9.10 the specific variances $\Psi = \Sigma - LL'$ (1)

a. Skull length : 0.5976

Skull breadth: 0.758

Femur length: 0.122

Tibia Length: 0

Hemerus Length: 0.0095

Ulna Length : 0.0938

b. $h_i^2 = l_{i1}^2 + l_{i2}^2 + \cdots + l_{im}^2$

→ Skull Length = $0.602^2 + 0.2^2 = 0.4024$

Skull breadth = $0.467^2 + 0.154 = 0.2418$

Femur length = $0.926^2 + 0.143^2 = 0.879$

Tibia Length = $1^2 + 0 = 1$

Hemerus Length = $0.874^2 + 0.476^2 = 0.9905$

Ulna Length = $0.894^2 + 0.327^2 = 0.9062$

c. $\frac{1}{p} \sum_{j=1}^{p} l_{1j}^2 = \frac{1}{6} \sum_{j=1}^{6} l_{1j}^2 = \frac{1}{6} * 4.001 = 0.667$

$\frac{1}{6} \sum_{j=1}^{6} l_{2j}^2 = \frac{1}{6} * 0.41711 = 0.0695$

d.

$$R - LL' - \Psi = \begin{bmatrix} 0 & 0.193 & -0.017 & 0 & 0 & 0.003 \\ 0.193 & 0 & -0.033 & 0 & 0 & -0.018 \\ -0.017 & -0.033 & 0 & 0 & -0.001 & 0.0034 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -0.001 & 0 & 0 & 0 \\ 0.003 & -0.018 & 0.0034 & 0 & 0 & 0 \end{bmatrix}$$

9.11 Variance $= \frac{1}{p}\sum_{j=1}^{m}\left[\sum_{i=1}^{p}\tilde{l}_{ij}^{*4} - \left(\sum_{i=1}^{p}\tilde{l}_{ij}^{*2}\right)/p\right]$

$= \frac{1}{6}[4.951227 - 29.51167/6 + 0.086299 - 0.322097/6]$

$= 0.01087$ the varimax criterion for the un-rotated factor

also 0.0116896 is the varimax criterion for the rotated factor

rotated loadings have simple structure, however unrotated loadings have clear interpretation.

9.19

a. Maximum Likelihood Factor analysis of the correlation matrix.

m = 2	factor 1	factor 2	communality
z_1	0.777	0.572	0.931
z_2	0.798	0.542	0.930
z_3	0.621	0.700	0.877
z_4	0	1	1
z_5	0.420	0.591	0.525
z_6	0.605	0.147	0.388
z_7	0.895	0.413	0.971
Variance	2.9692	2.6518	5.6210
% Var	0.424	0.379	0.803

m = 3	factor 1	factor 2	factor 3	communality
z_1	0.943	0.269	-0.025	0.962
z_2	0.919	0.241	0.267	0.974
z_3	0.926	0.074	-0.135	0.901
z_4	0.776	-0.610	-0.091	0.981
z_5	0.755	-0.017	0.051	0.573
z_6	0.542	0.538	-0.607	0.952
z_7	0.868	0.399	0.188	0.948

	factor1	factor2	factor3	Communality
variance	4.8713	0.9562	0.5046	6.2921
% var	0.690	0.137	0.072	0.899

b.

$m=2$

unrotated factor loadings

	factor1	factor2	Communality
Z_1	0.777	0.572	0.931
Z_2	0.798	0.542	0.930
Z_3	0.621	0.700	0.877
Z_4	0	1	1
Z_5	0.420	0.591	0.525
Z_6	0.605	0.147	0.388
Z_7	0.895	0.413	0.971
variance	2.9692	2.6518	5.6210
% var	0.424	0.379	0.803

Rotated factor loadings

	factor 1	factor2	Communality
Z_1	0.852	0.452	0.931
Z_2	0.868	0.419	0.930
Z_3	0.717	0.602	0.877
Z_4	0.146	0.989	1
Z_5	0.502	0.523	0.525
Z_6	0.620	0.057	0.388
Z_4	0.946	0.277	0.971
variance	3.5469	2.0741	5.6210
var %	0.507	0.296	0.803

(a)

	factor 1	factor2	factor3	Communality
z_1	0.943	0.269	-0.025	0.962
z_2	0.919	0.241	0.267	0.974
z_3	0.936	0.074	-0.175	0.901
z_4	0.776	-0.610	-0.091	0.982
z_5	0.755	-0.017	0.051	0.573
z_6	0.542	0.538	-0.607	0.952
z_7	0.868	0.399	0.188	0.948
variance	4.8313	0.9562	0.5046	6.2921
% var	0.690	0.137	0.072	0.899

m=3
unrotated factor loadings

	factor1	factor2	factor3
z_1	0.788	-0.371	-0.451
z_2	0.916	-0.313	-0.193
z_3	0.641	-0.546	-0.444
z_4	0.260	-0.956	-0.025
z_5	0.571	-0.459	-0.191
z_6	0.296	-0.050	-0.928
z_7	0.903	-0.179	-0.318
variance	3.1682	1.6858	1.4381
% var	0.453	0.241	0.205

rotated factor loadings

c. $\psi_{ii} = 1 - h_i^2$, communalities already obtained in the previous steps

	when m=2, specific variance	when m=3
Sales growth =	0.069	0.038
Sales profitability =	0.07	0.026
new-account Sales =	0.123	0.099
creativity test =	0	0.018
mechanical reasoning =	0.475	0.427
Abstract reasoning =	0.612	0.048
Mathematics test =	0.029	0.052

$$LL' + \Psi = \begin{bmatrix} 1 & 0.925 & 0.906 & 0.570 & 0.706 & 0.671 & 0.921 \\ 0.925 & 1 & 0.842 & 0.542 & 0.703 & 0.466 & 0.944 \\ 0.906 & 0.842 & 0.999 & 0.693 & 0.699 & 0.629 & 0.817 \\ 0.570 & 0.542 & 0.693 & 1.001 & 0.592 & 0.148 & 0.413 \\ 0.706 & 0.703 & 0.699 & 0.592 & 1 & 0.369 & 0.658 \\ 0.671 & 0.466 & 0.629 & 0.148 & 0.369 & 1 & 0.571 \\ 0.921 & 0.944 & 0.817 & 0.413 & 0.658 & 0.571 & 1 \end{bmatrix}$$

the estimate in matrix $LL' + \Psi$ for $m=3$ matches well with the original correlation matrix, however it is difficult to provide interpretations for the factors, however "creative test" and "mathematical test" have the highest communality for rotated factor loadings

d.

$m < \frac{1}{2}(2p+1-\sqrt{8p+1}) \rightarrow m < 3.725$

for $m=3$ $\rightarrow (50-1-(2*7+4*3+5)/6)\ ln \frac{|\hat{L}\hat{L}'+\Psi|}{|R|} > \chi^2_{[(7-3)^2-7-3]/2}(\alpha)$

$\rightarrow 62.067 > 11.3$ we reject the null hypothesis

the value of the determinant is very small and rounding error could affect the calculation of the test.

e. $\tilde{f}' = [-0.538 \quad 0.443 \quad 0.651]$

10.2

a. $\Sigma_{11}^{-1/2} = \begin{bmatrix} 0.36667 & -0.06667 \\ -0.06667 & 0.46667 \end{bmatrix}$

$\Sigma_{22}^{-1} = \begin{bmatrix} 0.1842 & 0.05263 \\ 0.05263 & 0.1579 \end{bmatrix}$

$\Sigma_{11}^{-1/2} \Sigma_{12} \Sigma_{22}^{-1} \Sigma_{21} \Sigma_{11}^{-1/2} = \begin{bmatrix} 0.27557 & -0.032 \\ -0.032 & 0.2690 \end{bmatrix}$

$\lambda_1 = 0.30445 \;,\; \lambda_2 = 0.2404$

$\Rightarrow \rho_1^* = 0.55 \;,\; \rho_2^* = 0.49$

b. $e_1' = [0.742 \quad -0.670]$
$e_2' = [0.670 \quad 0.742]$

$a_1 = \Sigma_{11}^{-1/2} e_1 = \begin{bmatrix} 0.3168 \\ -0.362 \end{bmatrix}$

$\Rightarrow b_1 \propto \Sigma_{22}^{-1} \Sigma_{21} a_1 = k \Sigma_{22}^{-1} \Sigma_{21} a_1 \Rightarrow b_1 = k \begin{bmatrix} 0.20 \\ -0.05 \end{bmatrix}$

$k = \frac{1}{\operatorname{var}(V_1)} \;,\; b_1' \Sigma_{22} b_1 = 0.2975$

$\Rightarrow b_1 = \frac{1}{\sqrt{0.2975}} \begin{bmatrix} 0.20 \\ -0.05 \end{bmatrix} = \begin{bmatrix} 0.36 \\ -0.09 \end{bmatrix}$

$\therefore U_1 = a_1' X^{(1)} = 0.32 X_1^{(1)} - 0.36 X_2^{(1)}$
$V_1 = b_1' X^{(2)} = 0.36 X_1^{(2)} - 0.09 X_2^{(2)}$
$U_2 = a_2' X^{(1)} = 0.20 X_1^{(1)} + 0.30 X_2^{(1)}$
$V_2 = b_2' X^{(2)} = 0.23 X_1^{(2)} + 0.30 X_2^{(2)}$

c.

$$E\begin{bmatrix} u_1 \\ u_2 \\ \hline v_1 \\ v_2 \end{bmatrix} = \begin{bmatrix} 0.32 * (-3) - 0.36 * 2 \\ 0.20 * (-3) + 0.30 * 2 \\ \hline 0.36 * 0 - 0.09 * 1 \\ 0.23 * 0 + 0.30 * 1 \end{bmatrix} = \begin{bmatrix} -1.68 \\ 0 \\ \hline -0.09 \\ 0.30 \end{bmatrix}$$

$$Cov\begin{bmatrix} U \\ \hline V \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0.55 & 0 \\ 0 & 1 & 0 & 0.49 \\ 0.55 & 0 & 1 & 0 \\ 0 & 0.49 & 0 & 1 \end{bmatrix}$$

10.12

a. $\hat{\rho}_1^* = 0.69$, $\hat{\rho}_2^* = 0.19$

H_0: all $\rho_i^* = 0$

$$-\left(n-1-\frac{1}{2}(p+q+1)\right)\ln\left[(1-\hat{\rho}_1^{*2})(1-\hat{\rho}_2^{*2})\right] \geq \chi^2_{pq}(0.05)$$

$\rightarrow 45.09 \geq 12.592$ we reject the null hypothesis

H_0: $\rho_1^* \neq 0$, $\rho_2^* = 0$

$\rightarrow 2.426 < 5.991$ we do not reject the null hypothesis

b. $\hat{\omega}_1 = aZ_1^{(1)} + bZ_2^{(1)}$

$\rightarrow \hat{U}_1 = 0.7689 * \text{dining} + 0.2721 * \text{movies}$

$\hat{V}_1 = 0.0491 * \text{headage} + 0.8975 * \text{income} + 0.19 * \text{edu level}$

c.

$X^{(1)}$ variables	$\hat{U}_1$	$\hat{V}_1$
dining	0.99	0.68
movies	0.89	0.61

$X^{(2)}$ variables	U_1	V_1
age	0.29	0.42
income	0.68	0.98
edu level	0.35	0.51

d. $\hat{u}_1$ is a measure of family intertainment outside the home. $\hat{u}_1$ can be considered as family status

e. It can be seen that for the relation between demographic variables and consumption variables focus on R_{12} or R_{21}.

It suggest that annual frequency of dinning in arestaurant and attendin movies depends directly on the annual family income.