Exercise 1 :-

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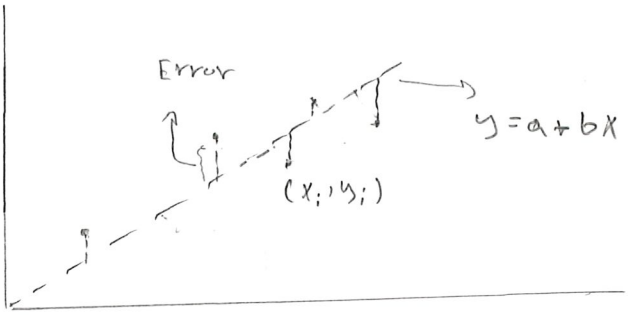
→ the target is to draw a fitting line between the points that minimize the error between points and Fitted line



Error "E" $= y_i - [a + bx_i]$

$$E = \sum_{i=1}^{N} [y_i - a - bx_i] \quad ; \text{ where "n" is the number of points}$$

→ To minimize "E" → we will take the MSE "mean square Error"

$$\therefore E = \frac{1}{N} \sum_{i=1}^{N} [y_i - a - bx_i]^2 \longrightarrow \text{then: } \nabla_{a,b} E = 0$$

(1-) $\frac{dE}{da} = 0 \longrightarrow \frac{1}{N} \sum_{i=1}^{N} [2(y_i - a - bx_i) * -1] = 0$

$$\therefore \frac{1}{N} \sum_{i=1}^{N} [-2y_i + 2a + 2bx_i] = 0$$

$$\rightarrow -2 \sum_{i=1}^{N} y_i + 2Na + 2b \sum_{i=1}^{N} x_i = 0$$

$$\rightarrow \underbrace{Na} + \underbrace{\left(\sum_{i=1}^{N} x_i\right) b} = \sum_{i=1}^{N} y_i \longrightarrow (1)$$

(2-) $\frac{dE}{db} = 0$

$$\sum_{i=1}^{N} [2(y_i - a - bx_i) * -x_i] = 0 \longrightarrow \sum_{i=1}^{N} [-2x_i y_i + 2ax_i + 2bx_i^2] = 0$$

$$\therefore a \sum_{i=1}^{N} x_i + b \sum_{i=1}^{N} x_i^2 = \sum_{i=1}^{N} x_i y_i$$

$$\rightarrow \underbrace{\left[\sum_{i=1}^{N} x_i\right] a} + \underbrace{\left[\sum_{i=1}^{N} x_i^2\right] b} = \sum_{i=1}^{N} x_i y_i \longrightarrow (2)$$

For Simplification

we will assume that $\sum_{i=1}^{N} x_i = G$, $\sum_{i=1}^{N} y_i = F$

$\sum_{i=1}^{N} x_i^2 = H$, $\sum_{i=1}^{N} x_i y_i = K$

by applying to eqn (1) & (2) and Forming ametrix

$$\begin{bmatrix} N & G \\ G & H \end{bmatrix} \begin{bmatrix} a \\ b \end{bmatrix} = \begin{bmatrix} F \\ K \end{bmatrix}$$

→ by using "Linsolve ( )" in matlab, we can get the following Formula For "a" & "b"

$$a = -\frac{(F * H) - (G * K)}{(G^2 - (H * N))}$$

$$b = \frac{F*G - K*N}{G^2 - H*N}$$