* Bonferroni intervals are narrower than the simultaneous T^2 intervals

5.5

a. $n(C\bar{x})'(CSC')^{-1}C\bar{x} \le \frac{(n-1)(q-1)}{(n-q+1)} F_{q-1,n-q+1}(\alpha)$ ①

the equality as $\mu_1 - \mu_2 = 0$ and $\mu_2 - \mu_3 = 0$

$$C = \begin{bmatrix} 1 & -1 & 0 \\ 0 & 1 & -1 \end{bmatrix}$$

$$C\bar{x} = \begin{bmatrix} 1 & -1 & 0 \\ 0 & 1 & -1 \end{bmatrix} \begin{bmatrix} 46.1 \\ 57.3 \\ 50.4 \end{bmatrix} = \begin{bmatrix} -11.2 \\ 6.9 \end{bmatrix}$$

$$CSC' = \begin{bmatrix} 1 & -1 & 0 \\ 0 & 1 & -1 \end{bmatrix} \begin{bmatrix} 101.3 & 63.0 & 71.0 \\ 63.0 & 80.2 & 55.6 \\ 71.0 & 55.6 & 97.4 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ -1 & 1 \\ 0 & -1 \end{bmatrix} = \begin{bmatrix} 1 & -1 & 0 \\ 0 & 1 & -1 \end{bmatrix} \begin{bmatrix} 38.3 & -8.0 \\ -17.2 & 24.6 \\ 15.4 & -41.8 \end{bmatrix}$$

$$CSC' = \begin{bmatrix} 55.5 & -32.6 \\ -32.6 & 66.4 \end{bmatrix} \Rightarrow (CSC')^{-1} = \frac{1}{2622.44} \begin{bmatrix} 66.4 & 32.6 \\ 32.6 & 55.5 \end{bmatrix} = \begin{bmatrix} 0.025 & 0.012 \\ 0.012 & 0.021 \end{bmatrix}$$

$$T^2 = 40[-11.2 \;\; 6.9] \begin{bmatrix} 0.025 & 0.012 \\ 0.012 & 0.021 \end{bmatrix} \begin{bmatrix} -11.2 \\ 6.9 \end{bmatrix} = 40[-11.2 \;\; 6.9] \begin{bmatrix} -0.197 \\ 0.0105 \end{bmatrix}$$

$$T^2 = 91.154$$

$$\frac{(n-1)(q-1)}{n-q+1} F_{q-1,n-q+1}(\alpha) = \frac{39 * 2}{38} * F_{2,38} = 6.671$$

$T^2 = 91.154 > 6.671$ $\Rightarrow$ we reject the null hypothesis at 5% level that $H_0: C\mu = 0$

b. first interval for $\mu_1 - \mu_2$.

$$c_1'\bar{x} \pm \sqrt{\frac{(n-1)(q-1)}{(n-q+1)} F_{q-1,\, n-q+1}(\alpha)} \sqrt{\frac{c_1' S c_1}{n}} = -11.2 \pm \sqrt{\frac{78}{38} \times 3.25} \times \sqrt{\frac{77.5}{40}}$$

$\Rightarrow -11.2 \pm 2.583 \times 1.388 = -11.2 \pm 3.584$

$\Rightarrow [-14.784, -7.616]$

second interval for $\mu_2 - \mu_3$

$$c_2'\bar{x} \pm \sqrt{\frac{(n-1)(q-1)}{(n-q+1)} F_{q-1,\, n-q+1}(\alpha)} \sqrt{\frac{c_2' S c_2}{n}} = 6.9 \pm 2.583 \times \sqrt{\frac{66.4}{40}}$$

$\Rightarrow 6.9 \pm 3.328 \Rightarrow [3.572, 10.228]$

6.8

a. $\bar{x}_{11} = \dfrac{6+5+8+4+7}{5} = 6$

$\bar{x}_{21} = \dfrac{3+1+2}{3} = 2$

$\bar{x}_{31} = \dfrac{2+5+3+2}{4} = 3$

$\bar{x}_1 = \dfrac{6+5+8+4+7+3+1+2+2+5+3+2}{12} = 4$

$\therefore x_{11_1} = \bar{x}_1 + (\bar{x}_{11} - \bar{x}_1) + (x_{11} - \bar{x}_{11}) = 4 + (6-4) + (6-6) = 4 + 2 + 0$

$$\begin{bmatrix} 6 & 5 & 8 & 4 & 7 \\ 3 & 1 & 2 & & \\ 2 & 5 & 3 & 2 & \end{bmatrix} = \begin{bmatrix} 4 & 4 & 4 & 4 & 4 \\ 4 & 4 & 4 & & \\ 4 & 4 & 4 & 4 & \end{bmatrix} + \begin{bmatrix} 2 & 2 & 2 & 2 & 2 \\ -2 & -2 & -2 & & \\ -1 & -1 & -1 & -1 & \end{bmatrix} + \begin{bmatrix} 0 & -1 & 2 & -2 & 1 \\ 1 & -1 & 0 & & \\ -1 & 2 & 0 & -1 & \end{bmatrix}$$

observation = sample mean + treatment effect + residual

$SS_{obs} = 246$

$SS_{mean} = 192$

$SS_{tr} = 36$

$SS_{res} = 18$

total $SS = SS_{obs} - SS_{mean} = 246 - 192 = 54$

$$\bar{x}_{12} = \frac{7+9+6+9+9}{5} = 8$$

$$\bar{x}_{22} = \frac{3+6+3}{3} = 4$$

$$\bar{x}_{32} = \frac{3+1+1+3}{4} = 2$$

$$\bar{x}_{2} = \frac{7+9+6+9+9+3+6+3+3+1+1+3}{12} = 5$$

$$\begin{bmatrix} 7 & 9 & 6 & 9 & 9 \\ 3 & 6 & 3 & & \\ 3 & 1 & 1 & 3 & \end{bmatrix} = \begin{bmatrix} 5 & 5 & 5 & 5 & 5 \\ 5 & 5 & 5 & & \\ 5 & 5 & 5 & 5 & \end{bmatrix} + \begin{bmatrix} 3 & 3 & 3 & 3 & 3 \\ -1 & -1 & -1 & & \\ -3 & -3 & -3 & -3 & \end{bmatrix} + \begin{bmatrix} -1 & 1 & -2 & 1 & 1 \\ -1 & 2 & -1 & & \\ 1 & -1 & -1 & 1 & \end{bmatrix}$$

observation = sample mean + treatment effect + residual

$SS_{obs} = 402$

$SS_{mean} = 300$

$SS_{tr} = 84$

$SS_{res} = 18$

total SS = 402 - 300 = 102

b.

Source of variation	Matrix of sum of squares and cross products	Degrees of freedom
treatment	$B = \begin{bmatrix} 36 & 48 \\ 48 & 84 \end{bmatrix}$	$3-1=2$
Residual	$W = \begin{bmatrix} 18 & -13 \\ -13 & 18 \end{bmatrix}$	$5+3+4-3=9$
Total (corrected)	$\begin{bmatrix} 54 & 35 \\ 35 & 102 \end{bmatrix}$	11

c. $$\Lambda^* = \frac{|W|}{|B+W|} = \frac{155}{4283} = 0.0362$$

$H_0: \tau_1 = \tau_2 = \tau_3 = 0$, H_1: at least one $\tau_i \neq 0$

$$\left(\frac{1-\sqrt{\Lambda^*}}{\sqrt{\Lambda^*}}\right)\left(\frac{\sum n_i - g - 1}{g-1}\right) = 17.024$$

$$F_{2p,\,2(\sum n_i - g - 1)}(0.01) = 4.77$$

$17.024 > 4.77$ reject the null hypothesis

Using Bartlett correction

$$-\left(n-1-\frac{p+g}{2}\right)\ln\left(\frac{|W|}{|B+W|}\right) = 28.209$$

$$\chi^2_{p(g-1)}(0.01) = 13.28$$

$28.209 > 13.28$ reject the null hypothesis

6.19

a. $\bar{x}_{11} = 12.219$, $\bar{x}_{12} = 8.113$, $\bar{x}_{13} = 9.590$

$\bar{x}_{21} = 10.106$, $\bar{x}_{22} = 10.762$, $\bar{x}_{23} = 18.168$

$$S_1 = \begin{bmatrix} 23.013 & 12.366 & 2.907 \\ 12.366 & 17.544 & 4.773 \\ 2.907 & 4.773 & 13.963 \end{bmatrix}, \quad S_2 = \begin{bmatrix} 4.362 & 0.760 & 2.362 \\ 0.760 & 25.851 & 7.686 \\ 2.362 & 7.686 & 46.654 \end{bmatrix}$$

$H_0: \mu_1 - \mu_2 = 0$, $H_a: \mu_1 - \mu_2 \neq 0$

$$T^2 = (\bar{x}_1 - \bar{x}_2)'\left[\left(\frac{1}{n_1}+\frac{1}{n_2}\right)S_{pooled}\right]^{-1}(\bar{x}_1 - \bar{x}_2)$$

$$= [2.113 \;\; -2.649 \;\; -8.578]\left[\left(\frac{1}{36}+\frac{1}{23}\right)\begin{bmatrix} 15.815 & 7.887 & 2.696 \\ 7.887 & 20.750 & 5.897 \\ 2.696 & 5.897 & 26.581 \end{bmatrix}\right]^{-1}\begin{bmatrix} 2.113 \\ -2.649 \\ -8.578 \end{bmatrix}$$

$= 50.913$

$$\frac{(n_1+n_2-2)p}{(n_1+n_2-p-1)} F_{p,\,n_1+n_2-p-1}(0.01) = 13$$

$50.913 > 13 \rightarrow$ reject the null hypothesis

b. $$\hat{a} \propto \left[\left(\frac{1}{n_1}+\frac{1}{n_2}\right) S_{pooled}\right]^{-1} (\bar{X}_1 - \bar{X}_2) = \begin{bmatrix} 3.575 \\ -1.879 \\ -4.474 \end{bmatrix}$$

c. $$\mu_{11}-\mu_{21} \div (\bar{X}_1 - \bar{X}_2) \pm \sqrt{\frac{(n_1+n_2-2)p}{(n_1+n_2-p-1)} F_{p,\,n_1+n_2-p-1}(0.01)} \sqrt{\left(\frac{1}{n_1}+\frac{1}{n_2}\right) S_{11,\,pooled}}$$

$$\rightarrow 2.113 \pm \sqrt{13}\sqrt{\left(\frac{1}{36}+\frac{1}{23}\right) * 15.815}$$

$$\rightarrow -1.714 \leq \mu_1 - \mu_2 \leq 5.94$$

$$\mu_{12}-\mu_{22} \rightarrow -2.649 \pm \sqrt{13}\sqrt{\left(\frac{1}{36}+\frac{1}{23}\right) * 20.75}$$

$$\rightarrow -7.033 \leq \mu_{12}-\mu_{22} \leq 1.735$$

$$\mu_{13}-\mu_{23} \rightarrow -8.578 \pm \sqrt{13}\sqrt{\left(\frac{1}{36}+\frac{1}{23}\right) * 26.581}$$

$$\rightarrow -13.54 \leq \mu_{13}-\mu_{23} \leq -3.616$$

the capital cost appear to have the largest differrence

d. in the previous steps we assumed $\Sigma_1 = \Sigma_2$, however there is big difference in variance that violate this assumption.

deleting the outiers and assuming large samples with different Σ

$\bar{X}_{11} = 11.312$, $\bar{X}_{12} = 7.633$, $\bar{X}_{13} = 9.561$

$\bar{X}_{21} = 10.293$, $\bar{X}_{22} = 11.018$, $\bar{X}_{23} = 18.321$

$$T^2 = (\bar{X}_1 - \bar{X}_2)' \left[\frac{1}{n_1} S_1 + \frac{1}{n_2} S_2\right]^{-1} (\bar{X}_1 - \bar{X}_2)$$

$$= [1.206 \;\; -3.129 \;\; -8.607] \left[\frac{1}{34}\begin{bmatrix} 9.025 & 5.156 & 3.207 \\ 5.156 & 14.239 & 4.319 \\ 3.207 & 4.319 & 11.987 \end{bmatrix} + \frac{1}{23}\begin{bmatrix} 4.362 & 0.760 & 2.362 \\ 0.760 & 25.851 & 7.686 \\ 2.362 & 7.686 & 46.654 \end{bmatrix}\right]^{-1} \begin{bmatrix} 1.206 \\ -3.129 \\ -8.607 \end{bmatrix}$$

⑥

$$= \begin{bmatrix} 1.206 & -3.129 & -8.607 \end{bmatrix} \begin{bmatrix} 2.358 & -0.238 & -0.149 \\ -0.238 & 0.712 & -0.118 \\ -0.149 & -0.118 & 0.455 \end{bmatrix} \begin{bmatrix} 1.206 \\ -3.129 \\ -8.607 \end{bmatrix}$$

$= 42.637$

$\chi^2_p(\alpha) = \chi^2_3(0.01) = 11.34$

$42.637 > 11.34$ we reject the null hypothesis

6.22

a. $\bar{X}_{11} = 0.397$, $\bar{X}_{12} = 5.330$, $\bar{X}_{13} = 3.688$, $\bar{X}_{14} = 49.420$

$\bar{X}_{21} = 0.314$, $\bar{X}_{22} = 5.179$, $\bar{X}_{23} = 2.315$, $\bar{X}_{24} = 38.155$

$$S_1 = \begin{bmatrix} 0.007 & 0.070 & 0.031 & 0.151 \\ 0.070 & 1.144 & 0.148 & 3.431 \\ 0.031 & 0.148 & 0.456 & 3.308 \\ 0.151 & 3.431 & 3.308 & 55.252 \end{bmatrix}, \quad S_2 = \begin{bmatrix} 0.010 & 0.154 & 0.004 & 0.030 \\ 0.154 & 1.144 & -0.039 & 1.281 \\ 0.004 & -0.039 & 0.121 & 1.098 \\ 0.030 & 1.280 & 1.098 & 23.261 \end{bmatrix}$$

$$T^2 = (\bar{X}_1 - \bar{X}_2)' \left[\left(\frac{1}{n_1} + \frac{1}{n_2} \right) S_{pooled} \right]^{-1} (\bar{X}_1 - \bar{X}_2)$$

$T^2 = 96.373$

$$\frac{(n_1+n_2-2)P}{(n_1+n_2-P-1)} * F_{P,n_1+n_2-P-1}(\alpha) = 11.003 \text{ at } \alpha = 0.05$$

$96.373 > 11.003$ we reject the null hypothesis $H_0: \mu_1 - \mu_2 = 0$

$$\hat{a} \propto \left[\left(\frac{1}{n_1} + \frac{1}{n_2} \right) S \right]^{-1} (\bar{X}_1 - \bar{X}_2) = \begin{bmatrix} 1242.487 \\ -79.700 \\ -77.852 \\ 9.886 \end{bmatrix}$$

b. $\mu_{1i} - \mu_{2i}$: $(\bar{X}_{1i} - \bar{X}_{2i}) \pm \sqrt{\frac{(n_1+n_2-2)p}{(n_1+n_2-p-2)} * F_{p,\, n_1+n_2-p-1}(0.05)} \sqrt{(\frac{1}{n_1}+\frac{1}{n_2}) S_{ii,pooled}}$

7

$\mu_{11} - \mu_{21}$: $0.084 \pm \sqrt{11.002} * 0.024$

$\rightarrow 0.004 \leq \mu_{11} - \mu_{21} \leq 0.163$

$\mu_{12} - \mu_{22}$: $0.151 \pm \sqrt{11.002} * 0.303$

$\rightarrow -0.853 \leq \mu_{12} - \mu_{22} \leq 1.154$

$\mu_{13} - \mu_{23}$: $1.372 \pm \sqrt{11.002} * 0.191$

$\rightarrow 0.739 \leq \mu_{13} - \mu_{23} \leq 2.006$

$\mu_{14} - \mu_{24}$: $11.266 \pm \sqrt{11.002} * 2.102$

$\rightarrow 4.292 \leq \mu_{14} - \mu_{24} \leq 18.239$

Bonferroni intervals:

$\mu_{1i} - \mu_{2i}$: $(\bar{X}_{1i} - \bar{X}_{2i}) \pm t_{n_1+n_2-2}(\frac{\alpha}{2p}) \sqrt{(\frac{1}{n_1}+\frac{1}{n_2}) S_{ii,pooled}}$

$\mu_{11} - \mu_{21}$: $0.84 \pm 2.6 * 0.024$

$0.045 \leq \mu_{11} - \mu_{21} \leq 0.122$

$\mu_{12} - \mu_{22}$: $0.151 \pm 2.6 * 0.303$

$-0.337 \leq \mu_{12} - \mu_{22} \leq 0.639$

$\mu_{13} - \mu_{23}$: $1.372 \pm 2.6 * 0.191$

$1.064 \leq \mu_{13} - \mu_{23} \leq 1.680$

$\mu_{14} - \mu_{24}$: $11.266 \pm 2.6 * 2.102$

$7.876 \leq \mu_{14} - \mu_{24} \leq 14.656$

c. this violate one of the assumptions of population comparision so this test can only be used in certain cases and can not be generalized

8.4

$$|\Sigma - \lambda I| = 0 \rightarrow \begin{bmatrix} \sigma^2-\lambda & \sigma^2\rho & 0 \\ \sigma^2\rho & \sigma^2-\lambda & \sigma^2\rho \\ 0 & \sigma^2\rho & \sigma^2-\lambda \end{bmatrix} = 0$$

$$(\sigma^2-\lambda)\left[(\sigma^2-\lambda)^2 - (\sigma^2\rho)^2\right] - \sigma^2\rho\left[(\sigma^2-\lambda) * \sigma^2\rho\right] = 0$$

$$\Rightarrow (\sigma^2-\lambda)\left[(\sigma^2-\lambda)^2 - (\sigma^2\rho)^2 - (\sigma^2\rho)^2\right] = 0$$

$$\Rightarrow (\sigma^2-\lambda) = 0 \Rightarrow \lambda_1 = \sigma^2$$

and $\left[(\sigma^2-\lambda)^2 - (\sigma^2\rho)^2 - (\sigma^2\rho)^2\right] = 0$

$\Rightarrow \left[(\sigma^2-\lambda)^2 - 2(\sigma^2\rho)^2\right] = 0$

$\Rightarrow \lambda_2 = \sigma^2(1+\rho\sqrt{2})$

and $\lambda_3 = \sigma^2(1-\rho\sqrt{2})$

$\Sigma e_1 = \lambda_1 e_1$

$$\begin{bmatrix} \sigma^2 & \sigma^2\rho & 0 \\ \sigma^2\rho & \sigma^2 & \sigma^2\rho \\ 0 & \sigma^2\rho & \sigma^2 \end{bmatrix}\begin{bmatrix} e_{11} \\ e_{12} \\ e_{13} \end{bmatrix} = \sigma^2\begin{bmatrix} e_{11} \\ e_{12} \\ e_{13} \end{bmatrix} \Rightarrow \begin{matrix} \sigma^2 e_1 + \sigma^2\rho e_2 = \sigma^2 e_1 \\ \sigma^2\rho e_1 + \sigma^2 e_2 + \sigma^2\rho e_3 = \sigma^2 e_2 \\ \sigma^2\rho e_2 + \sigma^2 e_3 = \sigma^2 e_3 \end{matrix}$$

$$e_1' = \left[\frac{1}{\sqrt{2}}, 0, -\frac{1}{\sqrt{2}}\right]$$

$$\begin{bmatrix} \sigma^2 & \sigma^2\rho & 0 \\ \sigma^2\rho & \sigma^2 & \sigma^2\rho \\ 0 & \sigma^2\rho & \sigma^2 \end{bmatrix}\begin{bmatrix} e_{21} \\ e_{22} \\ e_{23} \end{bmatrix} = \sigma^2(1+\rho\sqrt{2})\begin{bmatrix} e_{21} \\ e_{22} \\ e_{23} \end{bmatrix} \Rightarrow \begin{matrix} \sigma^2 e_{21} + \sigma^2\rho e_{22} = \sigma^2(1+\rho\sqrt{2})e_{21} \\ \sigma^2\rho e_{21} + \sigma^2 e_{22} + \sigma^2\rho e_{23} = \sigma^2(1+\rho\sqrt{2})e_{22} \\ \sigma^2\rho e_{22} + \sigma^2 e_{23} = \sigma^2(1+\rho\sqrt{2})e_{23} \end{matrix}$$

$$\Rightarrow e_2' = \left[\frac{1}{2}, \frac{-1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right]$$

$$\begin{bmatrix} \sigma^2 & \sigma^2\rho & 0 \\ \sigma^2\rho & \sigma^2 & \sigma^2\rho \\ 0 & \sigma^2\rho & \sigma^2 \end{bmatrix} \begin{bmatrix} e_{31} \\ e_{32} \\ e_{33} \end{bmatrix} = \sigma^2(1-\rho\sqrt{2}) \begin{bmatrix} e_{31} \\ e_{32} \\ e_{33} \end{bmatrix}$$

$$\rightarrow \sigma^2 e_{31} + \sigma^2\rho e_{32} = \sigma^2(1-\rho(\sqrt{2}))e_{31}$$

$$\sigma^2\rho e_{31} + \sigma^2 e_{32} + \sigma^2\rho e_{33} = \sigma^2(1-\rho\sqrt{2})e_{32}$$

$$\sigma^2\rho e_{32} + \sigma^2 e_{33} = \sigma^2(1-\rho\sqrt{2})e_{33}$$

$$\rightarrow e_3' = [\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}, -\frac{1}{2}]$$

$$Y_1 = e_1'X = \frac{1}{\sqrt{2}}X_1 - \frac{1}{\sqrt{2}}X_3$$

$$Y_2 = e_2'X = \frac{1}{2}X_1 + \frac{1}{\sqrt{2}}X_2 - \frac{1}{2}X_3$$

$$Y_3 = e_3'X = \frac{1}{2}X_1 + \frac{1}{\sqrt{2}}X_2 + \frac{1}{2}X_3$$

the variance explained by the first component

$$\frac{\lambda_1}{\lambda_1+\lambda_2+\lambda_3} = \frac{\sigma^2}{\sigma^2+\sigma^2(1+\rho\sqrt{2})+\sigma^2(1-\sqrt{2})} = \frac{1}{3} = 33.33\,\%$$

the variance explained by the second component

$$\frac{\lambda_2}{\lambda_1+\lambda_2+\lambda_3} = \frac{\sigma^2(1+\rho\sqrt{2})}{\sigma^2+\sigma^2(1+\rho\sqrt{2})+\sigma^2(1-\rho\sqrt{2})} = \frac{1}{3}(1+\rho\sqrt{2})$$

the varince explained by the third component

$$\frac{\lambda_3}{\lambda_1+\lambda_2+\lambda_3} = \frac{\sigma^2(1+\rho\sqrt{2})}{\sigma^2+\sigma^2(1+\rho\sqrt{2})+\sigma^2(1-\rho\sqrt{2})} = \frac{1}{3}(1-\rho\sqrt{2})$$

8.6

a. $|S - \lambda I| = 0 \longrightarrow$ $\lambda_1 = 7488.803$, $\lambda_2 = 13.837$

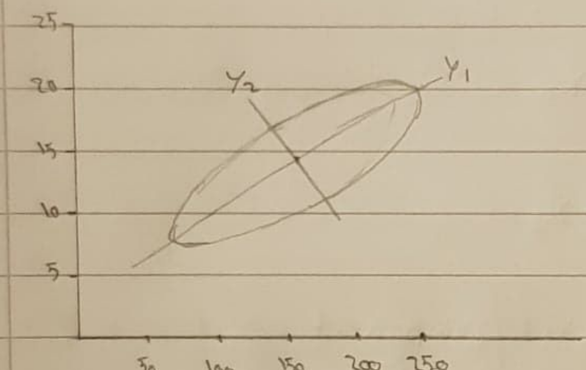
$e_1' = [-0.999, -0.041]$, $e_2' = [0.041, -0.999]$

$Y_1 = -0.999 X_1 - 0.041 X_2$

$Y_2 = 0.041 X_1 - 0.999 X_2$

b. $$\frac{\lambda_1}{\lambda_1 + \lambda_2} = \frac{7488.803}{7488.803 + 13.837} = 0.998$$

c.



d. $r_{Y_1, X_1} = 1$, $r_{Y_1, X_2} = 0.687$

X_1 has the largest correlation and that is due to X_1 = sales has much more variance.

8.10

a.

$$S = \begin{bmatrix} 4.333e{-}04 & 0.00028 & 1.590e{-}04 & 6.412e{-}05 & 8.897e{-}05 \\ 2.757e{-}04 & 0.00044 & 1.800e{-}04 & 1.815e{-}04 & 1.233e{-}04 \\ 1.590e04 & 0.00018 & 2.339e{-}04 & 7.341e{-}05 & 6.055e{-}05 \\ 6.412e{-}05 & 0.00018 & 7.341e{-}05 & 7.225e{-}04 & 5.083e{-}04 \\ 8.897e{-}05 & 0.00012 & 6.055e{-}05 & 5.083e{-}04 & 7.657e{-}04 \end{bmatrix}$$

$\lambda_1 = 0.00137$, $\lambda_2 = 0.00070$, $\lambda_3 = 0.00025$, $\lambda_4 = 0.00014$, $\lambda_5 = 0.00012$

$e_1' = [0.223, 0.307, 0.155, 0.639, 0.651]$ ⑪

$e_2' = [0.625, 0.570, 0.345, -0.248, -0.322]$

$e_3' = [0.326, -0.250, -0.038, -0.642, 0.646]$

$e_4' = [0.663, -0.414, -0.497, 0.309, -0.216]$

$e_5' = [0.118, -.589, 0.780, 0.148, -0.094]$

$Y_1 = 0.223\,X_1 + 0.307\,X_2 + 0.155\,X_3 + 0.639\,X_4 + 0.651\,X_5$

$Y_2 = 0.625\,X_1 + 0.570\,X_2 + 0.345\,X_3 - 0.248\,X_4 - 0.322\,X_5$

$Y_3 = 0.326\,X_1 - 0.250\,X_2 - 0.038\,X_3 - 0.642\,X_4 + 0.646\,X_5$

$Y_4 = 0.663\,X_1 - 0.414\,X_2 - 0.497\,X_3 + 0.309\,X_4 - 0.216\,X_5$

$Y_5 = 0.118\,X_1 - 0.589\,X_2 + 0.780\,X_3 + 0.148\,X_4 - 0.094\,X_5$

b. $$\frac{\lambda_1 + \lambda_2 + \lambda_3}{\lambda_1 + \lambda_2 + \lambda_3 + \lambda_4 + \lambda_5} \approx 0.8988$$

c. $$\frac{\hat{\lambda}_i}{1 + z(\alpha/2)\sqrt{2/n}} \leq \lambda_i \leq \frac{\hat{\lambda}_i}{1 - z(\alpha/2)\sqrt{2/n}}$$

$$\rightarrow \frac{0.00137}{1 + z(0.1/2)\sqrt{2/103}} \leq \lambda_i \leq \frac{0.00137}{1 - z(0.1/2)\sqrt{2/103}}$$

$\rightarrow 0.00106 \leq \lambda_1 \leq 0.00178$

$$\frac{0.00070}{1 + z(0.1/2)\sqrt{2/103}} \leq \lambda_2 \leq \frac{0.00070}{1 - z(0.1/2)\sqrt{2/103}}$$

$0.00057 \leq \lambda_2 \leq 0.00091$

$$\frac{0.00025}{1 + z(0.1/2)\sqrt{2/103}} \leq \lambda_3 \leq \frac{0.00025}{1 - z(0.1/2)\sqrt{2/103}}$$

$\rightarrow 0.00019 \leq \lambda_3 \leq 0.00032$

d. Stock returns can be summarized to 2 or 3 principal components that can explain 80% or 89% of the variation