Deep network classification by scattering and homotopy dictionary learning

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Overview

- public github repo
  - papers;
  - presentations
  - report pdf

- Lipschitz continuity condition :
  - determin if the transformation is stable to additive noise
  - there exists C $>$ 0 such that for all x and x' such that :

$$\|\Phi x' - \Phi x\| \leq C \|\mathrm{x}' - x\| \tag{1}$$

- Lipschitz continuous to deformations :
  - Lipschitz continuity relative to deformations is obtained if there exists C $>$ 0 such that for all τ and x:

$$\|\Phi x_\tau - \Phi x\| \leq C \|\mathrm{x}\| \sup_u |\nabla(\tau(u))| \tag{2}$$

- wavelet transform vs Fourier transform :
  - wavelets are stable to deformations (+) but they are translation covariant (-)
  - Fourier sinusoidal waves are not stable to deformations (-) but they are translation invariant (+)

- Notation
  - G : group of rotations **r** of angles $2k\pi/K$ for $0 \leq k \leq K$
  - Two-dimensional directional wavelets are obtained by rotating a single band-pass filter ψ by $r \in G$ and dilating it by 2^j for $j \in \mathbb{Z}$.

$$\psi_\lambda(u) = 2^{-2j}\psi(2^{-j}r^{-1}u) \textit{ with } \lambda = 2^{-j}r \tag{3}$$

- A wavelet transform commutes with translations, and is therefore not translation invariant.
  - Solution : introduce nonlinearity : Q

$$Qx = M(x * \psi_\lambda) \tag{4}$$

  - The operator M should commute with the action of any diffeomorphism