Assignment 1
Due Date: October 15; 2022

Name:	I.D.:
Total Marks:	

1. Write the statements in symbolic form using the symbols ~, ∨, and ∧ and the indicated letters to represent component statements. Let h = "John is healthy," w = "John is wealthy," and s = "John is wise."

(a)
John is healthy and wealthy but not wise.

$h \vee w \wedge \sim s$ $\quad (h \vee w) \wedge \sim s$ $\quad (h \wedge w) \wedge \sim s$ $\quad (h \vee w) \vee \sim s$ $\quad (h \wedge w) \vee \sim s$

(b)
John is not wealthy, but he is healthy and wise.

$\sim w \vee (h \vee s)$ $\quad \sim w \wedge (h \vee s)$ $\quad \sim w \wedge (h \wedge s)$ $\quad \sim w \wedge h \vee s$ $\quad \sim w \vee (h \wedge s)$

(c)
John is neither healthy, wealthy, nor wise.

$\sim w \vee \sim h \wedge \sim s$ $\quad \sim w \wedge \sim h \vee \sim s$ $\quad \sim w \wedge \sim h \wedge \sim s$ $\quad \sim(w \wedge h) \wedge \sim s$ $\quad \sim w \vee \sim h \vee \sim s$

(d)
John is neither wealthy nor wise, but he is healthy.

$(\sim w \vee \sim s) \wedge h$ $\quad \sim w \vee \sim s \wedge h$ $\quad (\sim w \wedge \sim s) \wedge h$ $\quad (\sim w \vee \sim s) \vee h$ $\quad (\sim w \wedge \sim s) \vee h$

(e)
John is wealthy, but he is not both healthy and wise.

$w \vee \sim(h \wedge s)$ $\quad w \wedge \sim(h \vee s)$ $\quad w \wedge \sim h \wedge \sim s$ $\quad w \vee \sim(h \vee s)$ $\quad w \wedge \sim(h \wedge s)$

2. Assume that c is a particular integer.

(a) Is $-8c$ an even integer?

- ○ Yes
- ◉ No

(b) Is $6c + 5$ an odd integer?

- ◉ Yes
- ○ No

(c) Is $(c^2 + 3) - (c^2 - 3) - 6$ an even integer?

- ○ Yes
- ◉ No

3. Consider the following statement. "The difference between any odd integer and any even integer is an odd integer". Which of the following could begin a direct proof of the statement?

- ○ Suppose that for any odd integer m and any even integer n, $m - n$ is an odd integer.
- ○ Observe that if $m = 3$, an odd integer, and $n = 2$, an even integer, $m - n = 1$, an odd integer.
- ◉ Suppose there exists an odd integer m and an even integer n such that $m - n$ is an odd integer.
- ○ Suppose that for any odd integer m and any even integer n, $m - n$ is an even integer.
- ○ Suppose m and n are any integers such that m is odd and n is even.

4 Fill in the blank.

It must be shown that [---Select--- ▾] at the end of the proof.

- ---Select---
- no such m and n exist
- both m and n are even
- both m and n are odd
- m − n is odd
- m − n is even

5. With an idea of where to start and where to finish, complete the next few steps that follow the first by filling in the blanks.

By definition, since m is an odd integer, there exists an ______ r such that $m =$ ______ .

Similarly, by definition, since n is an even integer, there exists an ______ such that $n =$ ______ .

Because the statement to be proved deals with the difference $m - n$, use the above information to rewrite the difference. Substitute the mathematical expressions found for m and n into the equation. (Enter your answer in terms of r and s.)

$m - n =$ ______

6. Consider how to algebraically manipulate the expression on the right side of the equation to reach the final goal. Finish the proof in essay form.

7. Consider the following theorem.

The sum of any even integer and any odd integer is odd.

Only six statements in the following scrambled list belong in the proof.

- By substitution, $m + n = 2t + 1$, where t is an integer.
- Let $m + n$ be any odd integer.
- So by definition of even, t is even.
- Let $t = r + s$. Then t is an integer because it is a sum of integers.
- By definition of even and odd, there are integers r and s such that $m = 2r$ and $n = 2s + 1$.
- Suppose m is any even integer and n is any odd integer.
- Hence, $m + n$ is twice an integer plus one. So by definition of odd, $m + n$ is odd.
- By substitution and algebra, $m + n = 2r + (2s + 1) = 2(r + s) + 1$.
- By definition of even and odd, there is an integer r such that $m = 2r$ and $n = 2r + 1$.

Construct the proof by choosing the appropriate statements from the list and putting them in the correct order.

1.
2.
3.
4.
5.
6.

8. Consider the following theorem.

The product of any even integer and any odd integer is even.

Here is a proof of the theorem with at least one incorrect step.

1. Suppose m is any even integer and n is any odd integer.
2. If $m \cdot n$ is even, then by definition of even there exists an integer r such that $m \cdot n = 2r$.
3. By definition of even and odd, there exist integers p and q such that $m = 2p$ and $n = 2q + 1$.
4. Therefore, by substitution, the product $m \cdot n = (2p)(2q + 1) = 2r$.
5. But r is an integer by the assumption above.
6. Thus $m \cdot n$ is two times an integer, so by definition of even, the product is even.

Identify the mistakes in the proof. (Select all that apply.)

- [ ] Step 1
- [ ] Step 2
- [ ] Step 3
- [ ] Step 4
- [ ] Step 5
- [ ] Step 6

9. Write the rational number 5.6073 as a ratio of two integers. (Simplify your answer completely.)

10. Consider the expression below. Assume m is an integer.

$$6m(3m + 24)$$

Complete the equality. (Simplify your answer completely. Enter an expression in the variable m. If no expression exists, enter DNE.)

$$6m(3m + 24) = 9\left(\boxed{} \right)$$

Given that m is an integer, is $6m(3m + 24)$ divisible by 9?

- ◯ Yes
- ◯ No