 & 4 \end{bmatrix}$

- The rank of the projection matrix is 1.

- The projection of  $\mathbf{x}$  onto  $U$  is  $\begin{bmatrix} 5 \\ 2 \\ -1 \end{bmatrix}$

✓ **Correct**

Well done.

- The projection matrix is  $\frac{1}{6} \begin{bmatrix} 5 & 2 & -1 \\ 2 & 2 & 2 \\ -1 & 2 & 5 \end{bmatrix}$

✓ **Correct**

Well done!

- The projection matrix is not symmetric.

- The projection matrix is symmetric.

✓ **Correct**

Projection matrices are always symmetric.



2. Project  $\begin{bmatrix} 3 \\ 2 \\ 2 \end{bmatrix}$  onto the subspace spanned by  $\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$  and  $\begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix}$ .

1 / 1 point

You can use the formula slide that comes with the corresponding lecture.

$\begin{bmatrix} 2 \\ 1 \\ 1 \end{bmatrix}$

$\begin{bmatrix} 3 \\ 2 \\ 2 \end{bmatrix}$

$\begin{bmatrix} 6 \\ 4 \\ 4 \end{bmatrix}$

$\begin{bmatrix} 2 \\ 1 \\ 2 \end{bmatrix}$

**Correct**

Absolutely! The original vector is already in the subspace, so the projection has no effect.

3. 1. Project  $\begin{bmatrix} 12 \\ 0 \\ 0 \end{bmatrix}$  onto the subspace  $U_1$  spanned by  $\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$ ,  $\begin{bmatrix} 0 \\ 1 \\ 2 \end{bmatrix}$ .

1 / 1 point

2. Project the result from 1. onto the subspace spanned by  $\begin{bmatrix} -10\sqrt{6} \\ -4\sqrt{6} \\ 2\sqrt{6} \end{bmatrix}$ . What is the final projection?

*Hint: For step 2. you do not necessarily need to compute anything.*

You can use the formula slide that comes with the corresponding lecture.

$\begin{bmatrix} 5 \\ 2\sqrt{6}+1 \\ -\sqrt{6}+2 \end{bmatrix}$

$\begin{bmatrix} 5 \\ 2\sqrt{6} \\ -1\sqrt{6} \end{bmatrix}$

$\begin{bmatrix} 10 \\ 4 \\ -2 \end{bmatrix}$

**Correct**

Good job! The first projection already lies in the second subspace. Therefore, the second projection does not do anything.