16

TENTAMEN (EXAMINATION)

Tentamensdatum/*Examination date:* 2020-01-14
(åå-mm-dd/*yy-mm-dd*)

	Ifylles av student	Ifylles av vakt
AID-nummer ***AID number***	2 9 8 5	2 9 8 5
	Completed by student	*Completed by supervisor*

Utbildningskod/*Education code:* 732A97 **Modul/*Module:*** TENT

Kursnamn/*Course title:* Multivariate Statistics

Institution/*Department:* IDA

Jag intygar att varken mobil eller något annat otillåtet hjälpmedel finns tillgängligt under tentamen.
I confirm that no mobile or other non-permitted aids are available during the examination. ☑

Inlämnat: antal lösblad 6 **tentamensformulär** ☐
Enclosed: number of sheets ***exam booklet***

Markera behandlade uppgifter med X/*Mark tasks attempted with an X*

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
X här/*here*	X	X	X	X											
Erhållna poäng *Points obtained*	1/2	3	2 1/2	2											
	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30
X här/*here*															
Erhållna poäng *Points obtained*															

Anvisningar/*Instructions*

1. Skriv AID-nummer, datum, utb.kod, modul på varje blad som lämnas in/*Write AID number, date, edu.code and module on every sheet that is handed in*
2. På varje papper får högst en uppgift lösas om inget annat anges/ *Maximum one task per sheet unless otherwise instructed*
3. Skriv endast på papprets ena sida om inget annat anges/ *Use only one side of each sheet unless otherwise instructed*
4. Numrera de papper som lämnas in/*Number every sheet that is handed in*
5. Använd inte röd penna/*Do not use a red pen/pencil*

Sen inlämning ☐
Late hand in

Klockslag
Time

Orsak
Reason

∑ Poäng/*Points:* 8 Betyg/*Grade:* F

Examinator/*Examiner:* [signature]

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Problem 1

$\frac{1}{2}$

$$\Sigma = (1-\alpha)\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ \vdots & \vdots & \vdots \\ 0 & 0 & 1 \end{bmatrix}_{p\times p} + \alpha \begin{bmatrix} 1 \\ 1 \\ \vdots \\ 1 \end{bmatrix}_{p\times 1} \begin{bmatrix} 1 & 1 & \cdots & 1 \end{bmatrix}_{1\times p}$$

Σ is positive semi definite matrix if for any vector $a \in R^{p\times 1}$,
$a'\Sigma a \geq 0$

let $a' = [a_1 \; a_2 \cdots a_p]$

$$a'\Sigma a = (1-\alpha)[a_1 \; a_2 \cdots a_p]\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ \vdots & \vdots & \vdots \\ 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} a_1 \\ a_2 \\ \vdots \\ a_p \end{bmatrix} + \alpha[a_1 \; a_2 \cdots a_p]\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ \vdots & \vdots & \vdots \\ 0 & 0 & 1 \end{bmatrix} * \begin{bmatrix} a_1 \\ a_2 \\ \vdots \\ a_p \end{bmatrix}$$

$$= (1-\alpha)(a_1^2 + a_2^2 + \cdots + a_p^2) + \alpha(a_1^2 + a_2^2 + \cdots + a_p^2)$$

and since $\alpha \geq 0$ and $a^2 \geq 0 \Rightarrow a'\Sigma a \geq 0$

Therefore $\Sigma \geq 0$ is semi positive matrix

also $$\Sigma = (1-\alpha)\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ \vdots & \vdots & \vdots \\ 0 & 0 & 1 \end{bmatrix}_{p\times p} + \alpha \begin{bmatrix} 1 \\ 1 \\ \vdots \\ 1 \end{bmatrix}_{p\times 1} \begin{bmatrix} 1 & 1 & \cdots & 1 \end{bmatrix}_{1\times p}$$

$$= \begin{bmatrix} 1-\alpha & 0 & 0 \\ 0 & 1-\alpha & 0 \\ \vdots & \ddots & \vdots \\ 0 & 0 & 1-\alpha \end{bmatrix}_{p\times p} + \begin{bmatrix} a & 0 & 0 \\ 0 & a & 0 \\ \vdots & \ddots & \vdots \\ 0 & 0 & a \end{bmatrix}_{p\times p}$$

$$= \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ \vdots & \ddots & \vdots \\ 0 & 0 & 1 \end{bmatrix}$$

Since the off-diagonal numbers are all zero (i.e: identical)
therfore Σ is semitric

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Problem 2

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$$\mu_{GX} = G\mu = G\begin{bmatrix} 0 \\ 0 \\ \vdots \\ 0 \end{bmatrix} = 0 = \mu$$

$$\Sigma_{GX} = G'\Sigma G$$

and since G is orthogonal $G'G = G^{-1}G = I$

$$\Rightarrow \Sigma_{GX} = \cancel{G}^{-1}\Sigma\cancel{G} = \Sigma$$

$\Rightarrow GX \sim N(\mu, \Sigma)$ GX has the same distribution as X

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2½

Problem 3

wrong test

a. $T^2 = n(\bar{x}-\mu)' S^{-1} (\bar{x}-\mu)$

$$= 25 \begin{bmatrix} 3.72 & 1.84 \end{bmatrix} \begin{bmatrix} 0.022 & -0.015 \\ -0.015 & 0.021 \end{bmatrix} \begin{bmatrix} 3.72 \\ 1.84 \end{bmatrix}$$

2

$$= 25 \begin{bmatrix} 3.72 & 1.84 \end{bmatrix} \begin{bmatrix} 0.05424 \\ -0.01716 \end{bmatrix}$$

$$= 4.25496$$

because the n is small we use $\frac{(n-1)p}{(n-p)} F_{p,n-p}(\alpha)$

$$\frac{(n-1)p}{(n-p)} F_{p,n-p}(\alpha) = \frac{19 * 2}{18} * F_{2,18}(0.05)$$

$$= 2.111 * 3.55 = 7.494$$

since $T^2 = 4.25496 < 7.494$ we do Not reject the null hypothesis $H_0: \mu' = (182, 182)$

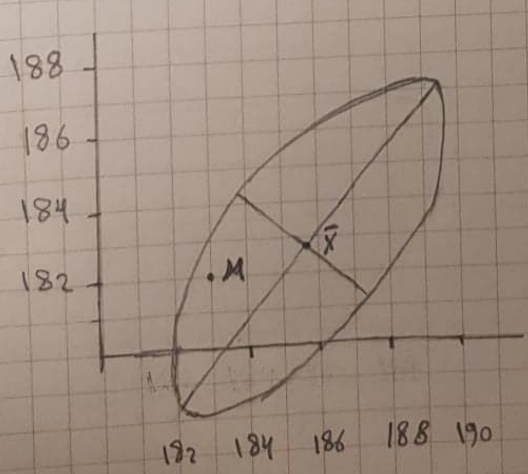
b. length of half axes $= \sqrt{\lambda_i} \sqrt{\frac{p(n-1)}{n(n-p)} F_{p,n-p}(\alpha)}$

½

long axis $= 12.69 * \sqrt{\frac{3.55}{25}} = 4.782$

short axis $= 5.215 * \sqrt{\frac{3.55}{25}} = 1.965$

direction of axes?



μ lie inside the 95% confidence interval

why?

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c. The assumed couarince matrix claim that the two variables are uncorrelated since $\sigma_{12} = 0$

However, given the sample data there exist a significant correlation between the two variables, since $S_{12} = 66.875 \rightarrow \rho_{12} = 0.71$

it does not seem reasonable that the measurements come from a distribution with such a variance - covariance matrix, Σ_1.

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Problem 4

a. $T^2 = n(\bar{x}-\mu)' \Sigma^{-1} (\bar{x}-\mu)$

$$= [3.72 \quad 1.84] \begin{bmatrix} 1/75 & -1/150 \\ -1/150 & 1/75 \end{bmatrix} \begin{bmatrix} 3.72 \\ 1.84 \end{bmatrix}$$

$= 1.13888 - 0.00049$

$= 1.13839$

1

since $(\bar{x}-\mu)\Sigma^{-1}(\bar{x}-\mu)$ is distributed as χ^2_p we test with χ_p^2

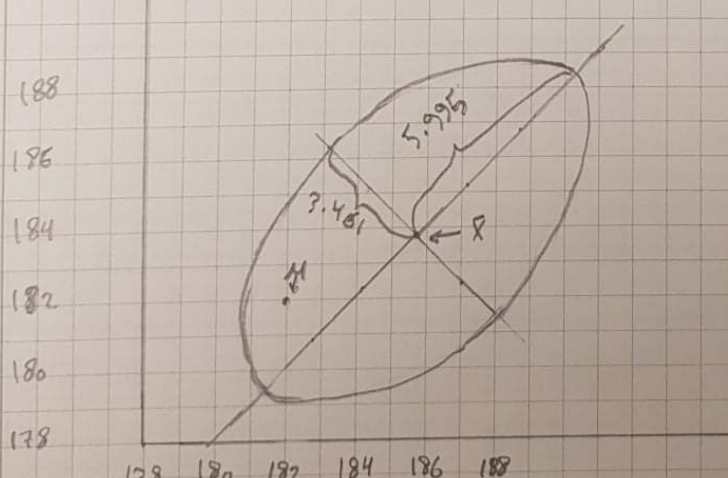
$\chi^2_2(0.05) = 5.99$

since $T^2 = 1.13839 < 5.99$ we do not reject the null hypothesis

b. half length of long axis $= 12.247 * \sqrt{\frac{5.99}{25}} = 5.995$

half length of short axis $= 7.071 * \sqrt{\frac{5.99}{25}} = 3.461$

direction?



1/2

M lie inside the 95 % confidence interval

why?

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c. The new Σ_2 assume correlated variables since $\sigma_{12} = 50$ and this is more reasonable because it is close to the sample data

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