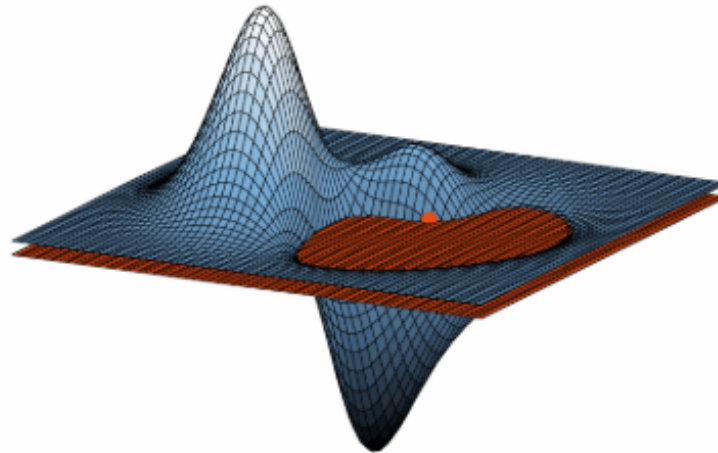


1. Now you have seen how to calculate the multivariate Taylor series, and what zeroth, first and second order approximations look like for a function of 2 variables. In this course we won't be considering anything higher than second order for functions of more than one variable.

1 / 1 point

In the following questions you will practise recognising these approximations by thinking about how they behave with different  $x$  and  $y$ , then you will calculate some terms in the multivariate Taylor series yourself.

The following plot features a surface and its Taylor series approximation in red, around a point given by a red circle. What order is the Taylor series approximation?



- ☒ Zeroth order
- ☐ First order
- ☐ Second order
- ☐ None of the above

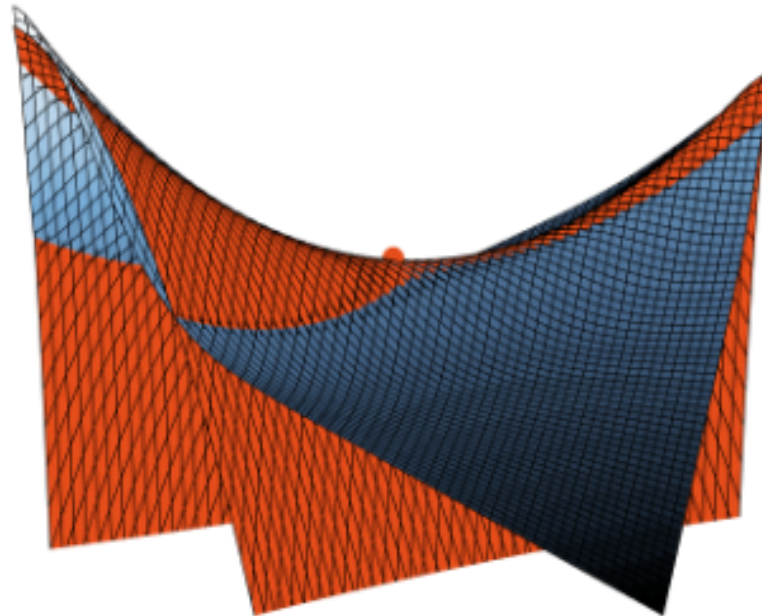
✓ Correct

The red surface is constant everywhere and so has no terms in  $\Delta x$  or  $\Delta x^2$



2. What order Taylor series approximation, expanded around the red circle, is the red surface in the following plot?

1 / 1 point



- ☐ Zeroth order
- ☐ First order
- ☒ Second order
- ☐ None of the above

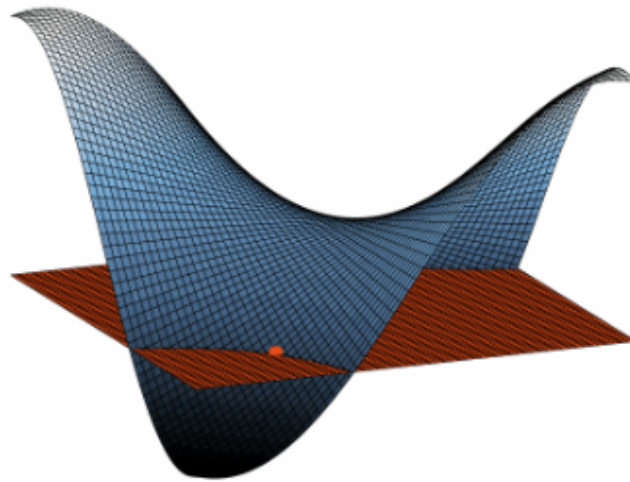
✓ Correct

The gradient of the surface is not constant, so we must have a term of higher order than  $\Delta x$ .

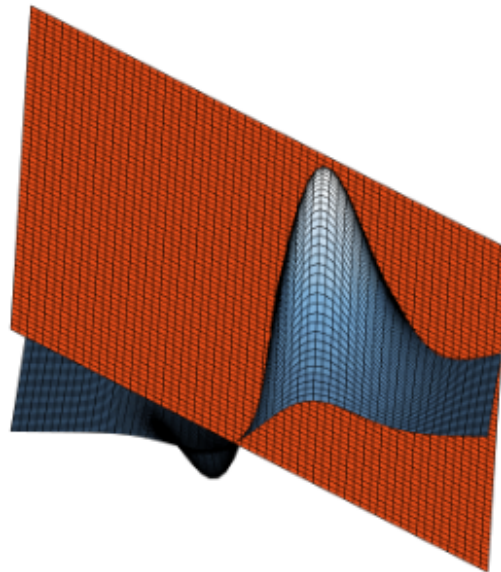
3. Which red surface in the following images is a first order Taylor series approximation of the blue surface? The original functions are given, but you don't need to do any calculations.

1 / 1 point

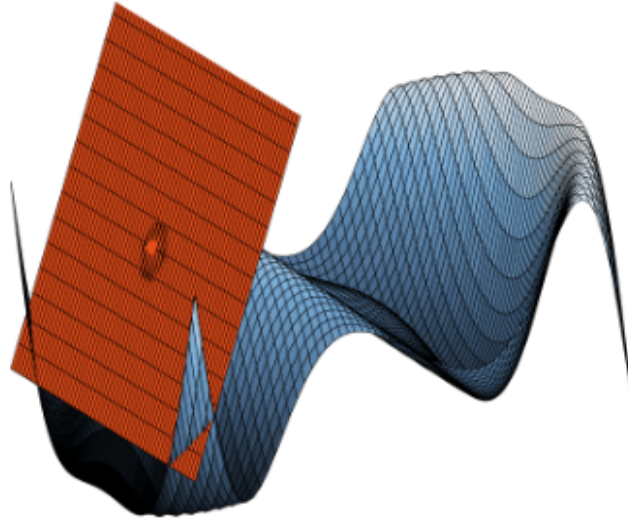
☐  $f(x, y) = \sin(xy/5)$



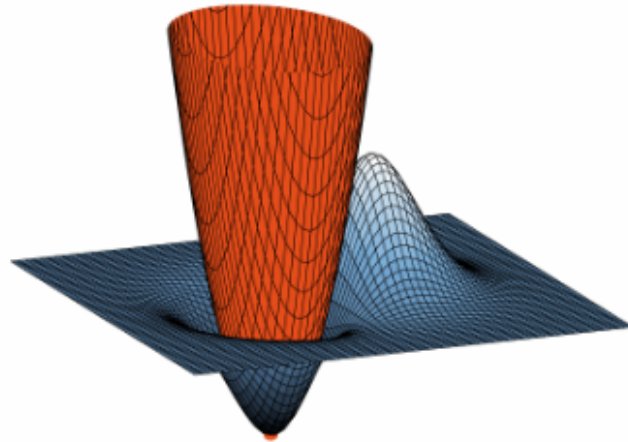
☐  $f(x, y) = (x^2 + 2x)e^{-x^2 - y^2/5}$



☒  $f(x, y) = x \sin(x^2/2 + y^2/4)$



☐  $f(x, y) = xe^{-x^2-y^2}$



☒ Correct

The gradient of the red surface is non-zero and constant, so the  $\Delta \mathbf{x}$  terms are the highest order.

4. Recall that up to second order the multivariate Taylor series is given by

1 / 1 point

$$f(\mathbf{x} + \Delta \mathbf{x}) = f(\mathbf{x}) + J_f \Delta \mathbf{x} + \frac{1}{2} \Delta \mathbf{x}^T H_f \Delta \mathbf{x} + \dots$$

Consider the function of 2 variables,  $f(x, y) = xy^2 e^{-x^4 - y^2/2}$ . Which of the following is the first order Taylor series expansion of  $f$  around the point  $(-1, 2)$ ?

- ☐  $f_1(-1 + \Delta x, 2 + \Delta y) = -4e^{-3} - 4e^{-3}\Delta x + 4e^{-3}\Delta y$
- ☒  $f_1(-1 + \Delta x, 2 + \Delta y) = -4e^{-3} - 12e^{-3}\Delta x + 4e^{-3}\Delta y$
- ☐  $f_1(-1 + \Delta x, 2 + \Delta y) = -4e^{-3} + 16e^{-3}\Delta x - 8e^{-3}\Delta y$
- ☐  $f_1(-1 + \Delta x, 2 + \Delta y) = 2e^{-33/2} - 63e^{-33/2}\Delta x - 2e^{-33/2}\Delta y$

✔ Correct

5. Now consider the function  $f(x, y) = \sin(\pi x - x^2 y)$ . What is the Hessian matrix  $H_f$  that is associated with the second order term in the Taylor expansion of  $f$  around  $(1, \pi)$ ?

1 / 1 point

- ☐  $H_f = \begin{pmatrix} -\pi^2 & \pi \\ \pi & -1 \end{pmatrix}$
- ☐  $H_f = \begin{pmatrix} -2 & -2\pi \\ 2\pi & 1 \end{pmatrix}$
- ☒  $H_f = \begin{pmatrix} -2\pi & -2 \\ -2 & 0 \end{pmatrix}$
- ☐  $H_f = \begin{pmatrix} -2\pi & -2 \\ -2 & 1 \end{pmatrix}$

✔ Correct

Good, you can check your second order derivatives here:

$$\partial_{xx} f(x, y) = -2y \cos(\pi x - x^2 y) - (\pi - 2xy)^2 \sin(\pi x - x^2 y)$$

$$\partial_{xy} f(x, y) = -2x \cos(\pi x - x^2 y) - x^2(\pi - 2xy) \sin(\pi x - x^2 y)$$

$$\partial_{yx} f(x, y) = -2x \cos(\pi x - x^2 y) - x^2(\pi - 2xy) \sin(\pi x - x^2 y)$$

$$\partial_{yy} f(x, y) = -x^4 \sin(\pi x - x^2 y)$$