5.1

a. $T^2 = n(\bar{x}-\mu)' S^{-1} (\bar{x}-\mu)$

$n=4$, $\mu' = [7, 11]$, $\bar{x}' = [6, 10]$, $X = \begin{bmatrix} 2 & 12 \\ 8 & 9 \\ 6 & 9 \\ 8 & 10 \end{bmatrix}$

$X - 1\bar{x}' = \begin{bmatrix} -4 & 2 \\ 2 & -1 \\ 0 & -1 \\ 2 & 0 \end{bmatrix} \rightarrow S = \begin{bmatrix} 8 & -3.33 \\ -3.33 & 2 \end{bmatrix}$

$S^{-1} = \frac{1}{16 - 11.11} \begin{bmatrix} 2 & 3.33 \\ 3.33 & 8 \end{bmatrix} = \begin{bmatrix} 0.409 & 0.681 \\ 0.681 & 1.636 \end{bmatrix}$

$T^2 = 4[-1, -1] \begin{bmatrix} 0.409 & 0.681 \\ 0.681 & 1.636 \end{bmatrix} \begin{bmatrix} -1 \\ -1 \end{bmatrix} = [-4, -4] \begin{bmatrix} -1.09 \\ -2.317 \end{bmatrix} = 13.628$

b. T^2 is distributed as $\frac{(n-1)p}{n-p} F_{p,n-p}$

$= \frac{(4-1)2}{(4-2)} F_{2,2} = 3F_{2,2}$

c. T^2 at $\alpha = 0.05 = 3 * 19 = 57$

$T^2 = 13.628 < 57 \rightarrow$ we can't reject the null hypothesis, $H_0: \mu = \begin{bmatrix} 7 \\ 11 \end{bmatrix}$

5.3

a. $T^2 = \frac{(n-1)\left|\sum_{j=1}^{n} (x_j - \mu_0)(x_j - \mu_0)'\right|}{\left|\sum_{j=1}^{n} (x_j - \bar{x})(x_j - \bar{x})'\right|} - (n-1)$

$= \frac{(4-1)\left|\begin{bmatrix} -5 \\ 1 \end{bmatrix}[-5, 1] + \begin{bmatrix} 1 \\ -2 \end{bmatrix}[1, -2] + \begin{bmatrix} -1 \\ -2 \end{bmatrix}[-1, -2] + \begin{bmatrix} 1 \\ -1 \end{bmatrix}[1, -1]\right|}{\left|\begin{bmatrix} -4 \\ 2 \end{bmatrix}[-4, 2] + \begin{bmatrix} 2 \\ -1 \end{bmatrix}[2\ -1] + \begin{bmatrix} 0 \\ -1 \end{bmatrix}[0\ -1] + \begin{bmatrix} 2 \\ 0 \end{bmatrix}[2\ 0]\right|} - (4-1)$

$= \frac{3\left|\begin{bmatrix} 25 & -5 \\ -5 & 1 \end{bmatrix} + \begin{bmatrix} 1 & -2 \\ -2 & 4 \end{bmatrix} + \begin{bmatrix} 1 & 2 \\ 2 & 4 \end{bmatrix} + \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}\right|}{\left|\begin{bmatrix} 16 & -8 \\ -8 & 4 \end{bmatrix} + \begin{bmatrix} 4 & -2 \\ -2 & 1 \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix} + \begin{bmatrix} 4 & 0 \\ 0 & 0 \end{bmatrix}\right|} - 3 = \frac{3\left|\begin{bmatrix} 28 & -6 \\ -6 & 10 \end{bmatrix}\right|}{\left|\begin{bmatrix} 24 & -10 \\ -10 & 6 \end{bmatrix}\right|} - 3$

$$= \frac{3(280-36)}{144-100} - 3 = \frac{732}{44} - 3 = 13.636$$

$$b. \ \Lambda = \left(\frac{\left| \sum_{j=1}^{n} (x_j - \bar{x})(x_j - \bar{x})' \right|}{\left| \sum_{j=1}^{n} (x_j - \mu_0)(x_j - \mu_0)' \right|} \right)^{n/2} = \left(\frac{44}{244} \right)^{4/2} = 0.0325$$

$$\Lambda^{2/n} = (0.0325)^{1/2} = 0.1803$$

5.4

$$a. \quad \bar{x} = \begin{bmatrix} 4.640 \\ 45.400 \\ 9.965 \end{bmatrix}, \quad S = \begin{bmatrix} 2.879 & 10.010 & -1.810 \\ 10.010 & 199.788 & -5.640 \\ -1.810 & -5.640 & 3.628 \end{bmatrix}$$

$$\lambda = \begin{bmatrix} 200.462 \\ 4.532 \\ 1.301 \end{bmatrix}, \quad e_1 = \begin{bmatrix} -0.051 \\ -0.998 \\ 0.029 \end{bmatrix}, \quad e_2 = \begin{bmatrix} -0.574 \\ 0.053 \\ 0.817 \end{bmatrix}, \quad e_3 = \begin{bmatrix} 0.817 \\ -0.025 \\ 0.575 \end{bmatrix}$$

half length of 1st axis $= \sqrt{\lambda_1} \sqrt{\frac{(n-1)p}{n(n-p)} F_{p,n-p}(0.1)} = \sqrt{200.462} \sqrt{\frac{19*3}{20*17} F_{3,17}(0.1)}$

$$= 14.158 \sqrt{\frac{57}{340} * 2.44} = 9.055$$

half length of 2nd axis $= \sqrt{\lambda_2} \sqrt{\frac{(n-1)p}{n(n-p)} F_{p,n-p}(0.1)} = \sqrt{4.532} * \sqrt{0.409}$

$$= 1.362$$

half length of 3rd axis $= \sqrt{\lambda_3} \sqrt{\frac{(n-1)p}{n(n-p)} F_{p,n-p}(0.1)} = \sqrt{1.301} * \sqrt{0.409}$

$$= 0.730$$

1st axis $= \pm \sqrt{\lambda_1} \sqrt{\frac{(n-1)p}{n(n-p)} F_{p,n-p}(\alpha)} * e_1 = \pm 9.055 * \begin{bmatrix} -0.051 \\ -0.998 \\ 0.029 \end{bmatrix} = \begin{bmatrix} -0.462 \\ -9.037 \\ 0.263 \end{bmatrix}$

2nd axis $= \pm 1.362 * \begin{bmatrix} -0.574 \\ 0.053 \\ 0.817 \end{bmatrix} = \begin{bmatrix} -0.782 \\ 0.072 \\ 1.113 \end{bmatrix}$

3rd axis $= \pm 0.73 \begin{bmatrix} 0.817 \\ -0.025 \\ 0.575 \end{bmatrix} = \begin{bmatrix} 0.596 \\ -0.018 \\ 0.420 \end{bmatrix}$

5.7 Simultaneous 95% T^2 intervals

$$\bar{x}_p - \sqrt{\frac{p(n-1)}{(n-p)} F_{p,n-p}(\alpha)} \sqrt{\frac{S_{pp}}{n}} \leq \mu_p \leq \bar{x}_p + \sqrt{\frac{p(n-1)}{(n-p)} F_{p,n-p}(\alpha)} \sqrt{\frac{S_{pp}}{n}}$$

first intreval $4.64 - \sqrt{\frac{3 * 19}{17} * 3.2} \sqrt{\frac{2.879}{20}} \leq \mu_1 \leq 4.64 + \sqrt{\frac{3 * 19}{17} * 3.2} \sqrt{\frac{2.879}{20}}$

$\rightarrow 4.64 - 3.276 * 0.379 \leq \mu_1 \leq 4.64 + 3.276 * 0.379$

$\rightarrow 3.398 \leq \mu_1 \leq 5.872$

second interval $45.4 - 3.276 \sqrt{\frac{199.788}{2}} \leq \mu_2 \leq 45.4 + 3.276 \sqrt{\frac{199.788}{20}}$

$\rightarrow 45.4 - 10.354 \leq \mu_2 \leq 45.4 + 10.354$

$\rightarrow 35.046 \leq \mu_2 \leq 55.754$

third interval $9.965 - 3.276 \sqrt{\frac{3.628}{20}} \leq \mu_3 \leq 9.965 + 3.276 \sqrt{\frac{3.628}{20}}$

$\rightarrow 8.57 \leq \mu_3 \leq 11.36$

95% Bonferroni intervals

$$\bar{x}_p - t_{n-1}\left(\frac{\alpha}{2p}\right) \sqrt{\frac{S_{pp}}{n}} \leq \mu_p \leq \bar{x}_p + t_{n-1}\left(\frac{\alpha}{2p}\right) \sqrt{\frac{S_{pp}}{n}}$$

first interval $4.64 - t_{19}\left(\frac{0.05}{6}\right) \sqrt{\frac{2.879}{20}} \leq \mu_1 \leq 4.64 + t_{19}\left(\frac{0.05}{6}\right) \sqrt{\frac{2.879}{20}}$

$\rightarrow 4.64 - 2.625 * 0.379 \leq \mu_1 \leq 4.64 + 2.625 * 0.379$

$\rightarrow 3.645 \leq \mu_1 \leq 5.635$

second interval $45.4 - 2.625 * 3.16 \leq \mu_2 \leq 45.4 + 2.625 * 3.16$

$\rightarrow 37.103 \leq \mu_2 \leq 53.697$

third interval $9.965 - 2.625 * 0.426 \leq \mu_3 \leq 9.965 + 2.625 * 0.426$

$8.847 \leq \mu_3 \leq 11.083$

* Bonferroni intervals are narrower than the simultaneous T^2 intervals