Distance definition

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The Johnson & Wichern textbook uses the term *distance* to mean *metric* so we will adopt the following definitions.

Definition (p. 37 Johnson & Wichern). *A function* $d : S \times S \to \mathbb{R}$ *is called a* distance function *or* metric *if it satisfies*

1. $$\forall_{P,Q\in S}\ d(P,Q) \geq 0 \quad (\mathrm{non-negativity})$$

2. $$\forall_{P,Q\in S}\ d(P,Q) = 0 \text{ iff } P = Q \quad (\text{identity of indiscernibles})$$

3. $$\forall_{P,Q\in S}\ d(P,Q) = d(Q,P) \quad (\text{symmetricity})$$

4. $$\forall_{P,Q,R\in S}\ d(P,Q) \leq d(P,R) + d(R,P) \quad (\text{triangle inequality})$$

Definition.
(https://en.wikipedia.org/wiki/Metric_(mathematics)#Pseudoquasimetrics)
A function $d : S \times S \to \mathbb{R}$ *is called a* pseudosemimetric *if it satisfies*

1. $$\forall_{P,Q\in S}\ d(P,Q) \geq 0 \quad (\mathrm{non-negativity})$$

2. $$\forall_{P\in S}\ d(P,P) = 0 \quad (\text{but } P \neq Q \text{ does not imply } d(P,Q) > 0)$$

3. $$\forall_{P,Q\in S}\ d(P,Q) = d(Q,P) \quad (\text{symmetricity})$$