LINKÖPING UNIVERSITY   Bayesian Learning
Division of Statistics and Machine Learning
Department of Computer and Information Science
Mattias Villani and Per Sidén

Mathematical Exercises 4

Try to solve the problems before class. Don't worry if you fail, the important thing is trying.
You should not hand in any solutions.
This part of the course is not obligatory and is not graded.

1. BERNOULLI MEETS LAPLACE

   (a) Let $x_1, ..., x_n \overset{iid}{\sim} Bernoulli(\theta)$ and assume the prior $\theta \sim Beta(\alpha, \beta)$. Derive the marginal likelihood for this model.

   (b) Compute the marginal likelihood of the model in a) using the Laplace approximation.

   (c) Is this approximation accurate if $\alpha = \beta = 1$ and you have observed $s = 6$ success in $n = 10$ trials?

2. FILL IN THE BLANKS - AGAIN

   (a) Derive the marginal likelihood for the Poisson model with Gamma prior at the end of Slide 6 at Lecture 10.

3. PARETO

   (a) Let $x_1, \ldots, x_n \overset{iid}{\sim} \text{Uniform}(0, \theta)$. Let $\theta \sim \text{Pareto}(\alpha, \beta)$, that is

   $$p(\theta) = \frac{\alpha \beta^\alpha}{\theta^{\alpha+1}}, \ \theta \geq \beta.$$

   Show that this is a conjugate prior to this Uniform model and derive the posterior for θ. [Hint: Don't forget to include an indicator function when you write up the likelihood function. The Uniform$(0, \theta)$ distribution is zero for outcomes larger than θ.]

   (b) Derive the predictive distribution of x_{n+1} given $x_1, \ldots, x_n$. [Hint: It is wise to break up the integrals in two parts.]

Have fun!

- Mattias