732A75 Advanced Data Mining
TDDD41 Data Mining - Clustering and Association Analysis
Lecture 8: Constrained Frequent Itemset Mining

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Outline

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  - Monotone and antimonotone constraints
  - Apriori algorithm + (anti) monotone constraints
  - FP grow algorithm + (anti) monotone constraints
  - Convertible (anti) monotone constraints
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- Literature
  - Course book. Second edition: 5.5. Third edition: 7.3.
  - Pei, J. and Han, J. Can We Push More Constraints into Frequent Pattern Mining ? In Proc. of the 2000 Int. Conf. on Knowledge Discovery and Data Mining, 2000.

Monotone and Antimonotone Constraints

- A constraint is a function that returns true or false for every itemset, meaning whether the itemset satisfies the constraint or not. For instance:
  - The itemset has support equal or greater than a given value.
  - The sum of the prices of the items in the itemset is greater than 5 units.
  - The most expensive item in the itemset costs less than 5 units.
  - The itemset contains the item "bread".
  - The itemset contains exactly three items.
- A constraint C is monotone when for every itemsets A and B such that $A \subseteq B$, if $C(A) = \mathit{true}$ then $C(B) = \mathit{true}$. For instance:
  - The itemset contains the item "bread".
- A constraint C is antimonotone when for every itemsets A and B such that $B \subseteq A$, if $C(A) = \mathit{true}$ then $C(B) = \mathit{true}$. For instance:
  - The support of the itemset is equal or greater than a given value.
- Alternatively, C is antimonotone when for every itemsets A and B such that $B \subseteq A$, if $C(B) = \mathit{false}$ then $C(A) = \mathit{false}$.
- Note that the apriori property applies to every antimonotone constraint, i.e. no need to check the constraint for supersets of A if $C(A) = \mathit{false}$.

Monotone and Antimonotone Constraints

- Examples of monotone constrains:
  - $sum(S) \geq v$ where S is the set of prices of the items in the itemset, and v is a given value.
  - Really monotone ? Only if the prices of all the items are non-negative.
  - $min(S) \leq v$
  - $range(S) \geq v$
- Examples of antimonotone constrains:
  - $sum(S) \leq v$ when the prices of all the items are non-negative.
  - $max(S) \leq v$
  - $range(S) \leq v$

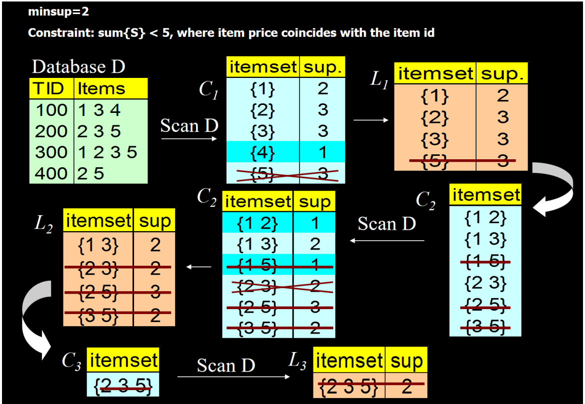
Monotone and Antimonotone Constraints

Constraint	Antimonotone	Monotone
$v \in S$	no	yes
$S \supseteq V$	no	yes
$S \subseteq V$	yes	no
$\min(S) \leq v$	no	yes
$\min(S) \geq v$	yes	no
$\max(S) \leq v$	yes	no
$\max(S) \geq v$	no	yes
$count(S) \leq v$	yes	no
$count(S) \geq v$	no	yes
$sum(S) \leq v$ ($a \in S, a \geq 0$)	yes	no
$sum(S) \geq v$ ($a \in S, a \geq 0$)	no	yes
$range(S) \leq v$	yes	no
$range(S) \geq v$	no	yes
$avg(S)\ \theta\ v, \theta \in \{=, \leq, \geq\}$	No but convertible	No but convertible
$support(S) \geq \xi$	yes	no
$support(S) \leq \xi$	no	yes

Apriori Algorithm + Antimonotone Constraint

Algorithm: apriori(D, *minsup*, $\boldsymbol{C}$)
Input: A transactional database D, *minsup*, **and an antimonotone constraint** $\boldsymbol{C}$.
Output: All the large itemsets in D **that satisfy** $\boldsymbol{C}$.

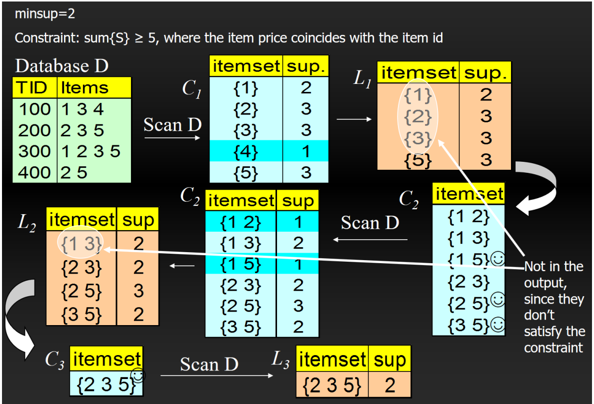
$L_1 = \{$ large 1-itemsets **that satisfy** $\boldsymbol{C}$ $\}$
for $(k = 2; L_{k-1} \neq \emptyset; k++)$ do
  C_k = apriori-gen(L_{k-1}) // Generate candidate large k-itemsets
  for all $t \in D$ do
    for all $c \in C_k$ such that $c \in t$ do
      $c.count++$
  $L_k = \{c \in C_k | c.count \geq minsup$ **and** $\boldsymbol{C(c) = true}\}$
return $\bigcup_k L_k$



Apriori Algorithm + Monotone Constraint

Algorithm: apriori(D, $minsup$, $\boldsymbol{C}$)
Input: A transactional database D, $minsup$, **and a monotone constraint $\boldsymbol{C}$**.
Output: All the large itemsets in D **that satisfy $\boldsymbol{C}$**.

$L_1 = \{$ large 1-itemsets $\}$
for $(k = 2; L_{k-1} \neq \varnothing; k++)$ do
  C_k = apriori-gen(L_{k-1}) // Generate candidate large k-itemsets
  for all $t \in D$ do
    for all $c \in C_k$ such that $c \in t$ do
      $c.count++$
  $L_k = \{c \in C_k | c.count \geq minsup\}$
return $\boldsymbol{\{c \in \bigcup_k L_k | C(c) = \mathit{true} \text{ or } C(d) = \mathit{true} \text{ for some } d \subseteq c\}}$

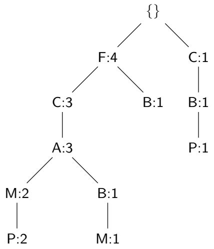


FP Grow Algorithm + Monotone Constraint

Algorithm: FP-grow(*Tree*, α, *minsup*, ***C***)
Input: A FP tree *Tree*, an itemset α, *minsup*, **and a monotone constraint *C***.
Output: All the itemsets in *Tree* that end with α, have *minsup* **and satisfy *C***.

if $C(\alpha)$ = *true* then
  replace C with C_{true} **// C_{true} is a constraint that always returns true**
for each item X in *Tree* do
  output the itemset $\beta = X \cup \alpha$ with support=$X.count$ **if $C(\beta)$ = *true***
  build the β conditional database and the corresponding FP tree $Tree_\beta$
  if $Tree_\beta$ is not empty then call FP-grow($Tree_\beta$, β, *minsup*, ***C***)

Transaction id	Items bought
1	F, C, A, M, P
2	F, C, A, B, M
3	F, B
4	C, B, P
5	F, C, A, M, P



Item	Conditional database
F	-
C	F:3
A	FC:3
B	FCA:1, F:1, C:1
M	FCA:2, FCAB:1
P	FCAM:2, CB:1

FP Grow Algorithm + Antimonotone Constraint

Algorithm: FP-tree(D, $minsup$, $\boldsymbol{C}$)
Input: A transactional database D, $minsup$, **and an antimonotone constraint** $\boldsymbol{C}$.
Output: The FP tree for D, $minsup$ **and** $\boldsymbol{C}$.

Count support for each item in D
Remove the infrequent items from the transactions in D
Remove the items that do not satisfy C from the transactions in D
Sort the items in each transaction in D in support descending order
Create a FP tree with a single node T with $T.name = NULL$
for each transaction $I \in D$ do
    insert-tree(I, T)

FP Grow Algorithm + Antimonotone Constraint

Algorithm: FP-grow(*Tree*, α, *minsup*, ***C***)
Input: A FP tree *Tree*, an itemset α, *minsup*, **and an antimonotone constraint *C***.
Output: All the itemsets in *Tree* that end with α, have *minsup* **and satisfy *C***.

let δ denote all the items in *Tree*
if $C(\alpha \cup \delta)$ = *true* then
  replace *C* with C_{true} **// C_{true} is a constraint that always returns true**
for each item X in *Tree* do
  if $C(X \cup \alpha)$ = *true* then
    output the itemset $\beta = X \cup \alpha$ with support=$X.count$
    build the β conditional database and the corresponding FP tree $Tree_\beta$
    if $Tree_\beta$ is not empty then call FP-grow($Tree_\beta$, β, *minsup*, ***C***)

Item	Conditional database
F	-
C	F:3
A	FC:3
B	FCA:1, F:1, C:1
M	FCA:2, FCAB:1
P	FCAM:2, CB:1

M-conditional database

Tid	Items bought
1	F, C, A
2	F, C, A
3	F, C, A, B

After sorting

Tid	Items bought
1	F, C, A
2	F, C, A
3	F, C, A

Output FM and re-start the process for the FM-conditional database ?

Convertible Monotone and Antimonotone Constraint

- $avg(S) \leq v$ and $avg(S) \geq v$ are neither monotone nor antimonotone.
- A constraint C is convertible monotone when there exists an item order R such that for every itemsets A and B respecting R and such that A is a suffix of B, if $C(A) = true$ then $C(B) = true$. For instance:
  - $avg(S) \geq v$ with respect to decreasing price order.
- A constraint C is convertible antimonotone when there exists an item order R such that for every itemsets A and B respecting R and such that B is a suffix of A, if $C(A) = true$ then $C(B) = true$. For instance:
  - $avg(S) \geq v$ with respect to increasing price order.
- Alternatively, C is convertible antimonotone when there exists an item order R such that for every itemsets A and B respecting R and such that B is a suffix of A, if $C(B) = false$ then $C(A) = false$.
- A constraint that is both convertible monotone and antimonotone is called strongly convertible.

Convertible Monotone and Antimonotone Constraints

Constraint	Convertible antimonotone	Convertible monotone	Strongly convertible
avg(S) $\leq$, $\geq$ v	Yes	Yes	Yes
median(S) $\leq$, $\geq$ v	Yes	Yes	Yes
sum(S) $\leq$ v (items could be of any value, v $\geq$ 0)	Yes	No	No
sum(S) $\leq$ v (items could be of any value, v $\leq$ 0)	No	Yes	No
sum(S) $\geq$ v (items could be of any value, v $\geq$ 0)	No	Yes	No
sum(S) $\geq$ v (items could be of any value, v $\leq$ 0)	Yes	No	No
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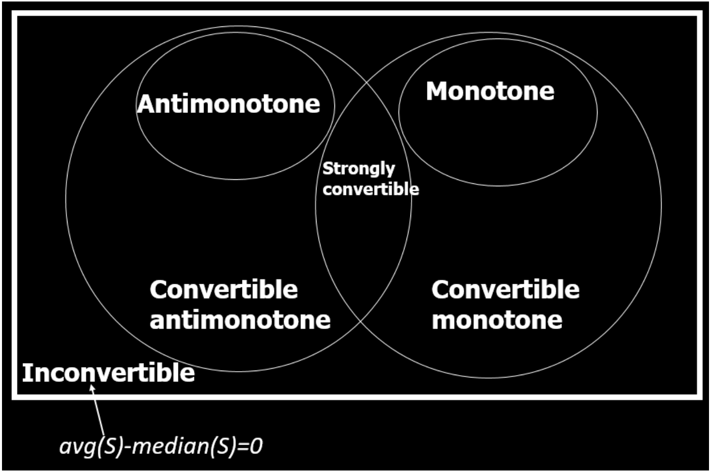
Summary



Antimonotone
Monotone
Strongly convertible
Convertible antimonotone
Convertible monotone
Inconvertible
avg(S)-median(S)=0