Written Solution

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Question 1:

a- \mathcal{Y} is a one hot vector ([0, 0, 0, 1, 0, 0,] where 1 corresponds to the target word. So this means that all the other terms from $-\log(\mathcal{Y})$ will be neglected except the term corresponding to the 1.

b-
$$\frac{\partial(\hat{J})}{\partial(v_c)} = u_o(\hat{\mathcal{Y}} - \mathcal{Y})$$

c-
$$\frac{\partial(\hat{J})}{\partial(u_c)} = v_c(\hat{\mathcal{Y}} - \mathcal{Y})$$

be neglected except the b-
$$\frac{\partial(J)}{\partial(v_c)} = u_o(\hat{\mathcal{Y}} - \mathcal{Y})$$

c- $\frac{\partial(J)}{\partial(u_c)} = v_c(\hat{\mathcal{Y}} - \mathcal{Y})$
d- $\frac{\partial(\sigma)}{\partial(x)} = \sigma(x)(1 - \sigma(x))$

$$\sigma(x) = \frac{1}{1+e^{-x}}$$

e-
$$J_{negative sampling}(vc, o, U) = -log(\sigma(u_o^T v_c)) - \sum_{k=1}^K log(\sigma(-u_o^T v_c))$$

Derivative is composed of two parts and we differentiate according to $\frac{\partial(J)}{\partial(v_c)}$ =

$$\frac{\partial(J)}{\partial(logterm)} \frac{\partial(logterm)}{\partial(\sigma)} \frac{\partial(\sigma)}{\partial(v_c)}$$

First part we will see that we differentiate using chain rule as above: $\frac{1}{\sigma(u_o^T v_c)} \sigma(u_o^T v_c) (1 - v_c)$ $\sigma(u_o^T v_c))u_o^T$

The $\sigma(u_o^T v_c)$ terms can be cancelled together. So final term is $-(1-\sigma(u_o^T v_c))u_o^T =$ $(\sigma(u_o^T v_c) - 1)u_o^T$

and Second part will be $\sum_{k=1}^{K} (1 - \sigma(-u_k^T v_c)) - u_k^T$

All Together
$$(\sigma(u_o^T v_c) - 1)u_o^T + \sum_{k=1}^K (1 - \sigma(-u_k^T v_c))u_k^T$$

f- i)
$$\frac{\partial (J_{skipgram})}{\partial (U)} = \sum_{-m < j < m, j \neq 0} \frac{\partial (J_{skipgram}(v_c, w_{j+1}, U))}{\partial (U)}$$

$$\begin{array}{l} \text{f- i)} \frac{\partial (J_{skipgram})}{\partial (U)} = \sum_{-m < j < m, j \neq 0} \frac{\partial (J_{skipgram}(v_c, w_{j+1}, U)}{\partial (U)} \\ \text{ii)} \frac{\partial (J_{skipgram})}{\partial (v_c)} = \sum_{-m < j < m, j \neq 0} \frac{\partial (J_{skipgram}(v_c, w_{j+1}, U)}{\partial (v_c)} \\ \text{iii)} \text{ w } ! = \text{c} \ \frac{\partial (J_{skipgram})}{\partial (v_w)} = 0 \end{array}$$

iii) w != c
$$\frac{\partial (J_s kipgram)}{\partial (v)} = 0$$