

1. Consider the function  $h : \mathbb{R} \rightarrow \mathbb{R}$ , where  $h(t) = (f \circ g)(t) = f(g(t))$  with

5 / 5 points

$$g(t) = \mathbf{x} = \begin{bmatrix} t \cos t \\ t \sin t \end{bmatrix}, \quad t \in \mathbb{R}$$

$$f(\mathbf{x}) = \exp(x_1 x_2^2), \quad \mathbf{x} = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} \in \mathbb{R}^2$$

$\frac{dh}{dt} = \frac{df}{dg} \frac{dg}{dt}$

 **Correct**

Yes, this is exactly what the chain-rule says.

$\frac{df}{d\mathbf{x}} = [x_2^2 \exp(x_1 x_2^2) \quad 2x_1 x_2 \exp(x_1 x_2^2)]$

 **Correct**

Yes, this is a row vector.

$\frac{df}{d\mathbf{x}} = [x_1 x_2^2 \quad 2x_2 x_1 x_2^2]$

$\frac{dg}{dt} = \begin{bmatrix} \cos t - t \sin t \\ \sin t + t \cos t \end{bmatrix}$

 **Correct**

Well done

$\frac{dg}{dt} = \begin{bmatrix} \sin t - t \cos t \\ \cos t + t \sin t \end{bmatrix}$

$\frac{dh}{dt} = \cos t - t \sin t + 2t \sin t (\sin t + t \cos t)$

$\frac{dh}{dt} = \exp(x_1 x_2^2) [x_2^2 (\cos t - t \sin t) + 2x_1 x_2 (\sin t + t \cos t)]$  with  
 $x_1 = t \cos t, x_2 = t \sin t$

 **Correct**

Yes, this is what we get when we apply the chain-rule. Well done!

2. Compute  $\frac{df}{dx}$  of the following function using the chain rule.

1 / 1 point

$$a = x^2$$

$$b = \exp(a)$$

$$c = a + b$$

$$d = \log(c)$$

$$e = \sin(c)$$

$$f = d + e$$

$$\frac{df}{dx} = \frac{(1 + \cos(x^2 + \exp(x^2))(x^2 + \exp(x^2)))(2x + 2x \exp(x^2))}{x^2 + \exp(x^2) + \log(x^3)}$$

$$\frac{df}{dx} = \frac{(1 + \cos(x^2 + \exp(x^2))(x^2 + \exp(x^2)))(2x + 2x \exp(x^2))}{x^2}$$

$$\frac{df}{dx} = \frac{(1 + \cos(x^2 + \exp(x^2))(x^2 + \exp(x^2)))(2x + 2x \exp(x^2))}{x^2 + \exp(x^2)}$$

 **Correct**

Excellent!

3. What is  $\frac{df}{dx}$  where

1 / 1 point

$$f = \cos(t^2)$$

$$t = x^3$$

$$-\sin(x^6)$$

$$6x^5 \sin(x^6)$$

$$-6x^5 \sin(x^6)$$

$$-6x \sin(x^6)$$

 **Correct**

Well done!