CS221 Final Exam

CS 221
December 10, 2012

Name: $\underbrace{\hspace{8cm}}_{\text{by writing my name I swear by the honor code}}$

SUNet ID (e.g., *psl*): ____________________

Read all of the following information before starting the exam:

- Show all work, clearly and in order, if you want to get full credit. We reserve the right to take off points if we cannot see how you arrived at your answer (even if your final answer is correct).
- Please keep your written answers brief; be clear and to the point. We will take points off for rambling and for incorrect or irrelevant statements.
- This test has 8 problems and is worth 76 points. It is your responsibility to make sure that you have all of the pages!
- Be sure to read all the problems, as some may be more difficult than others, and they are not ordered by difficulty.
- Good luck!

1. Touring Gendarra (*12 points*)

You have been employed by a tour company to schedule four-day itineraries for Gendarra, a country that has recently opened up to tourism after decades of isolation. There are three tour groups, and each tour group will visit some number of sites each of the four days. Below is the list of nine sites that each tour group can visit:

City Points of Interest	**Country Points of Interest**
Gendarran Castle East Wing *(ECastle)*	Base of Mt. Arah *(BaseArah)*
Gendarran Castle West Wing *(WCastle)*	Mt. Arah Summit *(SummitArah)*
Gendarran Castle North Wing *(NCastle)*	Village of Gendarra *(Village)*
Gendarran Castle South Wing *(SCastle)*	
Market of Gendarra *(Market)*	
Church of Gendarra *(Church)*	

There are several constraints on the itineraries (the first two are imposed by the government, and the last three are imposed by the tourists):

1. On any given day, each group can visit at most one of the four sites in Gendarran Castle.
2. On any given day, each group can only visit city sites or country sites but not both.
3. Over the course of the four days, $Group1$ must visit at least one site from the city and at least one site from the country.
4. Over the course of the four days, $Group2$ must visit all sites in the castle.
5. Over the course of the four days, $Group3$ must visit at least one site per day.

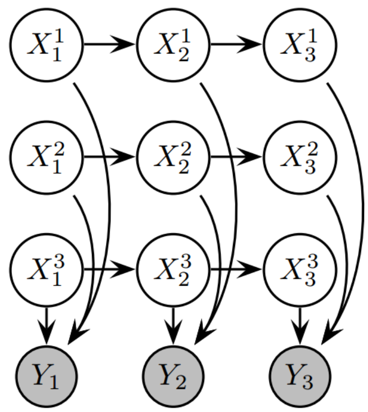
a. (*5 pts*) We will now formulate this problem as a CSP. For each group g, site s, and day d, define a variable $Visit(g, s, d)$. For each of the five constraints above, define the *smallest* number of factors capturing these constraints. How many factors are there, and what is the arity of each factor?

b. (*1 pt*) How many connected components does the factor graph corresponding to the CSP in part (b) have?

c. (*1 pt*) Being low on funds, you want to satisfy the constraints but also minimize the number of total sites visited by the three groups over the four days. Add a factor to the CSP (making it a weighted CSP) so that the maximum weight assignment meets this objective.

2. **Factorial HMMs** (*9 points*)

A factorial HMM is an example of a Bayesian network, shown below for $T = 3$ time steps:



The idea is that at each time slice, we have a state represented by $k = 3$ components; for example, at time 1, the components are $X_1 = (X_1^1, X_1^2, X_1^3)$. Each component i transitions over time independently according to $p_1(X_t^i \mid X_{t-1}^i)$. The components participate together to generate the corresponding observation according to $p_2(Y_t \mid X_t^1, \dots, X_t^k)$.

a. (*1 pt*) Assume that $Y_1, \dots, Y_T$ are unobserved. If we eliminate these variables, what is the tree-width of the resulting factor graph as a function of k and T?

b. (*1 pt*) Assume that $Y_1, \dots, Y_T$ are observed (we're conditioning on these variables). What is the tree-width of the resulting factor graph as a function of k and T?

c. (*1 pt*) Suppose we condition on all the variables except for $X_1^1, \ldots, X_T^1$. What is the tree-width of the resulting factor graph as a function of k and T?

d. (*2 pts*) Express the conditional distribution $\mathbb{P}(X_3^1, X_3^2 \mid Y_1, X_2^1, \ldots, X_2^k)$ in terms of p_1 and p_2 (you must simplify the expression if possible).

e. (*2 pts*) If instead of using a single shared transition distribution p_1, we change the model and use a separate transition distribution $p_{1,j}$ for each component $j = 1, \ldots, K$. The parameters of the new model are now $\theta' = (p_{1,1}, \ldots, p_{1,k}, p_2)$. Let L be the likelihood obtained by the maximum likelihood estimate of θ in the old model. and let L' be the likelihood obtained by the maximum likelihood estimate of θ' in the new model. Prove that $L' \geq L$.