Analysis Übung 2

Ahmed Bajra

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Übungen

8b)

$$\begin{aligned} a_n &= \sum_{1\leq k\leq n} \frac{k}{n^2} \\ &= \frac{1}{n^2}\frac{n(n+1)}{2} \\ &= \frac{1}{2} + \frac{1}{2n} \\ \lim_{n\to\infty} a_n &= \frac{1}{2} \Rightarrow \text{(Konvergiert)} \end{aligned}$$

$$a_{n+1} = \frac{1}{2} + \frac{1}{2(n+1)} < a_n \Rightarrow \text{Strikt monoton fallend}$$

8d)

$$\begin{aligned} a_1 &= 0 \\ a_2 &= 1 \\ a_n &= \frac{1}{2}(a_{n-1} + a_{n-2}) \\ a_{n+2} - a_{n+1} &= \frac{1}{2}(a_{n+1} + a_n) - a_{n+1} \\ &= \frac{1}{2}(a_n - a_{n+1}) \\ &= -\frac{1}{2}(a_{n+1} - a_n) \\ &= \left(-\frac{1}{2}\right)^2 (a_{\underline{n}} - a_{n-1}) \\ &= \left(-\frac{1}{2}\right)^{n-k+2} (a_k - a_{k-1}) \\ &= \left(-\frac{1}{2}\right)^n (a_2 - a_1) = \left(-\frac{1}{2}\right)^n \end{aligned}$$

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\begin{aligned}
a_{n+2} - a_{n+1} &= \left(-\frac{1}{2}\right)^n \\
a_n &= a_1 && +(a_2 - a_1) + \dots && +(a_{n-1} - a_{n-2}) + (a_n - a_{n-1}) \\
&= a_1 && +1 + \dots && + \left(-\frac{1}{2}\right)^{n-3} + \left(-\frac{1}{2}\right)^{n-2} \\
&= \sum_{0 \le k \le n-2} \left(-\frac{1}{2}\right)^k
\end{aligned}
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9a) Fibonacci

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\begin{aligned}
a_n &:= a_{n-1} + a_{n-2} \\
b_n &= \frac{a_n}{a_{n-1}} \\
&= \frac{a_{n-1} + a_{n-2}}{a_{n-1}} \\
&= 1 + \frac{a_{n-1}}{a_{n-2}} \\
&= 1 + \frac{1}{b_{n-1}} \\
b_1 &= 1 \le b_n \\
1 + \frac{1}{b_{n-1}} &\le 2 \\
&\Rightarrow 1 \le b_n \le 2
\end{aligned}
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