TENTAMEN (EXAMINATION)

2

Tentamensdatum/***Examination date:*** 2020-01-14
(åå-mm-dd/*yy-mm-dd*)

	Ifylles av student	Ifylles av vakt
AID-nummer / ***AID number***	3 0 0 2	3 0 0 2
	Completed by student	*Completed by supervisor*

Utbildningskod/***Education code:*** 732A97 **Modul**/***Module:*** TENT

Kursnamn/***Course title:*** Multivariate statistical method

Institution/***Department:*** IDA

Jag intygar att varken mobil eller något annat otillåtet hjälpmedel finns tillgängligt under tentamen.
I confirm that no mobile or other non-permitted aids are available during the examination. ☒

Inlämnat: antal lösblad 6 **tentamensformulär** ☐
Enclosed: number of sheets ***exam booklet***

Markera behandlade uppgifter med X/***Mark tasks attempted with an X***

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
X här/*here*	X	X	X	←	X										
Erhållna poäng *Points obtained*	2	2	4	4											
	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30
X här/*here*															
Erhållna poäng *Points obtained*															

Anvisningar/*Instructions*

1. Skriv AID-nummer, datum, utb.kod, modul på varje blad som lämnas in/*Write AID number, date, edu.code and module on every sheet that is handed in*
2. På varje papper får högst en uppgift lösas om inget annat anges/ *Maximum one task per sheet unless otherwise instructed*
3. Skriv endast på papprets ena sida om inget annat anges/ *Use only one side of each sheet unless otherwise instructed*
4. Numrera de papper som lämnas in/*Number every sheet that is handed in*
5. Använd inte röd penna/*Do not use a red pen/pencil*

Sen inlämning ☐
Late hand in

Klockslag ____
Time

Orsak ____
Reason

Σ Poäng/*Points:* 12 Betyg/*Grade:* D

Examinator/*Examiner:* K. Burdy

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Problem 1.

Prove: $\Sigma = (1-\alpha) I + \alpha \vec{1}_p \vec{1}_p^T$

$= A + B$

let $(1-\alpha) I$ equals A,

and $\alpha(\vec{1}_p \vec{1}_p^T)$ equals B.

So.

I prove symmetric.

① $A = (1-\alpha) I$

$\because I$ is the identity matrix ①

$\alpha \geq 0$ ②

$\Rightarrow A$ is symmetric.

② $B = \alpha \cdot \vec{1}_p \cdot \vec{1}_p^T$

$\because \alpha \geq 0$

$\vec{1}_p$ is a vector of p ones.

$\therefore \vec{1}_p \cdot \vec{1}_p^T$ is symmetric

$\therefore \alpha \cdot \vec{1}_p \cdot \vec{1}_p^T = B$ is symmetric.

From ~~above~~ provement above,

$\Sigma = A + B$

according to matrix calcu,

$\therefore \Sigma$ is symmetric.

II prove positive-semi-definite.

according to definition, positive-semi-definite means all the eigen-values of matrix should ≥ 0 No

① $A = (1-\alpha) I$, Not in general

so $A = P_A \cdot \Lambda_A \cdot P_A^{-1}$

and all the $\mathrm{diag}(\Lambda) > 0$,

② $B = \alpha \cdot \vec{1}_p \cdot \vec{1}_p^T$

$B = P_B \cdot \Lambda_B \cdot P_B^{-1}$

$\because \vec{1}_p$ is a vector of p ones,

$\therefore$ all the $\mathrm{diag}(B) \geq 0$

From provement above,

$\Sigma = A + B = P_\Sigma \cdot \Lambda_\Sigma \cdot P_\Sigma^{-1}$

$\Lambda_\Sigma = \Lambda_A + \Lambda_B$

all ~~elements~~ diagnal elements of $\Lambda_\Sigma \geq 0$.

$\therefore \Sigma$ is positive-semi-~~posi~~ definite.

No

$\therefore \Sigma$ is symmetric-positive-semi-definite.

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Problem 2:

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$\vec{X} \sim N(\vec{\mu}, \Sigma)$

$\therefore E(\vec{x}) = \vec{\mu}$

$V(\vec{x}) = \Sigma$

when $\vec{\mu} = \vec{0}$

$\Sigma = 6^2 I$

$\therefore E(\vec{x}) = \vec{0}$

$V(\vec{x}) = 6^2 I$

which I is the identity matrix of appro size.

$E(G\vec{x}) = 1 \cdot E(\vec{x})$

$= \vec{0}$

$\because G$ is an orthogonal matrix,

$\therefore$ All row and column vectors are orthogonal

$Var(G\vec{x}) = G \cdot Var(\vec{x}) \cdot G'$

$= 1 \cdot Var(\vec{x}) \cdot 1$

$= 6^2 I$

$\therefore E(\vec{x}) = E(G\vec{x}) = \vec{0}$

$V(\vec{x}) = V(G\vec{x}) = 6^2 I$

$\therefore \vec{X}$ and $G\vec{x}$ have the same distribution.

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Problem 3.

(a)

$H_0: \bar{x} = \mu$

$H_1: \bar{x} \neq \mu.$

$T^2 = (\bar{x} - \vec{\mu})^T (\frac{1}{n} S)^{-1} (\bar{x} - \vec{\mu})$

Why you are given Σ!

if $T^2 > \frac{(n-1)p}{n-p} F_{p,n-p}(\alpha)$

reject H_0.

$\mu = (182, 182)^T$ ①

$\bar{x} = (185.72, 183.84)^T$ ②

$n = 25$ ③

$S^{-1} = \begin{bmatrix} 0.022 & -0.015 \\ -0.015 & 0.021 \end{bmatrix}$ ④

$\Rightarrow$ put ①, ②, ③, ④ to T^2,

$$\Rightarrow T^2 = \left[\begin{pmatrix} 185.72 \\ 183.84 \end{pmatrix} - \begin{pmatrix} 182 \\ 182 \end{pmatrix}\right]^T \times 25$$

$$\times \begin{bmatrix} 0.022 & -0.015 \\ -0.015 & 0.021 \end{bmatrix}$$

$$\times \left[\begin{pmatrix} 185.72 \\ 183.84 \end{pmatrix} - \begin{pmatrix} 182 \\ 182 \end{pmatrix}\right]$$

$$= (3.72, 1.84) \times 25 \times \begin{bmatrix} 0.022 & -0.015 \\ -0.015 & 0.021 \end{bmatrix} \times \begin{pmatrix} 3.72 \\ 1.84 \end{pmatrix}$$

$$= 25 \times (0.05424, -0.01718) \times \begin{pmatrix} 3.72 \\ 1.84 \end{pmatrix}$$

$$= 25 \times 0.1701616$$

$$= 4.25404$$

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$n = 25 \quad \alpha = 0.05$

$p = 2$

$F_{2,23}(0.05) = 3.42$

$\therefore \frac{(n-1) \cdot p}{n-p} \cdot F_{p,n-p}(\alpha)$

$= \frac{24 \times 2}{23} \times 3.42$

≈ 7.13739

$\therefore T^2 < \frac{(n-1)p}{n-p} \cdot F_{p,n-p}(\alpha)$

$\therefore$ ~~accept~~ ~~except~~ fail to reject H_0, conclude the observed sample comes from the desired distribution.

(b)

$c^2 = 7.13739 = \frac{(n-1)p}{n-p} \cdot F_{p,n-p}(\alpha)$

$\sqrt{\lambda_1} \cdot \sqrt{\frac{1}{n} \cdot c^2}$

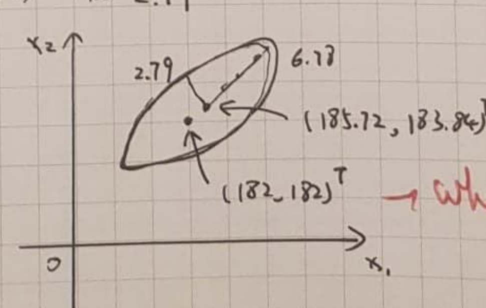
$= \sqrt{161.055} \times \sqrt{\frac{1}{25} \times 7.13739}$

≈ 6.78

$\sqrt{\lambda_2} \cdot \sqrt{\frac{1}{n} \cdot c^2}$

$= \sqrt{27.201} \times \sqrt{\frac{1}{25} \times 7.13739}$

≈ 2.79



→ why?

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From the graph and (a).

we know $\mu = (182, 182)^T$

~~[illegible]~~ falls in 95% confidence interval. so $(182, 182)^T$ must be in the ellipse.

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(c)

$$\Sigma_1 = \begin{bmatrix} 100 & 0 \\ 0 & 100 \end{bmatrix}$$

From the Σ_1, we see that $cov(X_1, X_2) = 0$, and the measurements assumed come from a Normal distribution.

$\therefore$ X_1 and X_2 are independent.

1

$$S = \begin{bmatrix} 91.481 & 66.875 \\ 66.875 & 96.775 \end{bmatrix}$$

the $cov(X_1, X_2) = 66.875$, which means X_1 and X_2 are highly correlated to each other.

So it is not reasonable that the sample comes from a distribution with var-cov matrix Σ_1.

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Problem 4.

(a) $H_0: \bar{x} = \mu$

1

$H_1: \bar{x} \neq \mu$

$T^2 = (\bar{x} - \mu)^T (\frac{1}{n} S)^{-1} (\bar{x} - \mu)$ 2

if $T^2 > \frac{(n-1) \cdot p}{n-p} F_{p, n-p}(\alpha)$,

reject H_0.

$\mu = (182, 182)^T$ ①

$\bar{x} = (185.72, 183.84)^T$ ②

$n = 25$ ③

$S^{-1} = \begin{bmatrix} 0.022 & -0.015 \\ -0.015 & 0.021 \end{bmatrix}$ ④

$\Rightarrow$ put ① ② ③ ④ to T^2.

$\Rightarrow T^2 = (3.72, 1.84) \times 25 \times \begin{bmatrix} 0.022 & -0.015 \\ -0.015 & 0.021 \end{bmatrix}$

$\times \begin{pmatrix} 3.72 \\ 1.84 \end{pmatrix}$

$= 4.25404$

$n = 25 \quad p = 2 \quad \alpha = 0.05$

$F_{2,23}(0.05) = 3.42$

$\therefore \frac{(n-1) \cdot p}{n-p} \cdot F_{p, n-p}(\alpha)$

$= \frac{24 \times 2}{23} \times 3.42$

≈ 7.13739

$\therefore T^2 < \frac{(n-1) p}{n-p} \cdot F_{p,n-p}(\alpha)$

Fail to reject

$\therefore$ ~~accept~~ H_0, conclude the observed sample comes from the desired distribution.

(b) $c^2 = 7.13739 = \frac{(n-1) p}{n-p} \cdot F_{p, n-p}(\alpha)$

$\sqrt{\lambda_1} \cdot \sqrt{\frac{1}{n} c^2}$

~~$= \sqrt{\cdots}$~~

$= \sqrt{161.055 \times \frac{1}{25} \times 7.13739}$

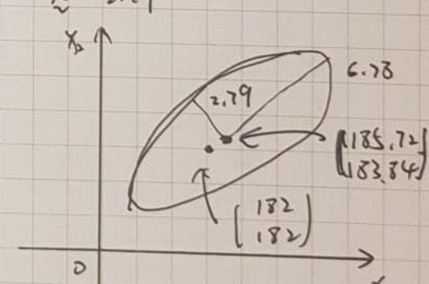
≈ 6.78

$\sqrt{\lambda_2} \cdot \sqrt{\frac{1}{n} c^2}$

$= \sqrt{27.201 \times \frac{1}{25} \times 7.13739}$

≈ 2.79

7



From the graph and (a), we know $\mu = (182, 182)^T$ falls in 95% confidence interval, so $(182, 182)^T$ must be in the ellipse.

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(c)

$$\Sigma_2 = \begin{bmatrix} 100 & 50 \\ 50 & 100 \end{bmatrix}$$

Compared to Σ_1, we can see the $cov(X_1, X_2) = 50$, which means these two variables are correlated.

✓ 1

$$S = \begin{bmatrix} 91.481 & 66.875 \\ 66.875 & 96.775 \end{bmatrix}.$$

the $cov(X_1, X_2) = 66.875$.

So it seems more reasonable that the sample shows similar variable relationship with the population.