Your grade: 100%

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Next item →

1. In this quiz, you will practice calculating the multivariate chain rule for various functions.

   1 / 1 point

   For the following functions, calculate the expression $\frac{df}{dt} = \frac{\partial f}{\partial \mathbf{x}} \frac{d\mathbf{x}}{dt}$ in matrix form, where $\mathbf{x} = (x_1, x_2)$.

   $f(\mathbf{x}) = f(x_1, x_2) = x_1^2 x_2^2 + x_1 x_2$

   $x_1(t) = 1 - t^2$

   $x_2(t) = 1 + t^2$

   ○ $\frac{df}{dt} = \frac{\partial f}{\partial \mathbf{x}} \frac{d\mathbf{x}}{dt} = [2x_1^2 x_2 + x_1, 2x_1 x_2^2 + x_2] \begin{bmatrix} -2t \\ 2t \end{bmatrix}$

   ○ $\frac{df}{dt} = \frac{\partial f}{\partial \mathbf{x}} \frac{d\mathbf{x}}{dt} = [2x_1 x_2^2 + x_2, 2x_1^2 x_2 + x_1] \begin{bmatrix} 2t \\ -2t \end{bmatrix}$

   ○ $\frac{df}{dt} = \frac{\partial f}{\partial \mathbf{x}} \frac{d\mathbf{x}}{dt} = [2x_1^2 x_2 + x_1, 2x_1 x_2^2 + x_2] \begin{bmatrix} 2t \\ -2t \end{bmatrix}$

   ◉ $\frac{df}{dt} = \frac{\partial f}{\partial \mathbf{x}} \frac{d\mathbf{x}}{dt} = [2x_1 x_2^2 + x_2, 2x_1^2 x_2 + x_1] \begin{bmatrix} -2t \\ 2t \end{bmatrix}$

   ✓ **Correct**
   Well done!

2. For the following functions, calculate the expression $\frac{df}{dt} = \frac{\partial f}{\partial \mathbf{x}} \frac{d\mathbf{x}}{dt}$ in matrix form, where $\mathbf{x} = (x_1, x_2, x_3)$.

   1 / 1 point

   $f(\mathbf{x}) = f(x_1, x_2, x_3) = x_1^3 \cos(x_2) e^{x_3}$

   $x_1(t) = 2t$

   $x_2(t) = 1 - t^2$

   $x_3(t) = e^t$

   ◉ $\frac{df}{dt} = \frac{\partial f}{\partial \mathbf{x}} \frac{d\mathbf{x}}{dt} = [3x_1^2 \cos(x_2) e^{x_3}, -x_1^3 \sin(x_2) e^{x_3}, x_1^3 \cos(x_2) e^{x_3}] \begin{bmatrix} 2 \\ -2t \\ e^t \end{bmatrix}$

   ○ $\frac{df}{dt} = \frac{\partial f}{\partial \mathbf{x}} \frac{d\mathbf{x}}{dt} = [3x_1^2 \cos(x_2) e^{x_3}, x_1^3 \cos(x_2) e^{x_3}, x_1^3 \sin(x_2) e^{x_3}] \begin{bmatrix} 2 \\ 2t \\ -e^t \end{bmatrix}$

   ○ $\frac{df}{dt} = \frac{\partial f}{\partial \mathbf{x}} \frac{d\mathbf{x}}{dt} = [3x_1^2 \cos(x_2) e^{x_3}, -x_1^3 \sin(x_2) e^{x_3}, x_1^3 \sin(x_2) e^{x_3}] \begin{bmatrix} 2 \\ 2t \\ e^t \end{bmatrix}$

   ○ $\frac{df}{dt} = \frac{\partial f}{\partial \mathbf{x}} \frac{d\mathbf{x}}{dt} = [3x_1^2 \cos(x_2) e^{x_3}, -x_1^3 \cos(x_2) e^{x_3}, x_1^3 \cos(x_2) e^{x_3}] \begin{bmatrix} 2 \\ 2t \\ e^t \end{bmatrix}$

   ✓ **Correct**

3. For the following functions, calculate the expression $\frac{df}{dt} = \frac{\partial f}{\partial x} \frac{\partial x}{\partial u} \frac{du}{dt}$ in matrix form, where $x = (x_1, x_2)$ 1/1 point

and $u = (u_1, u_2)$.

$$f(x) = f(x_1, x_2) = x_1^2 - x_2^2$$

$$x_1(u_1, u_2) = 2u_1 + 3u_2$$

$$x_2(u_1, u_2) = 2u_1 - 3u_2$$

$$u_1(t) = cos(t/2)$$

$$u_2(t) = sin(2t)$$

○ $\frac{df}{dt} = \frac{\partial f}{\partial x} \frac{\partial x}{\partial u} \frac{du}{dt} = [2x_1, 2x_2] \begin{bmatrix} 2 & -3 \\ 2 & 3 \end{bmatrix} \begin{bmatrix} sin(t/2) \\ 2cos(2t) \end{bmatrix}$

◉ $\frac{df}{dt} = \frac{\partial f}{\partial x} \frac{\partial x}{\partial u} \frac{du}{dt} = [2x_1, -2x_2] \begin{bmatrix} 2 & 3 \\ 2 & -3 \end{bmatrix} \begin{bmatrix} -sin(t/2)/2 \\ 2cos(2t) \end{bmatrix}$

○ $\frac{df}{dt} = \frac{\partial f}{\partial x} \frac{\partial x}{\partial u} \frac{du}{dt} = [2x_1, 2x_2] \begin{bmatrix} 2 & -3 \\ -2 & -3 \end{bmatrix} \begin{bmatrix} -cos(t/2)/2 \\ 2sin(2t) \end{bmatrix}$

○ $\frac{df}{dt} = \frac{\partial f}{\partial x} \frac{\partial x}{\partial u} \frac{du}{dt} = [-2x_1, -2x_2] \begin{bmatrix} -2 & 3 \\ -2 & -3 \end{bmatrix} \begin{bmatrix} -sin(t/2)/2 \\ 2cos(t) \end{bmatrix}$

⊘ Correct

Well done!

4. For the following functions, calculate the expression $\frac{df}{dt} = \frac{\partial f}{\partial x} \frac{\partial x}{\partial u} \frac{du}{dt}$ in matrix form, where $x = (x_1, x_2)$ 1/1 point

and $u = (u_1, u_2)$.

$$f(x) = f(x_1, x_2) = cos(x_1)sin(x_2)$$

$$x_1(u_1, u_2) = 2u_1^2 + 3u_2^2 - u_2$$

$$x_2(u_1, u_2) = 2u_1 - 5u_2^3$$

$$u_1(t) = e^{t/2}$$

$$u_2(t) = e^{-2t}$$

○ $\frac{df}{dt} = \frac{\partial f}{\partial x} \frac{\partial x}{\partial u} \frac{du}{dt} = [-sin(x_1)cos(x_2), cos(x_1)cos(x_2)] \begin{bmatrix} u_1 & 6u_2 - 1 \\ 2 & -u_2^2 \end{bmatrix} \begin{bmatrix} e^{t^2/2}/2 \\ -2e^{-2t} \end{bmatrix}$

○ $\frac{df}{dt} = \frac{\partial f}{\partial x} \frac{\partial x}{\partial u} \frac{du}{dt} = [-cos(x_1)sin(x_2), cos(x_1)cos(x_2)] \begin{bmatrix} u_1 & 6u_2 - 1 \\ 2 & -15u_2^2 \end{bmatrix} \begin{bmatrix} e^t \\ e^t \end{bmatrix}$

◉ $\frac{df}{dt} = \frac{\partial f}{\partial x} \frac{\partial x}{\partial u} \frac{du}{dt} = [-sin(x_1)sin(x_2), cos(x_1)cos(x_2)] \begin{bmatrix} 4u_1 & 6u_2 - 1 \\ 2 & -15u_2^2 \end{bmatrix} \begin{bmatrix} e^{t/2}/2 \\ -2e^{-2t} \end{bmatrix}$

○ $\frac{df}{dt} = \frac{\partial f}{\partial x} \frac{\partial x}{\partial u} \frac{du}{dt} = [-sin(x_1)cos(x_2), cos(x_1)cos(x_2)] \begin{bmatrix} 41u_1 & 6u_2 - 1 \\ 2 & -15u_2 \end{bmatrix} \begin{bmatrix} e^{t/2}/8 \\ -2e^{2t} \end{bmatrix}$

⊘ Correct

Well done!

5. For the following functions, calculate the expression $\frac{df}{dt} = \frac{\partial f}{\partial x}\frac{\partial x}{\partial u}\frac{du}{dt}$ in matrix form, where $\mathbf{x} = (x_1, x_2)$ and $\mathbf{u} = (u_1, u_2)$.

$$f(\mathbf{x}) = f(x_1, x_2, x_3) = \sin(x_1)\cos(x_2)e^{x_3}$$

$$x_1(u_1, u_2) = \sin(u_1) + \cos(u_2)$$

$$x_2(u_1, u_2) = \cos(u_1) - \sin(u_2)$$

$$x_3(u_1, u_2) = e^{u_1 + u_2}$$

$$u_1(t) = 1 + t/2$$

$$u_2(t) = 1 - t/2$$

○ $\frac{df}{dt} = \frac{\partial f}{\partial x}\frac{\partial x}{\partial u}\frac{du}{dt} =$

$$\left[\cos(x_1)\cos(x_2)e^{x_3}, -\sin(x_1)^2\sin(x_2)e^{x_3}, \sin(x_1)\cos(x_2)e^{x_3}\right] \begin{bmatrix} \sin(u_1) & -\sin(u_2) \\ -\sin(u_1) & -\cos(u_2) \\ 3e^{u_1+u_2} & e^{u_1+u_2} \end{bmatrix} \begin{bmatrix} -1/2 \\ -1/2 \end{bmatrix}$$

○ $\frac{df}{dt} = \frac{\partial f}{\partial x}\frac{\partial x}{\partial u}\frac{du}{dt} =$

$$\left[\cos(x_1)\cos(x_2)e^{x_3}, -\sin(x_1)\cos(x_2)e^{x_3}, \sin(x_1)\cos(x_2)e^{x_3}\right] \begin{bmatrix} \cos(u_1) & \sin(u_2) \\ -\sin(u_1) & -\cos(u_2) \\ e^{u_1+u_2} & -e^{u_1+u_2} \end{bmatrix} \begin{bmatrix} 1/2 \\ -1/2 \end{bmatrix}$$

○ $\frac{df}{dt} = \frac{\partial f}{\partial x}\frac{\partial x}{\partial u}\frac{du}{dt} =$

$$\left[\cos(x_1)\cos(x_2)e^{x_3}, \sin(x_1)\sin(x_2)e^{x_3}, \sin(x_1)\cos(x_2)e^{x_3}\right] \begin{bmatrix} -\cos(u_1) & -\sin(u_2) \\ -\sin(u_1) & -\cos(u_2) \\ e^{u_1+u_2} & 2e^{u_1+u_2} \end{bmatrix} \begin{bmatrix} 1/2 \\ 1/2 \end{bmatrix}$$

◉ $\frac{df}{dt} = \frac{\partial f}{\partial x}\frac{\partial x}{\partial u}\frac{du}{dt} =$

$$\left[\cos(x_1)\cos(x_2)e^{x_3}, -\sin(x_1)\sin(x_2)e^{x_3}, \sin(x_1)\cos(x_2)e^{x_3}\right] \begin{bmatrix} \cos(u_1) & -\sin(u_2) \\ -\sin(u_1) & -\cos(u_2) \\ e^{u_1+u_2} & e^{u_1+u_2} \end{bmatrix} \begin{bmatrix} 1/2 \\ -1/2 \end{bmatrix}$$

✓ Correct

Well done!