

GroupReport

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Assignment 1: Be careful with ‘==’

A pupil of a school is bad in arithmetic but good in programming. He writes a program to check if $1/3-1/4==1/12$:

1.1

The result of the program is “Teacher lied”.The computer calculates the value of $1/3$ which is not a ture value. The bits allocated to the significand would provide roughly 16 decimal places of precision.

```
## [1] "Teacher lied"
```

1.2

The code is modified to give the correct result as follows:

```
## [1] "Teacher said true"
```

There is also another method in which the problem could be solved.Here we would do it by assigning a set significand to 16 bits.

```
## [1] "Teacher said true"
```

Assignment 2: Derivative

A widely known way to compute the derivative of function $f(x)$ in point x is to use

$$f'(x) = \frac{f(x + \epsilon) - f(x)}{\epsilon}$$

2.1

The function computing the derivative of function $f(x)=x$ in this way is written below. Here we take $\epsilon = 10^{-15}$.

```
f<-function(x){
  e=10^-15
  a=x+e
  f_x=(a-x)/e
  print(f_x)
}
```

2.2

The derivative function at point $x=100000$ is computed.

```
## [1] 0
```

2.3

Here we get value 0, but the true value should be 1. From the function, when we calculate the $(x+10^{-15})-x$, it is not equal to 10^{-15} , it becomes 0, this is because the significand has changed.

Assignment 3: Variance

A known formula for estimating the variance is

$$Var(x) = \frac{1}{1-n} \left(\sum_{i=1}^n x_i^2 - \frac{1}{n} \left(\sum_{i=1}^n x_i \right)^2 \right)$$

3.1

The function `myvar()` estimating the variance is given below:

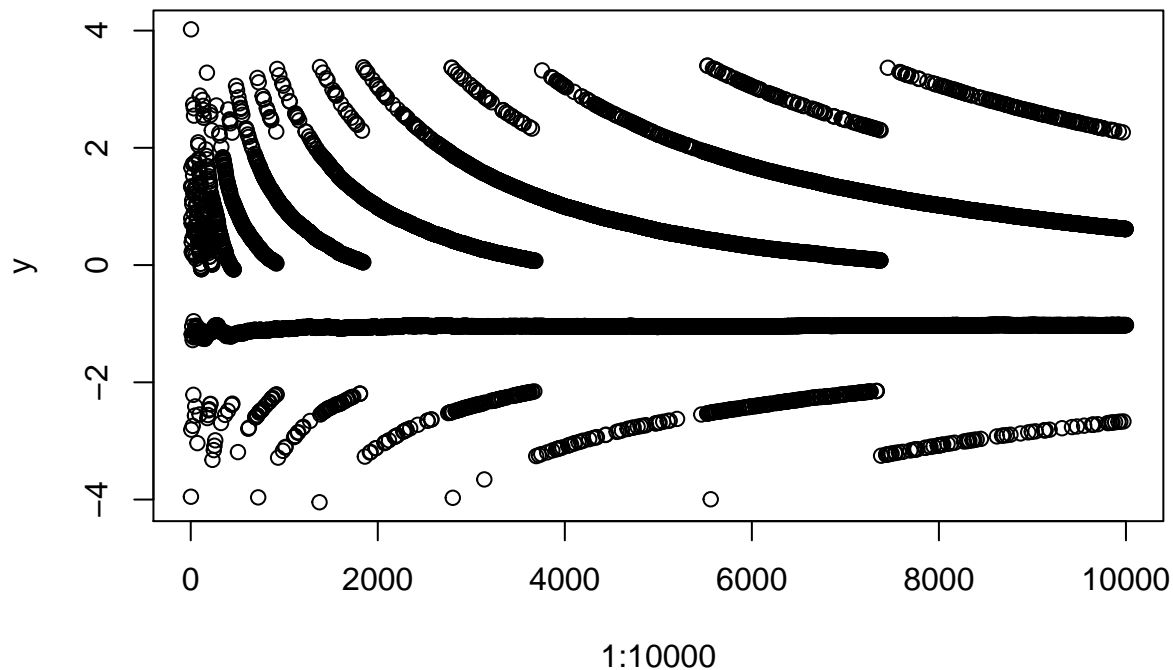
```
myvar <- function(x){  
  n <- length(x)  
  result <- (sum(x^2)-((sum(x)^2)/n))/(n-1)  
  return(result)  
}
```

3.2

A vector $x=(x_1 \dots x_{10000})$ with 10000 random numbers, normally distributed with mean 10^8 and variance 1 is generated.

3.3

The differences are round between $(-4, 4)$. The true result should be 0. The error comes from the summation part. The precision of the summation is lost. The numbers are all of the same sign and have roughly the same magnitude. In this case a pairwise “fan-in” method may yield good accuracy. And there is another error called cancellation, because the sum of square and square of sum has large magnitudes and small difference. The function does not go well for the above provided mean but ofcourse it works good for smaller mean values.



Assignment 4: Linear Algebra

The Excel file “tecator.xls” contains the results of a study aimed to investigate whether a near- infrared absorbance spectrum and the levels of moisture and fat can be used to predict the protein content of samples of meat. For each meat sample the data consists of a 100 channel spectrum of absorbance records and the levels of moisture (water), fat and protein. The absorbance is $-\log_{10}$ of the transmittance measured by the spectrometer. The moisture, fat and protein are determined by analytic chemistry. The worksheet you need to use is “data”. It contains data from 215 samples of finely chopped meat. The aim is to fit a linear regression model that could predict protein content.

4.1

The data set is successfully imported in R.

4.2

A and b has been computed for the given data set.

4.3

The kind of result we are getting is:

Error in solve.default(a = A, b = b) : system is computationally singular: reciprocal condition number = 7.78804e-17

The reciprocal condition number is equal to 7.78804e-17 which could be consider as 0. Matrices are well-conditioned if the reciprocal condition number is near 1 and ill-conditioned if it is near zero.

4.4

```
## [1] 8.523517e+14
```

The conditional number of matrix A is found to be 8.523517e+14. When the condition number is very large it is said to be ill-conditioned.

4.5

```
##                                [,1]
## Channel1      -110.6420066
## Channel2      -221.1899921
## Channel3       378.0067049
## Channel4      -129.7038290
## Channel5       413.4180272
## Channel6       -79.7519946
## Channel7      -203.0060010
## Channel8        82.7949648
## Channel9      -132.3810763
## Channel10       255.8233113
## Channel11      -328.6085020
## Channel12      -304.1498067
## Channel13       624.1267225
## Channel14      -298.8990298
## Channel15       40.7433672
## Channel16      -257.5359421
## Channel17       169.2389191
## Channel18       296.6629389
## Channel19      -325.0660101
## Channel20       -3.0077434
## Channel21       554.5604351
## Channel22     -1366.0281946
## Channel23      1860.3564505
## Channel24     -1416.1321099
## Channel25       631.8396619
## Channel26     -112.0439549
## Channel27       17.0167193
## Channel28     -228.9306325
## Channel29       444.2724259
## Channel30     -597.3805369
## Channel31       438.1484415
## Channel32       315.0394088
## Channel33     -349.8143768
## Channel34     -285.9109757
## Channel35       418.5810505
```

```
## Channel36      -79.1068236
## Channel37     -305.9413388
## Channel38       284.2543473
## Channel39     -435.5657863
## Channel40       819.7486282
## Channel41     -885.0073314
## Channel42       324.5893411
## Channel43       524.5895670
## Channel44     -583.4418903
## Channel45     -140.1709745
## Channel46       577.2361744
## Channel47     -294.2684468
## Channel48      -68.0751275
## Channel49     -90.4922841
## Channel50       404.1439524
## Channel51     -698.9990527
## Channel52      1258.8833783
## Channel53    -1672.7308401
## Channel54      1486.2299172
## Channel55     -812.3613443
## Channel56       192.4954799
## Channel57     -32.9120466
## Channel58        7.3752546
## Channel59     -88.6907142
## Channel60       344.8769021
## Channel61     -454.3518607
## Channel62       447.6205296
## Channel63     -197.4186847
## Channel64       222.3370792
## Channel65     -399.2558318
## Channel66       364.8665574
## Channel67     -367.1614830
## Channel68       243.9220622
## Channel69     -76.2948345
## Channel70     -318.1916071
## Channel71       327.6653346
## Channel72     -178.5231528
## Channel73       119.1856499
## Channel74       445.1150045
## Channel75     -20.0127319
## Channel76     -642.7509961
## Channel77       369.4809508
## Channel78      -74.9011366
## Channel79     -23.4854322
## Channel80     -676.8615334
## Channel81      1013.4538029
## Channel82     -889.7623189
## Channel83       403.0065633
## Channel84       424.0848303
## Channel85     -801.0956155
## Channel86       655.0134202
## Channel87       659.1829785
## Channel88    -2150.8325647
## Channel89      1671.8088853
```

```
## Channel90      298.6977068
## Channel91     -332.1727762
## Channel92     -487.3689759
## Channel93      278.6277368
## Channel94      201.6627353
## Channel95     -609.5081456
## Channel96      565.2851789
## Channel97     -133.3407595
## Channel98     -368.0087250
## Channel99      238.2015968
## Channel100     24.6418188
## Fat           -1.6666403
## Moisture      -0.9341099
```

```
## [1] 490471520662
```

For the original data the fat,moisture have large magnitude and the channels have smaller magnitude.After the scaling data set, the columns of the matrix have the same scale, which avoids artificial ill-conditioning.

Appendix - R-code

```
## ----echo=FALSE-----
x1<-1/3
x2<-1/4;
if (x1-x2==1/12){
print("Teacher said true")
} else{
print("Teacher lied")
}

## ---- echo=FALSE-----
x1<-1/3
x2<-1/4
if(all.equal(x1-x2,1/12)){
  print("Teacher said true")
} else {
  print("Teacher lied")
}

## ----echo=FALSE-----
x1<-1/3
x2<-1/4
if (round(x1-x2, digits = 16) == round(1/12, digits = 16)){
  print("Teacher said true")
} else{
  print("Teacher lied")
}

## -----
f<-function(x){
  e=10^-15
  a=x+e
  f_x=(a-x)/e
}
```

```

    print(f_x)
}

## ----echo=FALSE-----
f(100000)

## -----
myvar <- function(x){
  n <- length(x)
  result <- (sum(x^2)-((sum(x)^2)/n))/(n-1)
  return(result)
}

## ----echo=FALSE-----
x <- rnorm(10000,mean = 10^8, 1)

## ----echo=FALSE-----
y <- NULL
for(i in 1 : 10000){
  a <- myvar(x[1:i])
  b <- var(x[1:i])
  y[i] <- a - b
}
plot(x= 1: 10000,y=y)

## ----echo=FALSE-----
data <- read.csv("tecator.csv")

## ----echo=FALSE-----
X <- as.matrix(data[,c(2:102,104)])
Y <- as.matrix(data[,103])
X_1 <- as.matrix(cbind(1,X))
A <- t(X_1)%*%X_1
b <- t(X_1)%*%Y

## ----eval=FALSE,echo=FALSE-----
## beta <- solve(A, b)

## ----echo=FALSE-----
kappa(A)

## ----echo=FALSE-----
X1 <- scale(X, center = TRUE, scale = TRUE)
Y1 <- scale(Y, center=TRUE, scale=TRUE)
A1 <-t(X1)%*%X1
b1 <- t(X1)%*%Y1
beta1 <- solve(a = A1, b = b1)
beta1
kappa(A1)

## ----code=readLines(knitr::purl("GroupReport.Rmd", documentation = 1)), eval = FALSE----
## NA

```