Bonusaufgaben 3

Ahmed Bajra

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Aufgabe (a)

(i)

$$IG = \begin{pmatrix} 2 & 3 \\ 4 & 2 \end{pmatrix}$$
$$GI = \begin{pmatrix} 2 & 3 \\ 4 & 2 \end{pmatrix}$$
$$\Rightarrow GI = IG$$

(ii)

$$DH = \begin{pmatrix} 15 & 9 \\ 12 & 6 \end{pmatrix}$$
$$HD = \begin{pmatrix} 15 & 9 \\ 12 & 6 \end{pmatrix}$$
$$\Rightarrow DH = HD$$

(iii)

$$FH = \begin{pmatrix} 10 & 0 \\ 32 & 12 \end{pmatrix}$$
$$HF = \begin{pmatrix} 10 & 0 \\ 16 & 12 \end{pmatrix}$$
$$\Rightarrow FH \neq HF$$

(iv)

$$DF = \begin{pmatrix} 6 & 0 \\ 0 & 12 \end{pmatrix}$$
$$FD = \begin{pmatrix} 6 & 0 \\ 0 & 12 \end{pmatrix}$$
$$\Rightarrow DF = FD$$

(v)

$$\begin{aligned}
DG &= \begin{pmatrix} 6 & 9 \\ 12 & 6 \end{pmatrix} \\
GD &= \begin{pmatrix} 6 & 9 \\ 12 & 6 \end{pmatrix} \\
&\Rightarrow DG = GD
\end{aligned}$$

(vi)

$$\begin{aligned}
GH &= \begin{pmatrix} 10 & 9 \\ 28 & 6 \end{pmatrix} \\
HG &= \begin{pmatrix} 34 & 9 \\ 36 & 12 \end{pmatrix} \\
&\Rightarrow GH \neq HG
\end{aligned}$$

Aufgabe (b)

$$\begin{aligned}
AB = \begin{pmatrix} b_1 - 12 & 2b_2 - 4 \\ 3b_1 + 6 & -b - 12 \end{pmatrix} &= \\
BA = \begin{pmatrix} b_1 - 12 & 2b_1 + 4 \\ 3b_2 - 6 & -b_2 - 12 \end{pmatrix}& \\
\Rightarrow \forall b_2 \in \mathbb{R} : b_1 = 4 - b_2 \Rightarrow AB &= BA \\
\Leftrightarrow b_1 \neq 4 - b_2 \Rightarrow BA &\neq AB
\end{aligned}$$

$$AC = \begin{pmatrix} a+2c & b+2d \\ 4a-c & 3b-d \end{pmatrix} =$$
$$CA = \begin{pmatrix} a+3b & 2a-b \\ c+3d & 2c-d \end{pmatrix}$$
$$\Rightarrow$$
$$a+2c = a+3b,$$
$$b+2d = 2a-b,$$
$$4a-c = c+3d,$$
$$3b-d = 2c-d$$
$$\Leftrightarrow$$
$$3b-2c = 0$$
$$a-b-d = 0$$
$$4a-2c-3d = 0$$
$$3b-2c = 0$$
$$\Leftrightarrow$$
$$\forall \alpha \in \mathbb{R} : a = 0, b = \alpha, c = \frac{3\alpha}{2}, d = -\alpha \Leftrightarrow AC = CA$$

Aufgabe (c)

(i) $EFG + EFG = 2EFG$, da $EFG := A, EFG + EFG = A + A \Leftrightarrow (A+A)_{ij} = 2(A)_{ij}$

(ii) $EFG + EGF \neq 2EFG$, da $\exists E, F, G : EFG \neq EGF$

(iii) $G(H+E) = GE + GH$, da $G(H+E) = GH + GE = GE + GH$, (Assoziativgesetz über Addition)

(iv) $EFEFG + FEEFG + E^2F^2G \neq 3E^2F^2G$, da $\exists E, F : EF \neq FE$

(v) $EGHH + EGGH = (EGH + EG^2)H$, da $(B+C)A = BA + CA$ und $E(GG) = (EG)G$

(vi) $(GH)^2 \neq G^2H^2$, da $(GH)(GH) = GHGH \neq GGHH$