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Corporate Finance

Simple vs. Compound Interest

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Corporate Finance

- Topics Covered
  - Compound vs. simple interest
  - Annuities and perpetuities
  - Delayed perpetuities and growing perpetuities
  - Compounding within a year

Have \$100

$r = 7\%$, 2 years

Simple interest

$FV = 100 + 7 + 7$
$= 114$

Compound interest

$$FV = 100 + 7 + 7 + 7 \cdot (.07)$$

$$= 100 + 100r + 100r + 100r^2$$

interest on interest

$$= 100(1+r)(1+r)$$

$$= 100(1+r)^2$$

Compound interest, t years

$$FV = 100(1+r)^t$$

$t = 2$ Simple int $= \$114$

comp int $= \$114.49$

$t = 100$

Simple interest

$$FV = \$100(1 + .07(100))$$
$$= \$800$$

Compound interest

$$FV = 100(1.07)^{100} = 86,771$$

$$100 = PV(1+r)^t$$

$$PV = \frac{100}{(1+r)^t}$$

$$\frac{1}{(1+r)^t}$$: Discount Factor

DF

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Annuities and Perpetuities

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Annuity: Equal payments

Ex How much would you pay for $100 a year for 3 years?

$$PV = \frac{100}{1+r} + \frac{100}{(1+r)^2} + \frac{100}{(1+r)^3}$$

Annuity lasting for t years

$$PV = \frac{C}{1+r} + \frac{C}{(1+r)^2} + \cdots + \frac{C}{(1+r)^t}$$

$$= \frac{C}{r}\left[1 - \frac{1}{(1+r)^t}\right]$$

$$AF = \frac{1}{r}\left[1 - \frac{1}{(1+r)^t}\right]$$

$r = 5\%$, $t = 3$

$$PV = \frac{100}{.05}\left[1 - \frac{1}{(1.05)^3}\right]$$

$$= \$272.32$$

15-yr Mortgage

$r = 4\%$

Principal $= 0.5M$

$$0.5 = C \underbrace{\frac{1}{r}\left[1 - \frac{1}{(1+r)^{15}}\right]}_{AF}$$

$r = 4\%$

$AF^{15}_{.04} = 11.12$

$0.5 = C \times AF$

$\Rightarrow C = 0.5 / 11.12$

15-year mortgage

Loan amnt = 0.5M

r = 4%

$$0.5 = C\ AF^{15}_{.04}$$

$$AF^{15}_{.04} = 11.12 \Rightarrow C = \frac{0.5}{11.12} = 45K$$

$$FV = \qquad (1+r)^t$$

$$FV = \overbrace{C \frac{1}{r}\left[1 - \frac{1}{(1+r)^t}\right]} (1+r)^t$$

$$PV = \frac{C}{r}\left[1 - \frac{1}{(1+r)^t}\right]$$

$$= \frac{C}{1+r} + \frac{C}{(1+r)^2} + \cdots + \frac{C}{(1+r)^t}$$

$$r > 0 \Rightarrow \frac{1}{1+r} < 1$$

$$t \to \infty \Rightarrow \frac{1}{(1+r)^t} \to 0$$

PV of a Consol

$$PV = \frac{C}{r}$$

need $r > 0$

$$PV = \frac{C}{1+r} + \frac{C}{(1+r)^2} + \frac{C}{(1+r)^3} + \cdots$$

$r = 10\%$ $\quad C = \$100$

$$PV = \frac{C}{r} = \frac{100}{.1} = \$1000$$

$r = 20\%$ $\quad PV = \frac{100}{.2} = \$500$

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Growing Delayed Annuities and Perpetuities

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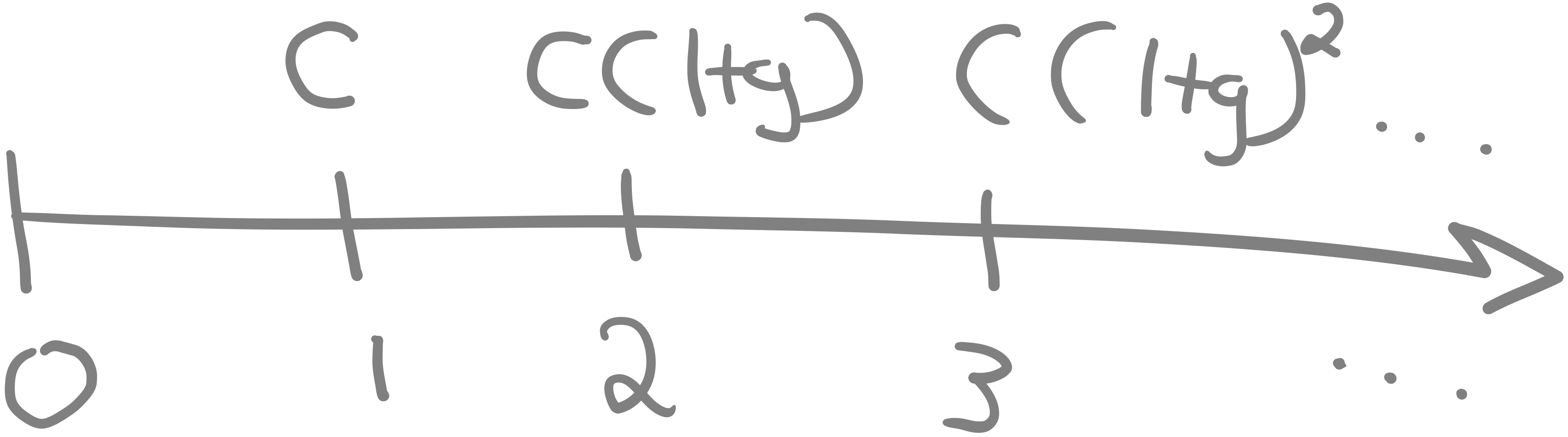
Growing Perpetuity



$$PV = \frac{C}{1+r} + \frac{C(1+g)}{(1+r)^2} + \frac{C(1+g)^2}{(1+r)^3} + \cdots$$

$$PV = \frac{C}{1+r} + \frac{C(1+g)}{(1+r)^2} + \cdots + \frac{C(1+g)^{t-1}}{(1+r)^t} + \cdots$$

If $r > g \Rightarrow$ . .

$$PV = \frac{C}{r-g}$$

$g = 0$

$$PV = \frac{C}{r}$$

· Delayed perpetuity:

Cash flow C, every year
starting 3 years from now

	0	0	C	C	…
0	1	2	3	4	…

$$PV = \frac{C}{r}$$

$$\boxed{PV = \frac{1}{(1+r)^2}\,\frac{C}{r}}$$

Perpetuity, CFs begin in
t years

$$PV = \frac{1}{(1+r)^{t-1}} \frac{C}{r}$$

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Compounding Within the Year and the Effective Annual Interest Rate

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Example

Bank offers a stated annual interest rate of 8%, compounded semi annually.

What do we have after 1 year?

SAIR = 8%

compounded semi-annually

$$FV = \$100(1.04)^{2}$$ ← frequency

↑

Period rate

$$FV = 100\left(1 + \frac{SAIR}{m}\right)^{mt}$$

$$PV = 100\left(1 + \frac{SAIR}{m}\right)^{-mt}$$

For t years

$$FV = \$100\left(1 + \frac{SAIR}{m}\right)^m$$

SAIR = 8%

m = 1, annual FV = $108

m = 2, semiannual $FV = 100(1.04)^2 = 108.16$

m = 12, monthly $= 100\left(1 + \frac{.08}{12}\right)^{12} = 108.30$

m = 365, daily $= 100\left(1 + \frac{.08}{365}\right)^{365} = 108.33$

108.32867

m = 31,536,000 secondly

= 108.32871

$m = \infty$

continuous compounding

$$\lim_{m \to \infty} \left(1 + \frac{SAIR}{m}\right)^m = e^{SAIR}$$

$e \approx 2.718 \ldots$

$r_a = \text{SAIR}$

FV of \$100 comp. cont. for t year

$$FV = 100\, e^{r_a t}$$

PV of 100 : $PV = 100\, e^{-r_a t}$

EAR: The rate that, when compounded annually produces the same return as the stAR, compounded mx / yr. $\leftarrow r_a$

$$1 + EAR = \left(1 + \frac{r_a}{m}\right)^m$$

$$EAR = \left(1 + \frac{r_a}{m}\right)^m - 1$$

$r_a = 6\%$

$m = 2$ $\quad EAR = (1 + .03)^2 - 1$

$= 6.09\%$

$m = 4$ $\quad EAR = (1 + .015)^4 - 1$

$= 6.14\%$

$m = \infty$

$$EAR = e^{.06} - 1$$

$$= 6.182$$

Period rate = 1.5%/month

APR = 1.5% × 12 = 18%

↑

SAIR

Spend \$100

After 1 month 101.50

" 2 " $100(1.015)^2$

⋮ ⋮ ⋮

" 12 months $100(1.015)^{12}$

$$= 119.56$$

$$EAR = \left(1 + \frac{.18}{12}\right)^{12} = 19.56\%$$

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