Matrix Computations and Optimization in Apache Spark

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Abstract

We describe matrix computations available in the cluster programming framework, Apache Spark. Out of the box, Spark provides abstractions and implementations for distributed matrices and optimization routines using these matrices. When translating single-node algorithms to run on a distributed cluster, we observe that often a simple idea is enough: separating matrix operations from vector operations and shipping the matrix operations to be ran on the cluster, while keeping vector operations local to the driver. In the case of the Singular Value Decomposition, by taking this idea to an extreme, we are able to exploit the computational power of a cluster, while running code written decades ago for a single core. Another example is our Spark port of the popular TFOCS optimization package, originally built for MATLAB, which allows for solving Linear programs as well as a variety of other convex programs. We conclude with a comprehensive set of benchmarks for hardware accelerated matrix computations from the JVM, which is interesting in its own right, as many cluster programming frameworks use the JVM. The contributions described in this paper are already merged into Apache Spark and available on Spark installations by default, and commercially supported by a slew of companies which provide further services.

1 Introduction

Modern datasets are rapidly growing in size and many datasets come in the form of matrices. There is a pressing need to handle large matrices spread across many machines with the same familiar linear algebra tools that are available for single-machine analysis. Several 'next generation' data flow engines that generalize MapReduce [5] have been developed for large-scale data processing, and building linear algebra functionality on these engines is a problem of great interest. In particular, Apache Spark [12] has emerged as a widely used open-source engine. Spark is a fault-tolerant and general-purpose cluster computing system providing APIs in Java, Scala, Python, and R, along with an optimized engine that supports general execution graphs.

In this work we present Spark's distributed linear algebra and optimization libraries, the largest concerted cross-institution effort to build a distributed linear algebra and optimization library. The library targets large-scale matrices that benefit from row, column, entry, or block sparsity to store and operate on distributed and local matrices. The library, named LINALG consists of fast and scalable implementations of standard matrix computations for common linear algebra operations including basic operations such as multiplication and more advanced ones such as factorizations. It also provides a variety of underlying primitives such as column and block statistics. Written in Scala and using native (C++ and fortran based) linear algebra libraries on each node, LINALG includes Java, Scala, and Python APIs, and is released as part of the Spark project under the Apache 2.0 license.

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1.1 Apache Spark

We restrict our attention to Spark, because it has several features that are particularly attractive for matrix computations:

1. The Spark storage abstraction called Resilient Distributed Datasets (RDDs) is essentially a distributed fault-tolerant vector on which programmers can perform a subset of operations expected from a regular local vector.
2. RDDs permit user-defined data partitioning, and the execution engine can exploit this to co-partition RDDs and co-schedule tasks to avoid data movement.
3. Spark logs the lineage of operations used to build an RDD, enabling automatic reconstruction of lost partitions upon failures. Since the lineage graph is relatively small even for long-running applications, this approach incurs negligible runtime overhead, unlike checkpointing, and can be left on without concern for performance. Furthermore, Spark supports optional in-memory distributed replication to reduce the amount of recomputation on failure.
4. Spark provides a high-level API in Scala that can be easily extended. This aided in creating a coherent API for both collections and matrix computations.

There exists a history of using clusters of machines for distributed linear algebra, for example [3]. These systems are often not fault-tolerant to hardware failures and assume random access to non-local memory. In contrast, our library is built on Spark, which is a dataflow system without direct access to non-local memory, designed for clusters of *commodity* computers with relatively slow and cheap interconnects, and abundant machines failures. All of the contributions described in this paper are already merged into Apache Spark and available on Spark installations by default, and commercially supported by a slew of companies which provide further services.

1.2 Challenges and Contributions

Given that we have access to RDDs in a JVM environment, four key challenges arise to building a distributed linear algebra library, each of which we address:

1. Data representation: how should one partition the entries of a matrix across machines so that subsequent matrix operations can be implemented as efficiently as possible? This led us to develop three different distributed matrix representations, each of which has benefits depending on the sparsity pattern of the data. We have built

   (a) CoordinateMatrix which puts each nonzero into a separate RDD entry.

   (b) BlockMatrix which treats the matrix as dense blocks of non-zeros, each block small enough to fit in memory on a single machine.

   (c) RowMatrix which assumes each row is small enough to fit in memory. There is an option to use a sparse or dense representation for each row.

   These matrix types and the design decisions behind them are outlined in Section 2.

2. Matrix Computations must be adapted for running on a cluster, as we cannot readily reuse linear algebra algorithms available for single-machine situations. A key idea that lets us distribute many operations is to separate algorithms into portions that require matrix operations versus vector operations. Since matrices are often quadratically larger than vectors, a reasonable assumption is that vectors fit in memory on a single machine, while matrices do not. Exploiting this idea, we were able to distribute the Singular Value Decomposition via code written decades ago in FORTRAN90, as part of the ARPACK [6] software package. By separating matrix from vector computations, and shipping the matrix computations to the cluster while keeping vector operations local to the driver, we were able to distribute two classes of optimization problems:

(a) Spectral programs: Singular Value Decomposition (SVD) and PCA

(b) Convex programs: Gradient Descent, LBFGS, Accelerate Gradient, and other unconstrained optimization methods. We provide a port of the popular TFOCS optimization framework [1], which covers Linear Programs and a variety of other convex objectives

Separating matrix operations from vector operations helps in the case that vectors fit in memory, but matrices do not. This covers a wide array of applications, since matrices are often quadratically larger. However, there are some cases for which vectors do not fit in memory on a single machine. For such cases, we use an RDD for the vector as well, and use BlockMatrix for data storage.

We give an outline of the most interesting of these computations in Section 3.

3. Many distributed computing frameworks such as Spark and Hadoop run on the Java Virtual Machine (JVM), which means that achieving hardware-specific acceleration for computation can be difficult. We provide a comprehensive survey of tools that allow matrix computations to be pushed down to hardware via the Basic Linear Algebra Subprograms (BLAS) interface from the JVM. In addition to a comprehensive set of benchmarks, we have made all the code for producing the benchmark public to allow for reproducibility.

   In Section 4 we provide results and pointers to code and benchmarks.

4. Given that there are many cases when distributed matrices and local matrices need to interact (for example multiplying a distributed matrix by a local one), we also briefly describe the local linear algebra library we built to make this possible, although the focus of the paper is distributed linear algebra.

2 Distributed matrix

Before we can build algorithms to perform distributed matrix computations, we need to lay out the matrix across machines. We do this in several ways, all of which use the sparsity pattern to optimize computation and space usage. A distributed matrix has long-typed row and column indices and double-typed values, stored distributively in one or more RDDs. It is very important to choose the right format to store large and distributed matrices. Converting a distributed matrix to a different format may require a global shuffle, which is quite expensive. Three types of distributed matrices have been implemented so far.

2.1 RowMatrix and IndexedRowMatrix

A RowMatrix is a row-oriented distributed matrix without meaningful row indices, backed by an RDD of its rows, where each row is a local vector. Since each row is represented by a local vector, the number of columns is limited by the integer range but it should be much smaller in practice. We assume that the number of columns is not huge for a RowMatrix so that a single local vector can be reasonably communicated to the driver and can also be stored / operated on using a single machine.

An IndexedRowMatrix is similar to a RowMatrix but with meaningful row indices. It is backed by an RDD of indexed rows, so that each row is represented by its index (long-typed) and a local vector.

2.2 CoordinateMatrix

A CoordinateMatrix is a distributed matrix backed by an RDD of its entries. Each entry is a tuple of (i: Long, j: Long, value: Double), where i is the row index, j is the column index, and value is the entry value.

A CoordinateMatrix should be used only when both dimensions of the matrix are huge and the matrix is very sparse. A CoordinateMatrix can be created from an RDD[MatrixEntry] instance, where MatrixEntry is a wrapper over (Long, Long, Double). A CoordinateMatrix can be converted to an IndexedRowMatrix with sparse rows by calling toIndexedRowMatrix.

2.3 BlockMatrix

A BlockMatrix is a distributed matrix backed by an RDD of MatrixBlocks, where a MatrixBlock is a tuple of ((Int, Int), Matrix), where the (Int, Int) is the index of the block, and Matrix is the sub-matrix at the given index with size rowsPerBlock $\times$ colsPerBlock. BlockMatrix supports methods such as add and multiply with another BlockMatrix. BlockMatrix also has a helper function validate which can be used to check whether the BlockMatrix is set up properly.

2.4 Local Vectors and Matrices

Spark supports local vectors and matrices stored on a single machine, as well as distributed matrices backed by one or more RDDs. Local vectors and local matrices are simple data models that serve as public interfaces. The underlying linear algebra operations are provided by Breeze and jblas. A local vector has integer-type and 0-based indices and double-typed values, stored on a single machine. Spark supports two types of local vectors: dense and sparse. A dense vector is backed by a double array representing its entry values, while a sparse vector is backed by two parallel arrays: indices and values. For example, a vector (1.0, 0.0, 3.0) can be represented in dense format as [1.0, 0.0, 3.0] or in sparse format as (3, [0, 2], [1.0, 3.0]), where 3 is the size of the vector.

3 Matrix Computations

We now move to the most challenging of tasks: rebuilding algorithms from single-core modes of computation to operate on our distributed matrices in parallel. Here we outline some of the more interesting approaches.

3.1 Singular Value Decomposition

The rank k singular value decomposition (SVD) of an $m \times n$ real matrix A is a factorization of the form $A = U\Sigma V^T$, where U is an $m \times k$ unitary matrix, Σ is an $k \times k$ diagonal matrix with non-negative real numbers on the diagonal, and V is an $n \times k$ unitary matrix. The diagonal entries Σ are known as the singular values. The k columns of U and the n columns of V are called the "left-singular vectors" and "right-singular vectors" of A, respectively.

Depending on whether the $m \times n$ input matrix A is tall and skinny ($m \gg n$) or square, we use different algorithms to compute the SVD. In the case that A is roughly square, we use the ARPACK package for computing eigenvalue decompositions, which can then be used to compute a singular value decomposition via the eigenvalue decomposition of A^TA. In the case that A is tall and skinny, we compute A^TA, which is small, and use it locally. In the following two sections with detail the approaches for each of these two cases. Note that the SVD of a wide and short matrix can be recovered from its transpose, which is tall and skinny, and so we do not consider the wide and short case.

There is a well known connection between the eigen and singular value decompositions of a matrix, that is the two decompositions are the same for positive semidefinite matrices, and A^TA is positive semidefinite with its singular values being squares of the singular values of A. So one can recover the SVD of A from the eigenvalue decomposition of A^TA. We exploit this relationship in the following two sections.

3.1.1 Square SVD with ARPACK

ARPACK is a collection of Fortran77 subroutines designed to solve eigenvalue problems [6]. Written many decades ago and compiled for specific architectures, it is surprising that it can be effectively distributed on a modern commodity cluster.

The package is designed to compute a few eigenvalues and corresponding eigenvectors of a general $n \times n$ matrix A. In the local setting, it is appropriate for sparse or structured matrices where structured means that a matrix-vector product requires order n rather than the usual order n^2 floating point operations and storage. APRACK is based upon an algorithmic variant of the Arnoldi process called the Implicitly Restarted

Matrix size	Number of nonzeros	Time per iteration (s)	Total time (s)
23,000,000 $\times$ 38,000	51,000,000	0.2	10
63,000,000 $\times$ 49,000	440,000,000	1	50
94,000,000 $\times$ 4,000	1,600,000,000	0.5	50

Table 1: Runtimes for ARPACK Singular Value Decomposition

Arnoldi Method (IRAM). When the matrix A is symmetric it reduces to a variant of the Lanczos process called the Implicitly Restarted Lanczos Method (IRLM). These variants may be viewed as a synthesis of the Arnoldi/Lanczos process with the Implicitly Shifted QR technique. The Arnoldi process only interacts with the matrix via matrix-vector multiplies.

APRACK is designed to compute a few, say k eigenvalues with user specified features such as those of largest real part or largest magnitude. Storage requirements are on the order of nk doubles with no auxiliary storage is required. A set of Schur basis vectors for the desired k-dimensional eigen-space is computed which is numerically orthogonal to working precision. The only interaction that ARPACK needs with a matrix is the result of matrix-vector multiplies.

By separating matrix operations from vector operations, we are able to distribute the computations required by ARPACK. An important feature of ARPACK is its ability to allow for arbitrary matrix formats. This is because it does not operate on the matrix directly, but instead acts on the matrix via prespecified operations, such as matrix-vector multiplies. When a matrix operation is required, ARPACK gives control to the calling program with a request for a matrix-vector multiply. The calling program must then perform the multiply and return the result to ARPACK. By using the distributed-computing utility of Spark, we can distribute the matrix-vector multiplies, and thus exploit the computational resources available in the entire cluster.

Since ARPACK is written in Fortran77, it cannot immediately be used on the Java Virtual Machine. However, through the netlib-java and breeze packages, we use ARPACK on the JVM on the driver node and ship the computations required for matrix-vector multiplies to the cluster. This also means that low-level hardware optimizations can be exploited for any local linear algebraic operations. As with all linear algebraic operations within MLlib, we use hardware acceleration whenever possible. This functionality has been available since Spark 1.1.

We provide experimental results using this idea. A very popular matrix in the recommender systems community is the Netflix Prize Matrix. The matrix has 17,770 rows, 480,189 columns, and 100,480,507 non-zeros. Below we report results on several larger matrices, up to 16x larger.

With the Spark implementation of SVD using ARPACK, calculating wall-clock time with 68 executors and 8GB memory in each, looking for the top 5 singular vectors, we can factorize larger matrices distributed in RAM across a cluster, in a few seconds, with times listed Table 1.

3.1.2 Tall and Skinny SVD

In the case that the input matrix has few enough columns that A^TA can fit in memory on the driver node, we can avoid shipping the eigen-decomposition to the cluster and avoid the associated communication costs.

First we compute Σ and V. We do this by computing A^TA, which can be done with one all-to-one communication, details of which are available in [11, 10]. Since $A^TA = V\Sigma^2V^T$ is of dimension $n \times n$, for small n (for example $n = 10^4$) we can compute the eigen-decomposition of A^TA directly and locally on the driver to retrieve V and Σ.

Once V and Σ are computed, we can recover U. Since in this case n is small enough to fit n^2 doubles in memory, then V and Σ will also fit in memory on the driver. U however will not fit in memory on a single node and will need to be distributed, and we still need to compute it. This can be achieved by computing $U = AV\Sigma^{-1}$ which is derived from $A = U\Sigma V^T$. Σ^{-1} is easy to compute since it is diagonal, and the pseudo-inverse of V is its transpose and also easy to compute. We can distribute the computation of $U = A(V\Sigma^{-1})$ by broadcasting $V\Sigma^{-1}$ to all nodes holding rows of U, and from there it is embarrassingly parallel to compute U.

The method COMPUTESVD on the ROWMATRIX class takes care of which of the tall and skinny or square versions to invoke, so the user does not need to make that decision.

3.2 Spark TFOCS: Templates for First-Order Conic Solvers

To allow users of single-machine optimization algorithms to use commodity clusters, we have developed Spark TFOCS, which is an implementation of the TFOCS convex solver for Apache Spark.

The original Matlab TFOCS library [1] provides building blocks to construct efficient solvers for convex problems. Spark TFOCS implements a useful subset of this functionality, in Scala, and is designed to operate on distributed data using the Spark. Spark TFOCS includes support for:

- Convex optimization using Nesterov's accelerated method (Auslender and Teboulle variant)
- Adaptive step size using backtracking Lipschitz estimation
- Automatic acceleration restart using the gradient test
- Linear operator structure optimizations
- Smoothed Conic Dual (SCD) formulation solver, with continuation support
- Smoothed linear program solver
- Multiple data distribution patterns. (Currently support is only implemented for RDD[VECTOR] row matrices.)

The name "TFOCS" is being used with permission from the original TFOCS developers, who are not involved in the development of this package and hence not responsible for the support. To report issues or download code, please see the project's GitHub page

`https://github.com/databricks/spark-tfocs`

3.2.1 TFOCS

TFOCS is a state of the art numeric solver; formally, a first order convex solver [1]. This means that it optimizes functions that have a global minimum without additional local minima, and that it operates by evaluating an objective function, and the gradient of that objective function, at a series of probe points. The key optimization algorithm implemented in TFOCS is Nesterovs accelerated gradient descent method, an extension of the familiar gradient descent algorithm. In traditional gradient descent, optimization is performed by moving "downhill" along a function gradient from one probe point to the next, iteration after iteration. The accelerated gradient descent algorithm tracks a linear combination of prior probe points, rather than only the most recent point, using a clever technique that greatly improves asymptotic performance.

TFOCS fine-tunes the accelerated gradient descent algorithm in several ways to ensure good performance in practice, often with minimal configuration. For example TFOCS supports backtracking line search. Using this technique, the optimizer analyzes the rate of change of an objective function and dynamically adjusts the step size when descending along its gradient. As a result, no explicit step size needs to be provided by the user when running TFOCS.

Matlab TFOCS contains an extensive feature set. While the initial version of Spark TFOCS implements only a subset of the many possible features, it contains sufficient functionality to solve several interesting problems.

3.2.2 Example: LASSO Regression

A LASSO linear regression problem (otherwise known as L1 regularized least squares regression) can be described and solved easily using TFOCS. Objective functions are provided to TFOCS in three separate parts, which are together referred to as a composite objective function. The complete LASSO objective function can be represented as:

$$\frac{1}{2}||Ax - b||_2^2 + \lambda||x||_1$$

This function is provided to TFOCS in three parts. The first part, the linear component, implements matrix multiplication:

$$Ax$$

The next part, the smooth component, implements quadratic loss:

$$\frac{1}{2}|| \bullet - b||_2^2$$

And the final part, the nonsmooth component, implements L1 regularization:

$$\lambda||x||_1$$

The TFOCS optimizer is specifically implemented to leverage this separation of a composite objective function into component parts. For example, the optimizer may evaluate the (expensive) linear component and cache the result for later use.

Concretely, in Spark TFOCS the above LASSO regression problem can be solved as follows:

TFOCS.optimize(new SmoothQuad(b), new LinopMatrix(A), new ProxL1(lambda), x0)

Here, SmoothQuad is the quadratic loss smooth component, LinopMatrix is the matrix multiplication linear component, and ProxL1 is the $L1$ norm nonsmooth component. The x0 variable is an initial starting point for gradient descent. Spark TFOCS also provides a helper function for solving LASSO problems, which can be called as follows:

SolverL1RLS.run(A, b, lambda)

3.2.3 Example: Linear Programming

Solving a linear programming problem requires minimizing a linear objective function subject to a set of linear constraints. TFOCS supports solving smoothed linear programs, which include an approximation term that simplifies finding a solution. Smoothed linear programs can be represented as:

$$\begin{array}{ll} \text{minimize} & c^T x + \frac{1}{2}||x - x_0||_2^2 \\ \text{subject to} & Ax = b \\ & x \geq 0 \end{array}$$

A smoothed linear program can be solved in Spark TFOCS using a helper function as follows:

SolverSLP.run(c, A, b, mu)

A complete linear program example is presented here:

`https://github.com/databricks/spark-tfocs`

3.3 Convex Optimization

We now focus on Convex optimization via gradient descent for separable objective functions. That is, objective functions that can be written in the form of

$$F(w) = \sum_{i=1}^{n} F_i(w)$$

where w is a d-dimensional vector of parameters to be tuned and each $F_i(w)$ represents the loss of the model for the i'th training point. In the case that d doubles can fit in memory on the driver, the gradient of $F(w)$ can be computed using the computational resources on the cluster, and then collected on the driver, where it will also fit in memory. A simple gradient update can be done locally and then the new guess for w broadcast out to the cluster. This idea is essentially separating the matrix operations from the vector operations, since the vector of optimization variables is much smaller than the data matrix.

Given that the gradient can be computed using the cluster and then collected on the driver, all computations on the driver can proceed oblivious to how the gradient was computed. This means in addition to gradient descent, we can use tradtional single-node implementations of all first-order optimization methods that only use the gradient, such as accelerated gradient methods, LBFGS, and variants thereof. Indeed, we have LBFGS and accelerated gradient methods implemented in this way and available as part of MLlib. For the first time we provide convergence plots for these optimization primitives available in Spark, listed in Figure 1. We have available the following optimization algorithms, with convergence plots in Figure 1:

- gra: gradient descent implementation [7] using full batch
- acc: accelerated descent as in [1], without automatic restart [8]
- acc_r: accelerated descent, with automatic restart [1]
- acc_b: accelerated descent, with backtracking, without automatic restart [1]
- acc_rb: accelerated descent, with backtracking, with automatic restart [1]
- lbfgs: an L-BFGS implementation [13]

In Figure 1 the x axis shows the number of outer loop iterations of the optimization algorithm. Note that for backtracking implementations, the full cost of backtracking is not represented in this outer loop count. For non-backtracking implementations, the number of outer loop iterations is the same as the number of spark map reduce jobs. The y axis is the log of the difference from best determined optimized value. The optimization test runs were:

- linear: A scaled up version of the test data from TFOCS's 'test_LASSO.m' example [1], with 10000 observations on 1024 features. 512 of the features are actually correlated with result. Unregularized linear regression was used. As expected, the Scala/Spark acceleration implementation was observed to be consistent with the TFOCS implementation on this dataset.
- linear l1: The same as 'linear', but with L1 regularization
- logistic: Each feature of each observation is generated by summing a feature gaussian specific to the observation?s binary category with a noise gaussian. 10000 observations on 250 features. Unregularized logistic regression was used.
- logistic l2: Same as 'logistic', but using L2 regularization

For all runs, all optimization methods were given the same initial step size. We now note some observations. First, acceleration consistently converges more quickly than standard gradient descent, given the same initial step size. Second, automatic restarts are indeed helpful for accelerating convergence. Third, Backtracking can significantly boost convergence rates in some cases (measured in terms of outer loop iterations), but the full cost of backtracking was not measured in these runs. Finally, LBFGS generally outperformed accelerated gradient descent in these test runs.

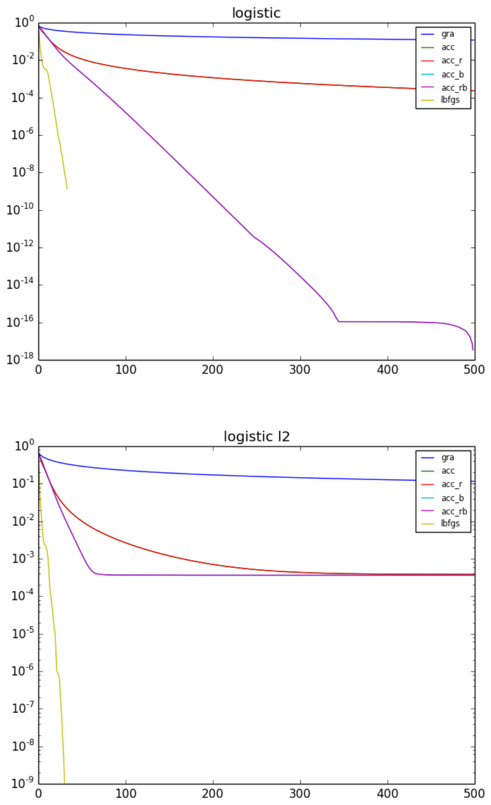


Figure 1: Error per iteration for optimization primitives. From left to right, top to bottom: logistic regression, least squares regression, L2 regularized logistic regression, L1 regularized least squares (LASSO).

3.4 Other matrix algorithms

There are several matrix computations on distributed matrices that use algorithms previously published, so we only cite them here:

- RowMatrix provides QR decomposition as described in [2]
- RowMatrix provides optimized computation of $A^T A$ via DIMSUM [11]
- BlockMatrix will provide large linear model parallelism [4, 9]

4 Hardware Acceleration

4.1 CPU and GPU acceleration

To allow full use of hardware-specific linear algebraic operations on a single node, we use the BLAS (Basic Linear Algebra Subroutines) interface with relevant libraries for CPU and GPU acceleration. Native libraries can be used in Scala as follows. First, native libaries must have a C BLAS interface or wrapper. The latter is called through the Java native interface implemented in Netlib-java library and wrapped by the Scala library called Breeze. We consider the following implementations of BLAS like routines:

- f2jblas - Java implementation of Fortran BLAS
- OpenBLAS - open source CPU-optimized C implementation of BLAS
- MKL - proprietary CPU-optimized C and Fortran implementation of BLAS by Intel
- cuBLAS - proprietary GPU-optimized implementation of BLAS like routines by nVidia. nVidia provides a Fortran BLAS wrapper for cuBLAS called NVBLAS. It can be used in Netlib-java through CBLAS interface.

In addition to the mentioned libraries, we also consider the BIDMat matrix library that can use MKL or cuBLAS. Our benchmark includes matrix-matrix multiplication routine called GEMM. This operation comes from BLAS Level 3 and can be hardware optimized as opposed to the operations from the lower BLAS levels. We benchmark GEMM with different matrix sizes both for single and double precision. The system used for benchmark is as follows:

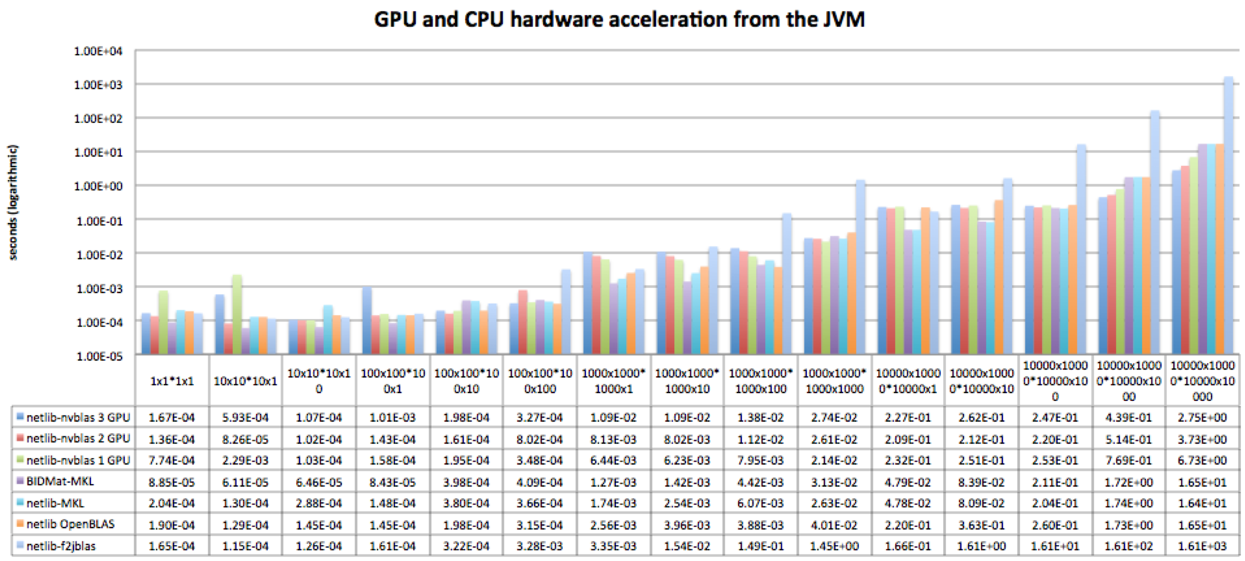
- CPU - 2x Xeon X5650 @ 2.67GHz (32 GB RAM)
- GPU - 3x Tesla M2050 3GB, 575MHz, 448 CUDA cores
- Software - RedHat 6.3, Cuda 7, nVidia driver 346.72, BIDMat 1.0.3

The results for the double precision matrices are depicted on Figure 2. A full spreadsheet of results and code is available at `https://github.com/avulanov/scala-blas`.

Results show that MKL provides similar performance to OpenBLAS except for tall matrices when the latter is slower. Most of the time GPU is less effective due to overhead of copying matrices to/from GPU. However, when multiplying sufficiently large matrices, i.e. starting from 10000×10000 by 10000×1000, the overhead becomes negligible with respect to the computation complexity. At that point GPU is several times more effective than CPU. Interestingly, adding more GPUs speeds up the computation almost linearly for big matrices.

Because it is unreasonable to expect all machines that Spark is run on to have GPUs, we have made OpenBlas the default method of choice for hardware acceleration in Spark's local matrix computations. Note that these performance numbers are useful anytime the JVM is used for Linear Algebra, including Hadoop, Storm, and popular commodity cluster programming frameworks. As an example of BLAS usage in Spark, Neural Networks available in MLlib use the interface heavily, since the forward and backpropagation steps in neural networks are a series of matrix-vector multiplies.

A full spreadsheet of results and code is available at



	1x1*1x1	10x10*10x1	10x10*10x10	100x100*100x1	100x100*100x10	100x100*100x100	1000x1000*1000x1	1000x1000*1000x10	1000x1000*1000x100	1000x1000*1000x1000	10000x10000*10000x1	10000x10000*10000x10	10000x10000*10000x100	10000x10000*10000x1000	10000x10000*10000x10000
netlib-nvblas 3 GPU	1.67E-04	5.93E-04	1.07E-04	1.01E-03	1.98E-04	3.27E-04	1.09E-02	1.09E-02	1.38E-02	2.74E-02	2.27E-01	2.62E-01	2.47E-01	4.39E-01	2.75E+00
netlib-nvblas 2 GPU	1.36E-04	8.26E-05	1.02E-04	1.43E-04	1.61E-04	8.02E-04	8.13E-03	8.02E-03	1.12E-02	2.61E-02	2.09E-01	2.12E-01	2.20E-01	5.14E-01	3.73E+00
netlib-nvblas 1 GPU	7.74E-04	2.29E-03	1.03E-04	1.58E-04	1.95E-04	3.48E-04	6.44E-03	6.23E-03	7.95E-03	2.14E-02	2.32E-01	2.51E-01	2.53E-01	7.69E-01	6.73E+00
BIDMat-MKL	8.85E-05	6.11E-05	6.46E-05	8.43E-05	3.98E-04	4.09E-04	1.27E-03	1.42E-03	4.42E-03	3.13E-02	4.79E-02	8.39E-02	2.11E-01	1.72E+00	1.65E+01
netlib-MKL	2.04E-04	1.30E-04	2.88E-04	1.48E-04	3.80E-04	3.66E-04	1.74E-03	2.54E-03	6.07E-03	2.63E-02	4.78E-02	8.09E-02	2.04E-01	1.74E+00	1.64E+01
netlib OpenBLAS	1.90E-04	1.29E-04	1.45E-04	1.45E-04	1.98E-04	3.15E-04	2.56E-03	3.96E-03	3.88E-03	4.01E-02	2.20E-01	3.63E-01	2.60E-01	1.73E+00	1.65E+01
netlib-f2jblas	1.65E-04	1.15E-04	1.26E-04	1.61E-04	3.22E-04	3.28E-03	3.35E-03	1.54E-02	1.49E-01	1.45E+00	1.66E-01	1.61E+00	1.61E+01	1.61E+02	1.61E+03

Figure 2: Benchmarks for hardware acceleration from the JVM. Full results and all numbers are available at `https://github.com/avulanov/scala-blas`

`https://github.com/avulanov/scala-blas`

4.2 Sparse Single-Core Linear Algebra

The BLAS interface is made specifically for *dense* linear algebra. There are not many libraries on the JVM that efficiently handle sparse matrix operations, or even provide the option to store a local matrix in sparse format. MLlib provides SparseMatrix, which provides memory efficient storage in Compressed Column Storage (CCS) format. In this format, a row index and a value is stored for each non-zero element in separate arrays. The columns are formed by storing the first and the last indices of the elements for that column in a separate array.

MLlib has specialized implementations for performing Sparse Matrix $\times$ Dense Matrix, and Sparse Matrix $\times$ Dense Vector multiplications, where matrices can be optionally transposed. These implementations outperform libraries such as Breeze, and are competitive against libraries like SciPy, where implementations are backed by C. Benchmarks available at `https://github.com/apache/spark/pull/2294`.

Conclusions

We described the distributed and local matrix computations available in Apache Spark, a widely distributed cluster programming framework. By separating matrix operations from vector operations, we are able to distribute a large number of traditional algorithms meant for single-node usage. This allowed us to solve Spectral and Convex optimization problems, opening to the door to easy distribution of many machine learning algorithms. We conclude by providing a comprehensive set of benchmarks on accessing hardware-level optimizations for matrix computations from the JVM.

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`https://github.com/apache/spark/graphs/contributors`

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References

[1] Stephen R Becker, Emmanuel J Candès, and Michael C Grant. Templates for convex cone problems with applications to sparse signal recovery. *Mathematical Programming Computation*, 3(3):165–218, 2011.

[2] Austin R Benson, David F Gleich, and James Demmel. Direct qr factorizations for tall-and-skinny matrices in mapreduce architectures. In *Big Data, 2013 IEEE International Conference on*, pages 264–272. IEEE, 2013.

[3] L Susan Blackford, Jaeyoung Choi, Andy Cleary, Eduardo D'Azevedo, James Demmel, Inderjit Dhillon, Jack Dongarra, Sven Hammarling, Greg Henry, Antoine Petitet, et al. *ScaLAPACK users' guide*, volume 4. siam, 1997.

[4] Weizhu Chen, Zhenghao Wang, and Jingren Zhou. Large-scale l-bfgs using mapreduce. In *Advances in Neural Information Processing Systems*, pages 1332–1340, 2014.

[5] Jeffrey Dean and Sanjay Ghemawat. Mapreduce: simplified data processing on large clusters. *Communications of the ACM*, 51(1):107–113, 2008.

[6] Richard B Lehoucq, Danny C Sorensen, and Chao Yang. *ARPACK users' guide: solution of large-scale eigenvalue problems with implicitly restarted Arnoldi methods*, volume 6. Siam, 1998.

[7] Xiangrui Meng, Joseph Bradley, Burak Yavuz, Evan Sparks, Shivaram Venkataraman, Davies Liu, Jeremy Freeman, DB Tsai, Manish Amde, Sean Owen, Doris Xin, Reynold Xin, Michael J. Franklin, Reza Zadeh, Matei Zaharia, and Ameet Talwalkar. Mllib: Machine learning in apache spark. *arXiv preprint arXiv:1505.06807*, 2015.

[8] Brendan ODonoghue and Emmanuel Candes. Adaptive restart for accelerated gradient schemes. *Foundations of computational mathematics*, 15(3):715–732, 2013.

[9] Reza Bosagh Zadeh. Large linear model parallelism via a join and reducebykey. `https://issues.apache.org/jira/browse/SPARK-6567`, 2015. Accessed: 2015-08-09.

[10] Reza Bosagh Zadeh and Gunnar Carlsson. Dimension independent matrix square using mapreduce. In *Foundations of Computer Science (FOCS 2013) - Poster*, 2013.

[11] Reza Bosagh Zadeh and Ashish Goel. Dimension independent similarity computation. *The Journal of Machine Learning Research*, 14(1):1605–1626, 2013.

[12] Matei Zaharia, Mosharaf Chowdhury, Michael J Franklin, Scott Shenker, and Ion Stoica. Spark: cluster computing with working sets. In *Proceedings of the 2nd USENIX conference on Hot topics in cloud computing*, volume 10, page 10, 2010.

[13] Ciyou Zhu, Richard H Byrd, Peihuang Lu, and Jorge Nocedal. Algorithm 778: L-bfgs-b: Fortran subroutines for large-scale bound-constrained optimization. *ACM Transactions on Mathematical Software (TOMS)*, 23(4):550–560, 1997.

Appendix A - BLAS references

To find information about the implementations used, here we provide links to implementations used in Figure 2.

1. Netlib, Reference BLAS and CBLAS `http://www.netlib.org/blas/`
2. Netlib-java `https://github.com/fommil/netlib-java`
3. Breeze `https://github.com/scalanlp/breeze`
4. BIDMat `https://github.com/BIDData/BIDMat/`
5. OpenBLAS `https://github.com/xianyi/OpenBLAS`
6. CUDA `http://www.nvidia.com/object/cuda_home_new.html`
7. NVBLAS `http://docs.nvidia.com/cuda/nvblas`