We consider r.v. $\{X_v\}_{v \in V}$

Scope of factor – its r.v.s

Given

- UGM $(G, \{\psi^c\})$, i.e., $\{\psi^c\}$ is a set of factors and ψ^c is assoc. with the clique c.
- Junction tree (T, B) of width w
- evidence x_e

We will write $\psi^c(\bar{X})$ even if the scope of ψ^c is a subset of $\bar{X}$.

We want to compute $\sum_{x_{V \setminus e}} \prod_c \psi^c(x_{V \setminus e}, x_e)$.

Notice, each clique of G is a subset of some bag of (T, B), i.e., some $B(t)$. That is,

$\forall \psi^c$. $\exists t$ s/t $B(t)$ "contains the scope of ψ^c.

Root T in an arbitrary vertex, called, r

Simplifications:

(1) Evidence, e, can be incorporated into factors by setting $\psi^c_*(X_{V\setminus e}) \leftarrow \psi^c(X_{V\setminus e}, x_e)$.
So, we assume no evidence.

(2) Associate a single factor with each vertex of T by setting

$$\psi_t(X_{B(t)}) \leftarrow \prod_{\substack{c \subseteq B(t) \\ \text{and } t \text{ closest} \\ \text{to } r \text{ among} \\ \text{such vertices}}} \psi^c_*(X_{B(t)})$$

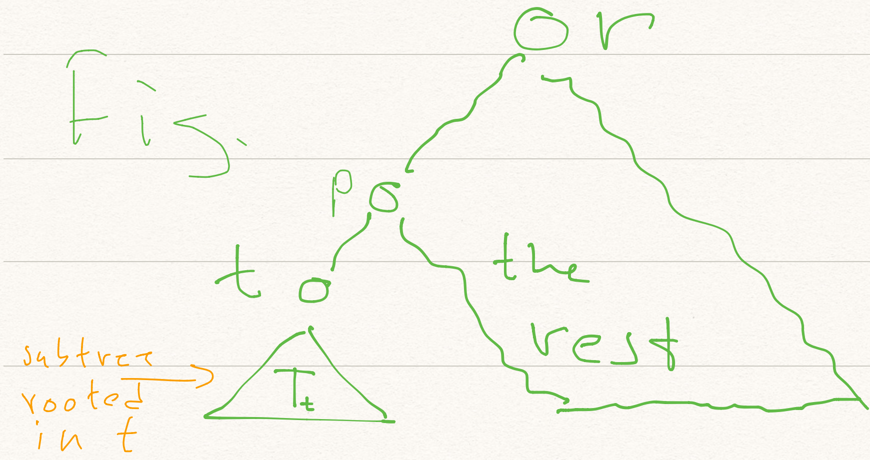
Notice we now want to compute

$$\prod_{t \in V(T)} \psi_t(X_{B(t)}) = \prod_c \psi^c(\bar{X}),$$

which we denote $\psi(\bar{X})$.

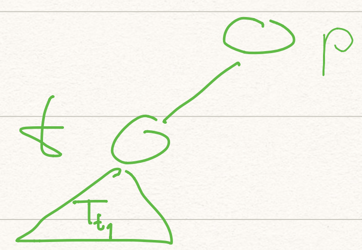
Subproblems and subsolutions

Fig.

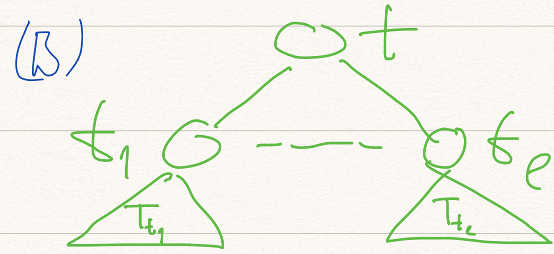


The subproblems are rooted subtrees of the 2 types below:

(A)



(B)



Let $B^*(t) = \bigcup_{t' \in V(T_t)} B(t)$

(A) the subsolution for T_t, with parent p, is

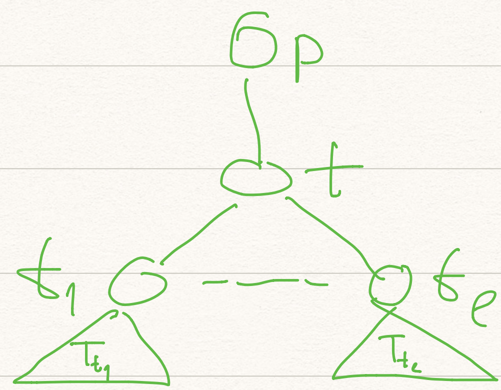
$$m_{t\to p}(x_{B(p)}) := \sum \prod \psi_s(x_{B(p)}, x_{B^*(t)\setminus B(p)})$$

$$\sum_{X_{B^*(t)\setminus B(p)}} \prod_{s\in U(T_t)}$$

$$\text{(B)}\quad \psi^*_t(X_{B(t)}) := \sum_{X_{B^*(t)\setminus B(t)}} \prod_{s\in U(T_t)} \psi_s(X_{B(t)}, X_{B^*(t)\setminus B(t)})$$

$$\left(\text{So } m_{t\to p}(X_{B(p)}) = \sum_{X_{B(t)\setminus B(p)}} \psi^*(X_{B(t)}).\right)$$

Consider configuration:



Note, $\forall i\neq j$ $\quad (B^*(t_i)\setminus B(t)) \cap (B^*(t_j)\setminus B(t)) = \emptyset$

(By running intersection.)

Notice also

$$\psi^*_t(X_{B(t)}) = \sum_{X_{B^*(t)\setminus B(t)}} \prod_{s\in U(T_t)} \psi_s(X_{B(t)}, X_{B^*(t)\setminus B(t)})$$

$$= \psi_t(X_{B(t)}) \sum \prod^{\ell} \prod \psi_s(X_{B(t)}, X_{B^*(t)\setminus B(t)})$$

$$X_{B^*(t)\setminus B(t)} \quad j=1 \quad s\in V(t_j)$$

$$= \psi_t(X_{B(t)}) \prod_{j=1}^{\ell} \sum_{X_{B^*(t_j)\setminus B(t)}} \prod_{s\in V(t_j)} \psi_s(X_{B(t)}, X_{B^*(t_j)\setminus B(t)})$$

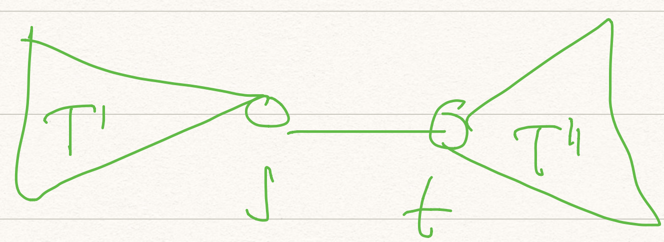
$$= \psi_t(X_{B(t)}) \prod_{j=1}^{\ell} m_{t_j\to t}(X_{B(t)})$$

Finally we can sum the result at the root

Notice

- We can root anywhere

- For conf



any root r' in T' gives the same $\psi_{t\to s}$ and

any root r'' in T'' gives the same $\psi_{t\to s}$

Conclusion

(1) only 2 possible messages for each edge

(2) all "bag marginals" can be comp. in time $O(|E(T)|\, c^w) = O(|V(G)|\, c^w)$ for some const. c (which dep. on # of values in our categorical CPDs)