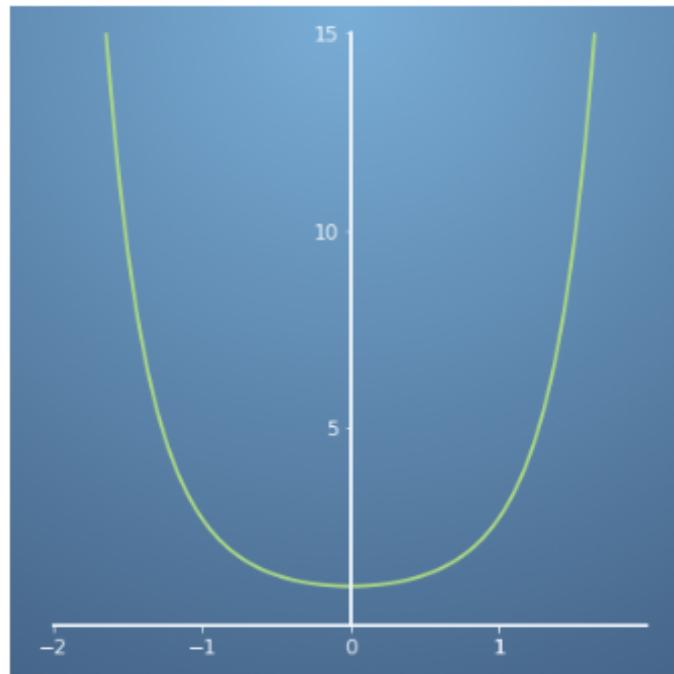


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1. In the two previous videos, we have shown the short mathematical proofs for the Taylor series, and for special cases when the starting point is  $x = 0$ , the Maclaurin series. In these set of questions, we will begin to work on applying the Taylor and Maclaurin series formula to obtain approximations of functions.

1 / 1 point

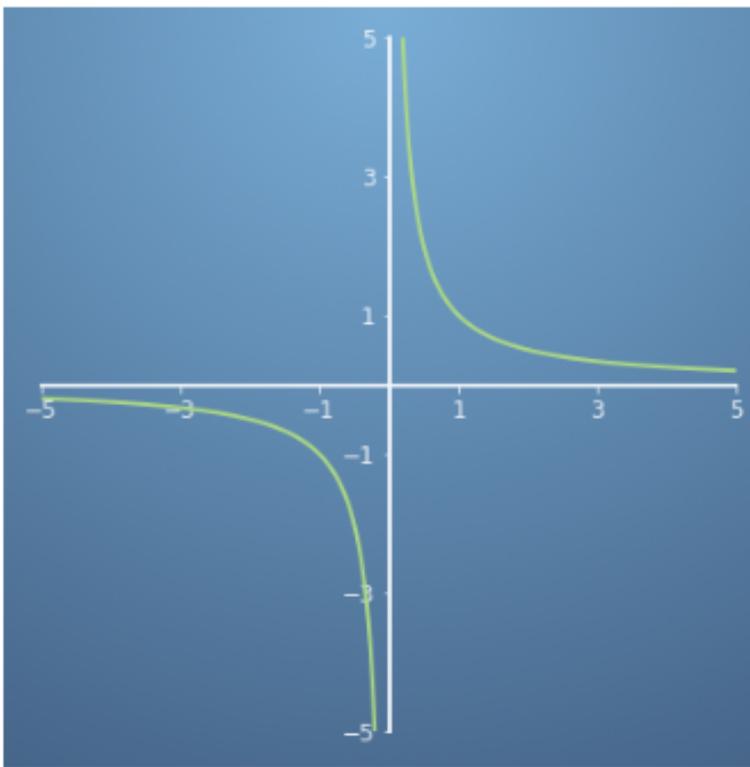


For the function  $f(x) = e^{x^2}$  about  $x = 0$ , using the the Maclaurin series formula, obtain an approximation up to the first three non zero terms.

- $f(x) = 1 + 2x + \frac{x^2}{2} + \dots$
- $f(x) = 1 - x^2 - \frac{x^4}{2} \dots$
- $f(x) = x^2 + \frac{x^4}{2} + \frac{x^6}{6} + \dots$
- $f(x) = 1 + x^2 + \frac{x^4}{2} + \dots$

✓ Correct

We find that only even powers of  $x$  appear in the Taylor approximation for this function.



2. Use the Taylor series formula to approximate the first three terms of the function  $f(x) = 1/x$ , expanded around the point  $p = 4$ .

$f(x) = \frac{1}{4} - \frac{(x-4)}{16} + \frac{(x-4)^2}{64} + \dots$

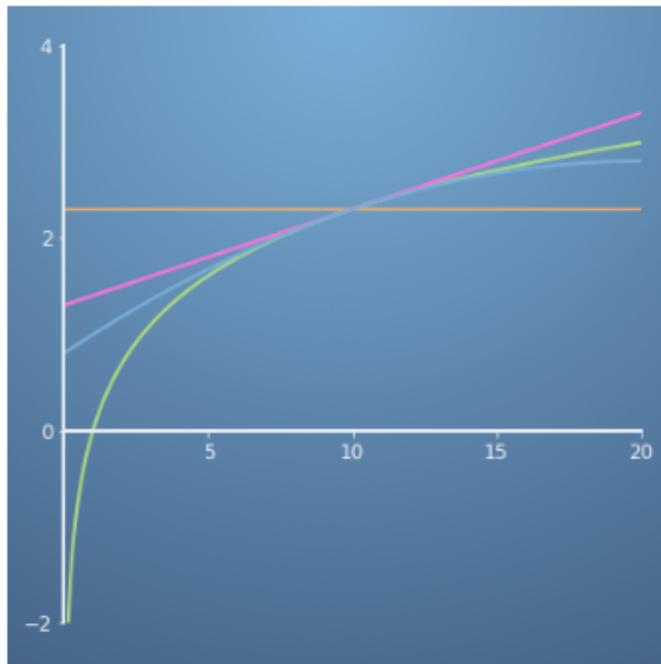
$f(x) = -\frac{1}{4} - \frac{(x+4)}{16} - \frac{(x+4)^2}{64} \dots$

$f(x) = \frac{(x-4)}{16} + \frac{(x-4)^2}{64} - \frac{(x-4)^3}{256} \dots$

$f(x) = \frac{1}{4} - \frac{(x+4)}{16} + \frac{(x+4)^2}{64} + \dots$

**Correct**

We find that only even powers of  $x$  appear in the Taylor approximation for this function.



3. By finding the first three terms of the Taylor series shown above for the function  $f(x) = \ln(x)$  (green line) about  $x = 10$ , determine the magnitude of the difference of using the second order taylor expansion against the first order Taylor expansion when approximating to find the value of  $f(2)$ .

- $\Delta f(2) = 1.0$
- $\Delta f(2) = 0.32$
- $\Delta f(2) = 0$
- $\Delta f(2) = 0.5$

 **Correct**

The second order Taylor approximation about the point  $x = 10$  is

$$f(x) = \ln(10) + \frac{(x-10)}{10} - \frac{(x-10)^2}{200} \dots$$

So the first order approximation is

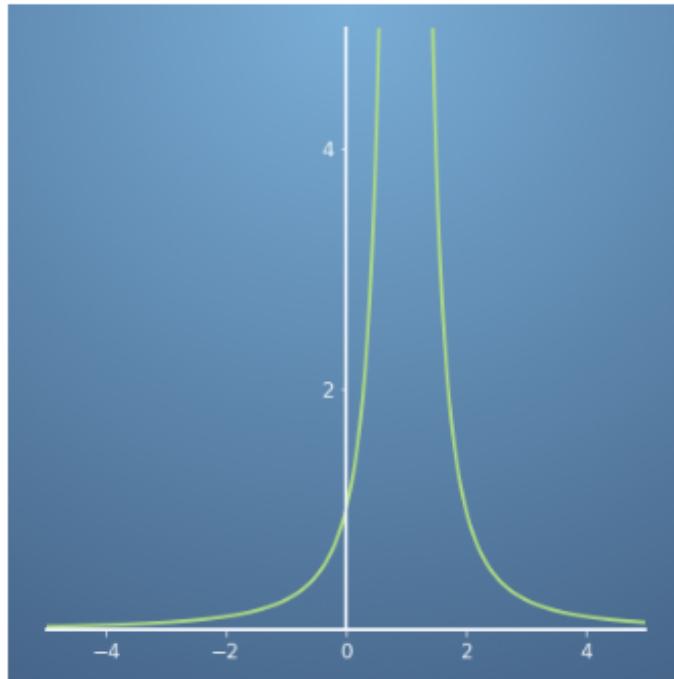
$$g_1 = \ln(10) + \frac{(x-10)}{10}$$

and the second order approximation is

$$g_2 = \ln(10) + \frac{(x-10)}{10} - \frac{(x-10)^2}{200}.$$

4. In some cases, a Taylor series can be expressed in a general equation that allows us to find a particular  $n^{th}$  term of our series. For example the function  $f(x) = e^x$  has the general equation  $f(x) = \sum_{n=0}^{\infty} \frac{x^n}{n!}$ . Therefore if we want to find the  $3^{rd}$  term in our Taylor series, substituting  $n = 2$  into the general equation gives us the term  $\frac{x^2}{2}$ . We know the Taylor series of the function  $e^x$  is  $f(x) = 1 + x + \frac{x^2}{2} + \frac{x^3}{3!} + \dots$ . Now let us try a further working example of using general equations with Taylor series.

1 / 1 point



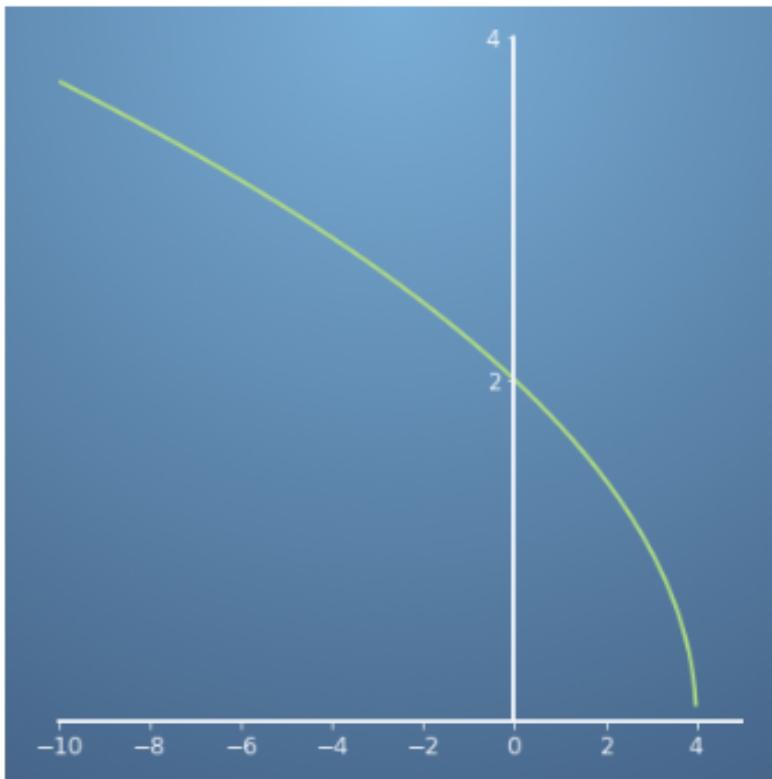
By evaluating the function  $f(x) = \frac{1}{(1-x)^2}$  about the origin  $x = 0$ , determine which general equation for the  $n^{th}$  order term correctly represents  $f(x)$ .

- $f(x) = \sum_{n=0}^{\infty} (2+n)(x)^n$
- $f(x) = \sum_{n=0}^{\infty} (1+n)(-x)^n$
- $f(x) = \sum_{n=0}^{\infty} (1+2n)(x)^n$
- $f(x) = \sum_{n=0}^{\infty} (1+n)x^n$

✓ **Correct**

By doing a Maclaurin series approximation, we obtain

$f(x) = 1 + 2x + 3x^2 + 4x^3 + 5x^4 + \dots$ , which satisfies the general equation shown.



5. By evaluating the function  $f(x) = \sqrt{4 - x}$  at  $x = 0$ , find the quadratic equation that approximates this function.

- $f(x) = 2 + x + x^2 \dots$
- $f(x) = \frac{x}{4} - \frac{x^2}{64} \dots$
- $f(x) = 2 - x - \frac{x^3}{64} \dots$
- $f(x) = 2 - \frac{x}{4} - \frac{x^2}{64} \dots$

Correct

The quadratic equation shown is the second order approximation.