Discrete Distributions

Following is a list of some discrete distributions, abbreviations, their probability functions, means, variances, and characteristic functions.

An asterisk (*) indicates that the expression is too complicated to present here; in some cases, a closed formula does not even exist.

Distribution, notation	Probability function	EX	$\operatorname{Var} X$	$\varphi_X(t)$
One point $\delta(a)$	$p(a) = 1$	a	0	e^{ita}
Symmetric Bernoulli	$p(-1) = p(1) = \frac{1}{2}$	0	1	$\cos t$
Bernoulli $\mathrm{Be}(p),\ 0 \le p \le 1$	$p(0) = q,\ p(1) = p;\ q = 1 - p$	p	pq	$q + pe^{it}$
Binomial $\mathrm{Bin}(n,p),\ n = 1, 2, \ldots,\ 0 \le p \le 1$	$p(k) = \binom{n}{k} p^k q^{n-k},\ k = 0, 1, \ldots, n;\ q = 1 - p$	np	npq	$(q + pe^{it})^n$
Geometric $\mathrm{Ge}(p),\ 0 \le p \le 1$	$p(k) = pq^k,\ k = 0, 1, 2, \ldots;\ q = 1 - p$	$\frac{q}{p}$	$\frac{q}{p^2}$	$\frac{p}{1 - qe^{it}}$
First success $\mathrm{Fs}(p),\ 0 \le p \le 1$	$p(k) = pq^{k-1},\ k = 1, 2, \ldots;\ q = 1 - p$	$\frac{1}{p}$	$\frac{q}{p^2}$	$\frac{pe^{it}}{1 - qe^{it}}$
Negative binomial $\mathrm{NBin}(n,p),\ n = 1, 2, 3, \ldots,$ $0 \le p \le 1$	$p(k) = \binom{n+k-1}{k} p^n q^k,\ k = 0, 1, 2, \ldots;$ $q = 1 - p$	$n\frac{q}{p}$	$n\frac{q}{p^2}$	$\left(\frac{p}{1 - qe^{it}}\right)^n$
Poisson $\mathrm{Po}(m),\ m > 0$	$p(k) = e^{-m}\frac{m^k}{k!},\ k = 0, 1, 2, \ldots$	m	m	$e^{m(e^{it} - 1)}$
Hypergeometric $H(N, n, p),\ n = 0, 1, \ldots, N,$ $N = 1, 2, \ldots,$ $p = 0, \frac{1}{N}, \frac{2}{N}, \ldots, 1$	$p(k) = \frac{\binom{Np}{k}\binom{Nq}{n-k}}{\binom{N}{n}},\ k = 0, 1, \ldots, Np;$ $q = 1 - p;$ $n - k = 0, \ldots, Nq$	np	$npq\frac{N - n}{N - 1}$	*

Continuous Distributions

Following is a list of some continuous distributions, abbreviations, their densities, means, variances, and characteristic functions.

An asterisk (*) indicates that the expression is too complicated to present here; in some cases, a closed formula does not even exist.

Distribution, notation	Density	EX	$\operatorname{Var} X$	$\varphi_X(t)$
Uniform/Rectangular $U(a,b)$	$f(x) = \frac{1}{b - a},\ a < x < b$	$\frac{1}{2}(a + b)$	$\frac{1}{12}(b - a)^2$	$\frac{e^{itb} - e^{ita}}{it(b - a)}$
$U(0,1)$	$f(x) = 1,\ 0 < x < 1$	$\frac{1}{2}$	$\frac{1}{12}$	$\frac{e^{it} - 1}{it}$
$U(-1,1)$	$f(x) = \frac{1}{2},\ \lvert x \rvert < 1$	0	$\frac{1}{3}$	$\frac{\sin t}{t}$

Distribution, notation	Density	EX	$\operatorname{Var} X$	$\varphi_X(t)$
Triangular Tri(a,b)	$f(x)=\frac{2}{b-a}\left(1-\frac{2}{b-a}\left\lvert x-\frac{a+b}{2}\right\rvert\right)$, $a<x<b$	$\frac{1}{2}(a+b)$	$\frac{1}{24}(b-a)^2$	$\left(\frac{e^{itb/2}-e^{ita/2}}{\frac{1}{2}it(b-a)}\right)^2$
Tri$(-1,1)$	$f(x)=1-\lvert x\rvert,\ \lvert x\rvert<1$	0	$\frac{1}{6}$	$\left(\frac{\sin\frac{t}{2}}{\frac{t}{2}}\right)^2$
Exponential Exp(a), $a>0$	$f(x)=\frac{1}{a}\,e^{-x/a},\ x>0$	a	a^2	$\frac{1}{1-ait}$
Gamma $\Gamma(p,a)$, $a>0$, $p>0$	$f(x)=\frac{1}{\Gamma(p)}\,x^{p-1}\,\frac{1}{a^p}\,e^{-x/a},\ x>0$	pa	pa^2	$\frac{1}{(1-ait)^p}$
Chi-square $\chi^2(n)$, $n=1,2,3,\ldots$	$f(x)=\frac{1}{\Gamma(\frac{n}{2})}\,x^{\frac{1}{2}n-1}\left(\frac{1}{2}\right)^{n/2}e^{-x/2},\ x>0$	n	$2n$	$\frac{1}{(1-2it)^{n/2}}$
Laplace $L(a)$, $a>0$	$f(x)=\frac{1}{2a}e^{-\lvert x\rvert/a},\ -\infty<x<\infty$	0	$2a^2$	$\frac{1}{1+a^2t^2}$
Beta $\beta(r,s)$, $r,s>0$	$f(x)=\frac{\Gamma(r+s)}{\Gamma(r)\Gamma(s)}\,x^{r-1}(1-x)^{s-1}$, $0<x<1$	$\frac{r}{r+s}$	$\frac{rs}{(r+s)^2(r+s+1)}$	*

Distribution, notation	Density	EX	$\operatorname{Var} X$	$\varphi_X(t)$
Weibull $W(\alpha,\beta)$, $\alpha,\beta>0$	$f(x)=\frac{1}{\alpha\beta}x^{(1/\beta)-1}\,e^{-x^{1/\beta}/\alpha},\ x>0$	$\alpha^\beta\,\Gamma(\beta+1)$	$\alpha^{2\beta}(\Gamma(2\beta+1)-\Gamma(\beta+1)^2)$	*
Rayleigh Ra(α), $\alpha>0$	$f(x)=\frac{2}{\alpha}\,x\,e^{-x^2/\alpha},\ x>0$	$\frac{1}{2}\sqrt{\pi\alpha}$	$\alpha(1-\frac{1}{4}\pi)$	*
Normal $N(\mu,\sigma^2)$, $-\infty<\mu<\infty$, $\sigma>0$	$f(x)=\frac{1}{\sigma\sqrt{2\pi}}\,e^{-\frac{1}{2}(x-\mu)^2/\sigma^2}$, $-\infty<x<\infty$	μ	σ^2	$e^{i\mu t-\frac{1}{2}t^2\sigma^2}$
$N(0,1)$	$f(x)=\frac{1}{\sqrt{2\pi}}\,e^{-x^2/2},\ -\infty<x<\infty$	0	1	$e^{-t^2/2}$
Log-normal $LN(\mu,\sigma^2)$, $-\infty<\mu<\infty$, $\sigma>0$	$f(x)=\frac{1}{\sigma x\sqrt{2\pi}}\,e^{-\frac{1}{2}(\log x-\mu)^2/\sigma^2},\ x>0$	$e^{\mu+\frac{1}{2}\sigma^2}$	$e^{2\mu}\left(e^{2\sigma^2}-e^{\sigma^2}\right)$	*
(Student's) t $t(n)$, $n=1,2,\ldots$	$f(x)=\frac{\Gamma(\frac{n+1}{2})}{\sqrt{\pi n}\,\Gamma(\frac{n}{2})}\cdot\frac{1}{(1+\frac{x^2}{n})^{(n+1)/2}}$, $-\infty<x<\infty$	0	$\frac{n}{n-2}$, $n>2$	*
(Fisher's) F $F(m,n)$, $m,n=1,2,\ldots$	$f(x)=\frac{\Gamma(\frac{m+n}{2})(\frac{m}{n})^{m/2}}{\Gamma(\frac{m}{2})\Gamma(\frac{n}{2})}\cdot\frac{x^{m/2-1}}{(1+\frac{mx}{n})^{(m+n)/2}}$, $x>0$	$\frac{n}{n-2}$, $n>2$	$\frac{n^2(m+2)}{m(n-2)(n-4)}-\left(\frac{n}{n-2}\right)^2$, $n>2$	*