Analysis Übung 4

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Aufgaben + Theorie

A1) $f : x \mapsto x^3$

$$\begin{aligned}
f'(x) &= \lim_{h\to 0} \frac{(x+h) - f(x)}{h} \\
&= \lim_{h\to 0} \frac{(x+h)^3 - x^3}{h} \\
&= \lim_{h\to 0} \frac{x^3 + 3x^2h + 3xh^2 + h^3 - x^3}{h} \\
&= \lim_{h\to 0} \frac{3x^2h + 3xh^2 + h^3}{h} = 3x^2 + 3xh + h^2 \\
&=3x^2
\end{aligned}$$

A2) Ist $f(x)$ differenzierbar

$$f(x) = \begin{cases} 0, & x \leq 0 \\ x, & x > 0 \end{cases}$$

$$\begin{aligned}
x \leq 0 : f'(x) &= \lim_{h\to 0} \frac{(x+h) - f(x)}{h} \\
&= 0 \\
x > 0 : f'(x) &= \lim_{h\to 0} \frac{(x+h) - f(x)}{h} \\
&= 1 \\
&\Rightarrow \text{Nicht differenzierbar}
\end{aligned}$$

A3)

1) $f_1(x) = (1-x)^5$
   $f_1'(x) = -5(1-x)^4$

2) $f_2(x) = (\sqrt{x} + \cos x)^1 8$
   $f_2'(x) = 18(\sqrt{x} + \cos x)^{17}(\frac{1}{2\sqrt{x}} - \sin x)$

3) $f_3(x) = x^x$
   $f_3'(x) = \frac{d}{dx} e^{x \log(x)} = (\log(x) + 1)x^x$

4) $f_4(x)$ is die Inverse von $h_4(x) = \frac{1}{2}\cos(x)$
   (Tipp: $\sin(\arccos(\alpha)) = \sqrt{1 - \cos(\arccos(\alpha))^2}$)
   $f_4'(x) = h_4^{-1}(x)'$
   $f_4'(x) = \frac{1}{-\frac{1}{2}\sin(\arccos(2x))} = -\frac{2}{\sqrt{1-4x^2}}$