

Sorting Algorithms

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Lecture 7

Sorting

- Arranging elements of set into order.
- Let A be a list of n elements A_1, A_2, \dots, A_n in memory. Sorting of list A refers to the operation of rearranging the contents of A so that they are in increasing (or decreasing) order (numerically or lexicographically); $A_1 < A_2 < A_3 < \dots < A_n$.
- Examples of Sorting:
 - Words in a dictionary are sorted.
 - Files in a directory are often listed in sorted order.
 - The index of a book is sorted.

Sorting Algorithms

➤ There are many, many different types of sorting algorithms, but the primary ones are:

- Bubble Sort
- Selection Sort
- Insertion Sort
- Merge Sort
- Quick Sort
- Shell Sort
- Radix Sort
- Swap Sort
- Heap Sort

BUBBLE SORT

➤ In bubble sort:

- Each element is compared with its adjacent element.
- If the first element is larger than the second one, then the positions of the elements are interchanged, otherwise it is not changed.
- Then next element is compared with its adjacent element and the same process is repeated for all the elements in the array until we get a sorted array.

BUBBLE SORT (STEPS)

➤ Step 1:

- Compare $A[1]$ and $A[2]$ and arrange them in the (or desired) ascending order, so that $A[1] < A[2]$.
- if $A[1]$ is greater than $A[2]$ then interchange the position of data by $\text{swap} = A[1]; A[1] = A[2]; A[2] = \text{swap}$.
- Then compare $A[2]$ and $A[3]$ and arrange them so that $A[2] < A[3]$. Continue the process until we compare $A[N - 1]$ with $A[N]$.

BUBBLE SORT (STEPS)

➤ Step 2:

- Repeat step 1 with one less comparisons that is, now stop comparison at $A[n - 1]$ and possibly rearrange $A[N - 2]$ and $A[N - 1]$ and so on.

➤ Step $n - 1$:

- Compare $A[1]$ with $A[2]$ and arrange them so that $A[1] < A[2]$.

BUBBLE SORT EXAMPLE

➤ First Pass:

- $(5\ 1\ 4\ 2\ 8)\ (1\ 5\ 4\ 2\ 8)$, Here, algorithm compares the first two elements, and swaps since $5 > 1$.
- $(1\ 5\ 4\ 2\ 8)\ (1\ 4\ 5\ 2\ 8)$, Swap since $5 > 4$
- $(1\ 4\ 5\ 2\ 8)\ (1\ 4\ 2\ 5\ 8)$, Swap since $5 > 2$
- $(1\ 4\ 2\ 5\ 8)\ (1\ 4\ 2\ 5\ 8)$, Now, since these elements are already in order ($8 > 5$), algorithm does not swap them.

BUBBLE SORT EXAMPLE

➤ Second Pass:

- $(1\ 4\ 2\ 5\ 8)(1\ 4\ 2\ 5\ 8)$
- $(1\ 4\ 2\ 5\ 8)(1\ 2\ 4\ 5\ 8)$, Swap since $4 > 2$
- $(1\ 2\ 4\ 5\ 8)(1\ 2\ 4\ 5\ 8)$
- $(1\ 2\ 4\ 5\ 8)(1\ 2\ 4\ 5\ 8)$
- Now, the array is already sorted, but our algorithm does not know if it is completed. The algorithm needs one **whole** pass without **any** swap to know it is sorted.

BUBBLE SORT EXAMPLE

➤ Third Pass:

- (1 2 4 5 8) (1 2 4 5 8)
- (1 2 4 5 8) (1 2 4 5 8)
- (1 2 4 5 8) (1 2 4 5 8)
- (1 2 4 5 8) (1 2 4 5 8)

BUBBLE SORT EXAMPLE 2

$i = 1$	j	0	1	2	3	4	5	6	7
	0	5	3	1	9	8	2	4	7
	1	3	5	1	9	8	2	4	7
	2	3	1	5	9	8	2	4	7
	3	3	1	5	9	8	2	4	7
	4	3	1	5	8	9	2	4	7
	5	3	1	5	8	2	9	4	7
	6	3	1	5	8	2	4	9	7
$i = 2$	0	3	1	5	8	2	4	7	9
	1	1	3	5	8	2	4	7	
	2	1	3	5	8	2	4	7	
	3	1	3	5	8	2	4	7	
	4	1	3	5	2	8	4	7	
	5	1	3	5	2	4	8	7	
$i = 3$	0	1	3	5	2	4	7	8	
	1	1	3	5	2	4	7		
	2	1	3	5	2	4	7		
	3	1	3	2	5	4	7		
	4	1	3	2	4	5	7		
$i = 4$	0	1	3	2	4	5	7		
	1	1	3	2	4	5			
	2	1	2	3	4	5			
	3	1	2	3	4	5			

BUBBLE SORT EXAMPLE 2

$i = 5$	0	1	2	3	4	5
---------	---	---	---	---	---	---

	1	1	2	3	4	
--	---	---	---	---	---	--

	2	1	2	3	4	
--	---	---	---	---	---	--

$i = 6$	0	1	2	3	4	
---------	---	---	---	---	---	--

	1	1	2	3		
--	---	---	---	---	--	--

$i = 7$	0	1	2	3		
---------	---	---	---	---	--	--

	1	2				
--	---	---	--	--	--	--

ALGORITHM

- Let A be a linear array of n numbers. Swap is a temporary variable for swapping (or interchange) the position of the numbers.
- 1. Input n numbers of an array A
- 2. Initialize $i = 0$ and repeat through step 4 if $(i < n)$
- 3. Initialize $j = 0$ and repeat through step 4 if $(j < n - i - 1)$
- 4. If $(A[j] > A[j + 1])$
 - (a) $\text{Swap} = A[j]$
 - (b) $A[j] = A[j + 1]$
 - (c) $A[j + 1] = \text{Swap}$
- 5. Display the sorted numbers of array A
- 6. Exit.

BubbleSort()

```
void bubbleSort (int a[ ], int size)
{
    int i, j, temp;
    for ( i = 0; i < size; i++ ) /* controls passes through the list */
    {
        for ( j = 0; j < size - 1; j++ ) /* performs adjacent comparisons */
        {
            if ( a[ j ] > a[ j+1 ] ) /* determines if a swap should occur */
            {
                temp = a[ j ]; /* swap is performed */
                a[ j ] = a[ j + 1 ];
                a[ j+1 ] = temp;
            }
        }
    }
}
```

Bubble Sort – Analysis

➤ **Best-case:** ➔ **$O(n)$**

- Array is already sorted in ascending order.
- The number of moves: 0 ➔ $O(1)$
- The number of key comparisons: $(n-1)$ ➔ $O(n)$

➤ **Worst-case:** ➔ **$O(n^2)$**

- Array is in reverse order:
- Outer loop is executed $n-1$ times,
- The number of moves: $3 \cdot (1+2+\dots+n-1) = 3 \cdot n \cdot (n-1)/2$ ➔ $O(n^2)$
- The number of key comparisons: $(1+2+\dots+n-1) = n \cdot (n-1)/2$ ➔ $O(n^2)$

➤ **Average-case:** ➔ **$O(n^2)$**

- We have to look at all possible initial data organizations.

➤ **So, Bubble Sort is $O(n^2)$**

SELECTION SORT

➤ Selection sort :

- It finds the smallest element of the array and interchanges it with the element in the first position of the array.
- Then it finds the second smallest element from the remaining elements in the array and places it in the second position of the array and so on.

SELECTION SORT(STEPS)

- Let A be a linear array of 'n' numbers, $A[1], A[2], A[3], \dots, A[n]$.
- **Step 1:**
 - Find the smallest element in the array of n numbers $A[1], A[2], \dots, A[n]$.
 - Let LOC is the location of the smallest number in the array.
 - Then interchange $A[LOC]$ and $A[1]$ by $\text{swap} = A[LOC]; A[LOC] = A[1]; A[1] = \text{Swap}$.

SELECTION SORT(STEPS)

➤ Step 2:

- Find the second smallest number in the sub list of $n - 1$ elements $A[2] A[3] \dots A[n - 1]$ (first element is already sorted).
- Now we concentrate on the rest of the elements in the array.
- Again $A[LOC]$ is the smallest element in the remaining array and LOC the corresponding location then interchange $A[LOC]$ and $A[2]$.
- Now $A[1]$ and $A[2]$ is sorted, since $A[1]$ less than or equal to $A[2]$.

SELECTION SORT(STEPS)

➤ Step $n - 1$:

- Find the $n - 1$ smallest number in the sub array of 2 elements (i.e., $A(n-1)$, $A(n)$).
- Consider $A[LOC]$ is the smallest element and LOC is its corresponding position.
- Then interchange $A[LOC]$ and $A(n - 1)$.
- Now the array $A[1], A[2], A[3], A[4], \dots, A[n]$ will be a sorted array.

ALGORITHM

➤ Let A be a linear array of n numbers $A[1], A[2], A[3], \dots, A[k], A[k+1], \dots, A[n]$. $Swap$ be a temporary variable for swapping (or interchanging) the position of the numbers. Min is the variable to store smallest number and Loc is the location of the smallest element.

1. Input n numbers of an array A
2. Initialize $i = 0$ and repeat through step 5 if $(i < n - 1)$
 - (a) $min = a[i]$
 - (b) $loc = i$
3. Initialize $j = i + 1$ and repeat through step 4 if $(j < n - 1)$
4. if $(a[j] < min)$
 - (a) $min = a[j]$
 - (b) $loc = j$
5. if $(loc \neq i)$
 - (a) $swap = a[i]$
 - (b) $a[i] = a[loc]$
 - (c) $a[loc] = swap$
6. display "the sorted numbers of array A "
7. Exit

Selection Sort Example(ascending)

➤ 70 75 89 61 37

- Smallest is 37
- Swap with index 0

➤ 37 75 89 61 70

- Smallest is 61
- Swap with index 1

➤ 37 61 89 75 70

- Smallest is 70
- Swap with index 2

➤ 37 61 70 75 89

- Smallest is 75
- Swap with index 3
 - Swap with itself

➤ 37 61 70 75 89

- Don't need to do last element because there's only one left

➤ 37 61 70 75 89

Selection Sort Example 2

[0]	[1]	[2]	[3]	[4]	
5	1	3	7	2	find min
1	5	3	7	2	swap to index 0
1	5	3	7	2	find min
1	2	3	7	5	swap to index 1
1	2	3	7	5	find min
1	2	3	7	5	swap to index 2
1	2	3	7	5	find min
1	2	3	5	7	swap to index 3

Selection Sort -- Analysis

- In general, we compare keys and move items (or exchange items) in a sorting algorithm (which uses key comparisons).
So, to analyze a sorting algorithm we should count the number of key comparisons and the number of moves.
 - Ignoring other operations does not affect our final result.
- In selectionSort function, the outer for loop executes $n-1$ times.
- We invoke swap function once at each iteration.

Total Swaps: $n-1$

Total Moves: $3*(n-1)$

(Each swap has three moves)

Selection Sort – Analysis (cont.)

- The inner for loop executes the size of the unsorted part minus 1 (from 1 to $n-1$), and in each iteration we make one key comparison.
 - # of key comparisons = $1+2+\dots+n-1 = n*(n-1)/2$
 - **So, Selection sort is $O(n^2)$**
- The best case, the worst case, and the average case of the selection sort algorithm are same. ➔ all of them are **$O(n^2)$**
 - This means that the behavior of the selection sort algorithm does not depend on the initial organization of data.
 - Since $O(n^2)$ grows so rapidly, the selection sort algorithm is appropriate only for small n .
 - Although the selection sort algorithm requires $O(n^2)$ key comparisons, it only requires $O(n)$ moves.
 - A selection sort could be a good choice if data moves are costly but key comparisons are not costly (short keys, long records).

INSERTION SORT

➤ Insertion sort algorithm:

- It sorts a set of values by inserting values into an existing sorted file.
- Compare the second element with first, if the first element is greater than second, place it before the first one.
- Otherwise place is just after the first one.
- Compare the third value with second and then with first and so on.

INSERTION SORT (STEPS)

➤ Step 1:

- *As the single element $A[1]$ by itself is sorted array.*

➤ Step 2:

- *$A[2]$ is inserted either before or after $A[1]$ by comparing it so that $A[1], A[2]$ is sorted array.*

INSERTION SORT (STEPS)

➤ Step 3:

- $A[3]$ is inserted into the proper place in $A[1], A[2]$, that is $A[3]$ will be compared with $A[1]$ and $A[2]$ and placed before $A[1]$, between $A[1]$ and $A[2]$, or after $A[2]$ so that $A[1], A[2], A[3]$ is a sorted array.

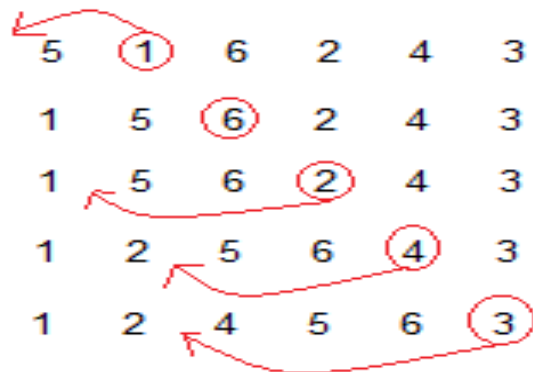
➤ Step n:

- $A[n]$ is inserted into its proper place in an array $A[1], A[2], A[3], \dots, A[n-1]$ so that $A[1], A[2], A[3], \dots, A[n]$ is a sorted array.

Example

5	1	6	2	4	3
---	---	---	---	---	---

Lets take this Array.



(Always we start with the second element as key.)

As we can see here, in insertion sort, we pick up a key, and compares it with elemnts ahead of it, and puts the key in the right place

5 has nothing before it.

1 is compared to 5 and is inserted before 5.

6 is greater than 5 and 1.

2 is smaller than 6 and 5, but greater than 1, so its is inserted after 1.

And this goes on...

54	26	93	17	77	31	44	55	20
----	----	----	----	----	----	----	----	----

Assume 54 is a sorted
list of 1 item

26	54	93	17	77	31	44	55	20
----	----	----	----	----	----	----	----	----

inserted 26

26	54	93	17	77	31	44	55	20
----	----	----	----	----	----	----	----	----

inserted 93

17	26	54	93	77	31	44	55	20
----	----	----	----	----	----	----	----	----

inserted 17

17	26	54	77	93	31	44	55	20
----	----	----	----	----	----	----	----	----

inserted 77

17	26	31	54	77	93	44	55	20
----	----	----	----	----	----	----	----	----

inserted 31

17	26	31	44	54	77	93	55	20
----	----	----	----	----	----	----	----	----

inserted 44

17	26	31	44	54	55	77	93	20
----	----	----	----	----	----	----	----	----

inserted 55

17	20	26	31	44	54	55	77	93
----	----	----	----	----	----	----	----	----

inserted 20

ALGORITHM

- Let A be a linear array of n numbers $A[1], A[2], A[3], \dots, A[n]$. Swap be a temporary variable to interchange the two values. Pos is the control variable to hold the position of each pass.
- 1. Input an array A of n numbers
- 2. Initialize $i = 1$ and repeat through steps 4 by incrementing i by one.
 - (a) If $(i \leq n - 1)$
 - (b) $\text{Swap} = A[i]$,
 - (c) $\text{Pos} = i - 1$
- 3. Repeat the step 3 if $(\text{Swap} < A[\text{Pos}] \text{ and } (\text{Pos} \geq 0))$
 - (a) $A[\text{Pos} + 1] = A[\text{Pos}]$
 - (b) $\text{Pos} = \text{Pos} - 1$
- 4. $A[\text{Pos} + 1] = \text{Swap}$
- 5. Exit