CS202 – Data Structures

LECTURE-08

Binary Trees and Search Trees

Tree properties, traversal, Binary Search Trees

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Topics

- Binary Trees
- Binary Tree Properties
- Local Search and Binary Search Trees (BSTs)

Binary Tree

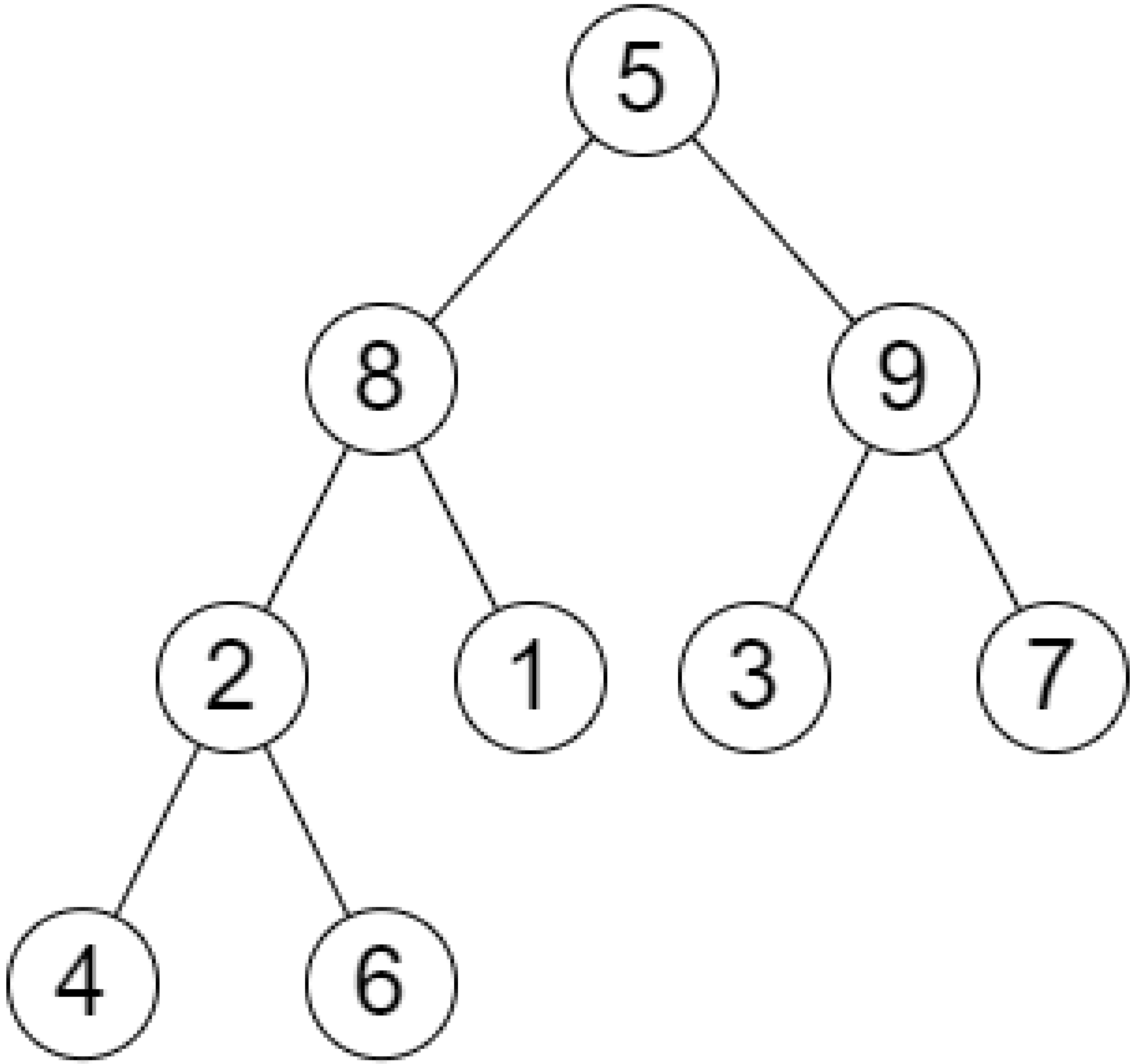
Binary Trees – Definitions Revisited

- A binary tree is an ordered tree in which every node has at most two children nodes
  - A left child precedes the right child in the ordering (by convention
  - A node is called an internal node if it has one or more children and external node if it has no children

- A binary tree is proper if each node has either zero or two children (it is called improper otherwise)

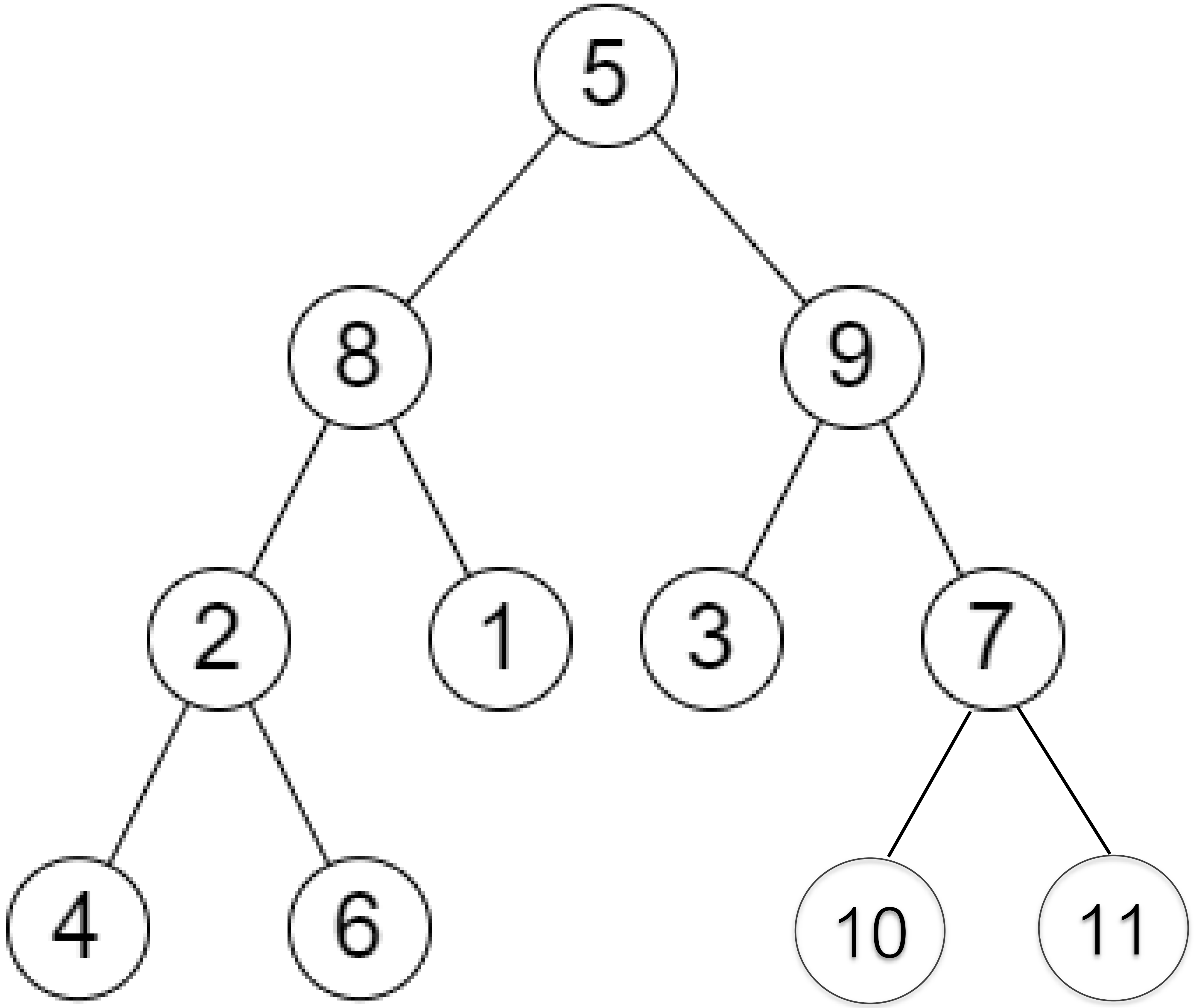
Binary Trees – More Definitions

- A complete binary tree is a binary tree in which all levels are fully filled except possibly the last level, which is filled from left to right.



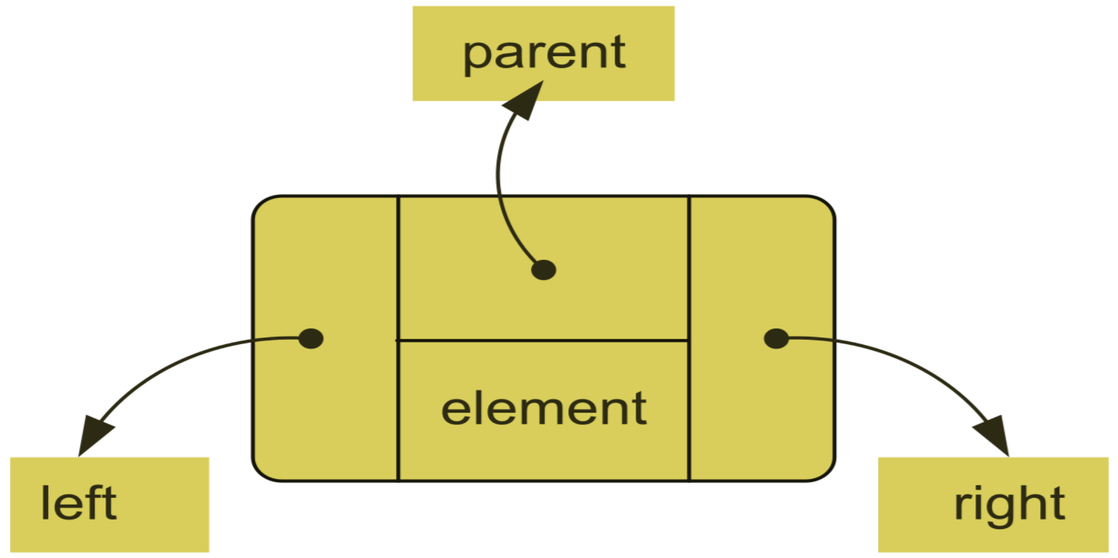
Binary Trees – More Definitions

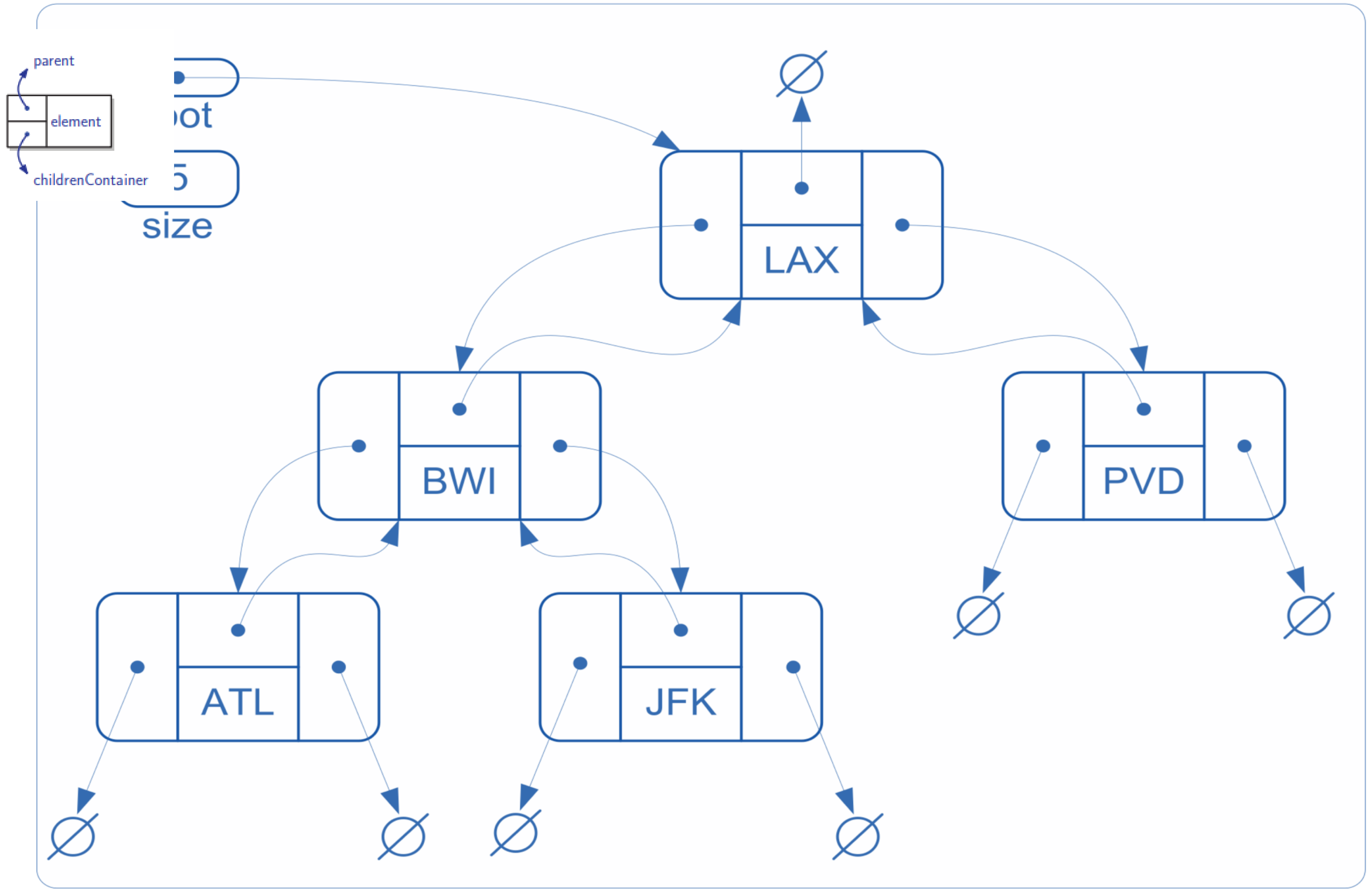
- A full binary tree is a binary tree in which every node has either 0 or 2 children.



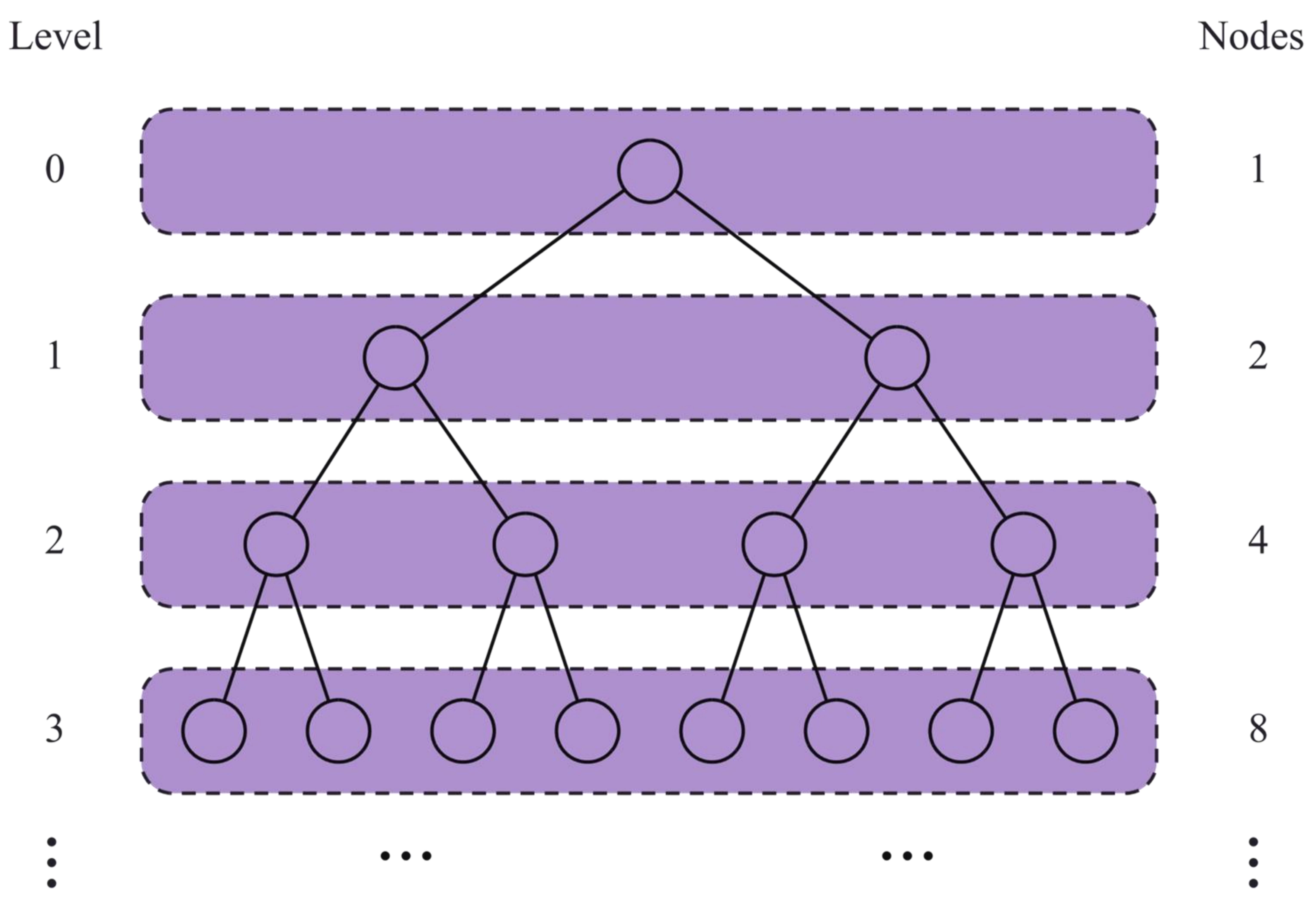
Representing Binary Trees

```
struct TreeNode {
    string element;
    TreeNode* parent;
    TreeNode* left;
    TreeNode* right;
}

class BinaryTree{
   public:
      BinaryTree();
      void insert(string data);
      TreeNode *search(int data);

   private:
      int size;
      TreeNode* root;
};
```





Height vs. Maximum Number of Nodes



More Properties

- Let T be a non-empty binary tree, and let n, n_E, n_I and h denote the number of nodes, number of external nodes, number of internal nodes, and height of T, respectively.
- Then T has the following properties:

1. $a \leq n \leq b$
2. $c \leq h \leq d$

What is a and b (in terms of h)?

What is c and d (in terms of n)?

$a \leq n \leq b$

If I give you a binary tree of height h then what is the minimum and maximum number of nodes can it have?

$c \leq h \leq d$

If I give you a binary tree with n nodes, then what is the minimum and maximum height can it assume?

More Properties

- Let T be a non-empty binary tree, and let n, n_E , n_I and h denote the number of nodes, number of external nodes, number of internal nodes, and height of T, respectively.
- Then T has the following properties:

1. $h + 1 \leq n \leq 2^{h+1} - 1$
2. $\log(n + 1) - 1 \leq h \leq n - 1$

More Properties

- Let T be a non-empty binary tree, and let n, n_E , n_I and h denote the number of nodes, number of external nodes, number of internal nodes, and height of T, respectively.
- Then T has the following properties:

$$A \le n_E \le B$$
$$C \le n_I \le D$$

More Properties

- Let T be a non-empty binary tree, and let n, n_E , n_I and h denote the number of nodes, number of external nodes, number of internal nodes, and height of T, respectively.
- Then T has the following properties:

3. $1 \leq n_E \leq 2^h$
4. $h \leq n_I \leq 2^h-1$

Tree Traversals – Summary

- Preorder Traversal → root – left – right
- Post order traversal → left – right – root
- In-order Traversal → left – root – right
- Level order traversals → level by level

Implementing Level Order Traversal

- Level order traversal is naturally not recursive!
- Use a queue, which initially only contains the root

```
Initially, the queue contains the root node
Repeat:
   Dequeue a node
   Visit it
   Enqueue its children nodes(left→right)
Until queue is empty
```

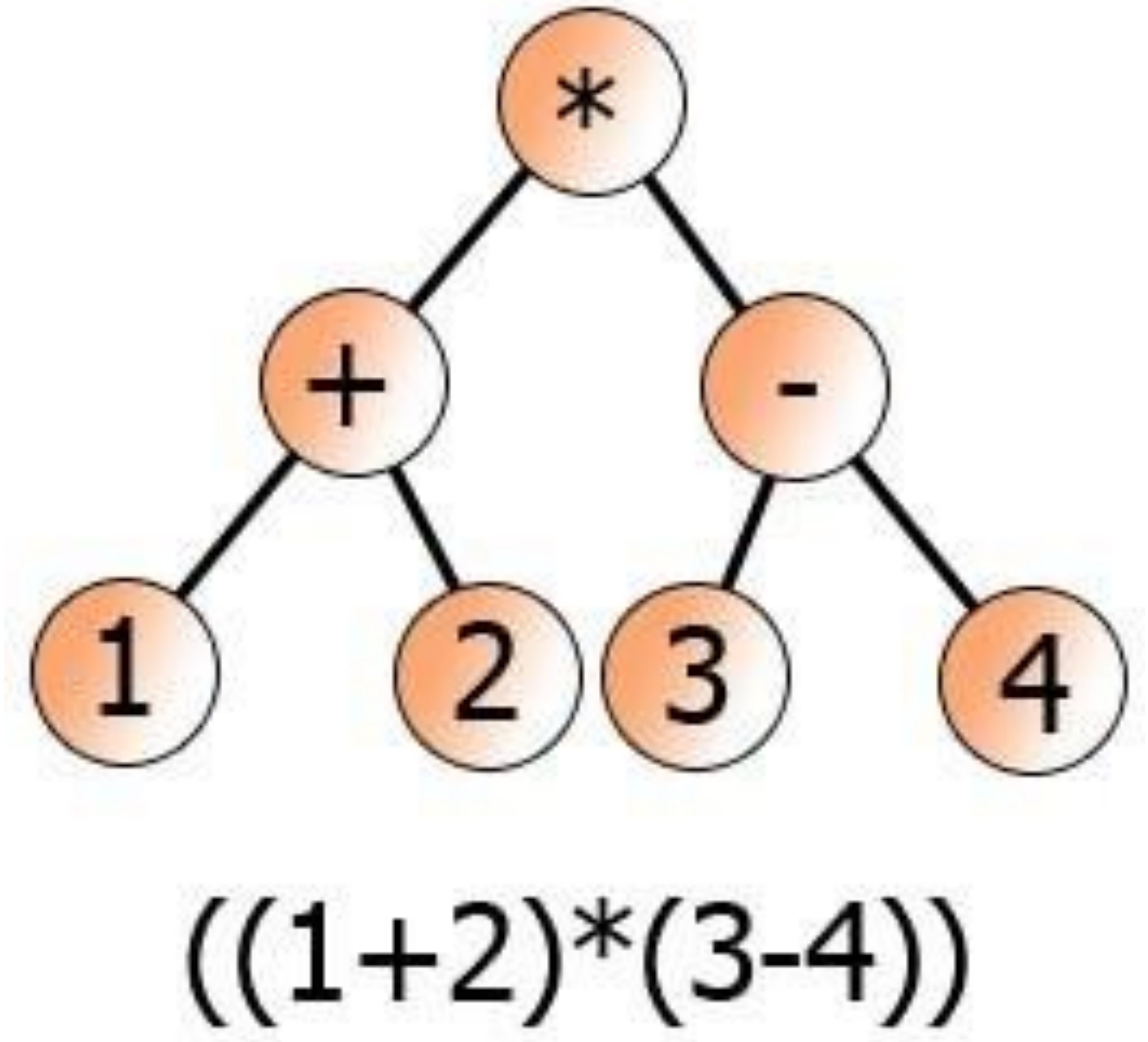
Applications of Tree Traversal

Customize and move the “do something,” and that’s the basis for dozens of algorithms and applications

```
void BinaryTree::traverse(TreeNode *node) {
  if (node != NULL) {
     traverse(node->left);
     // “do something”
     traverse(node->right);
  }
}
```

Evaluating Expression

- Inorder (infix): 1+2*3-4
- Preorder (prefix):*+12-34
- Postorder (postfix):12+34-*

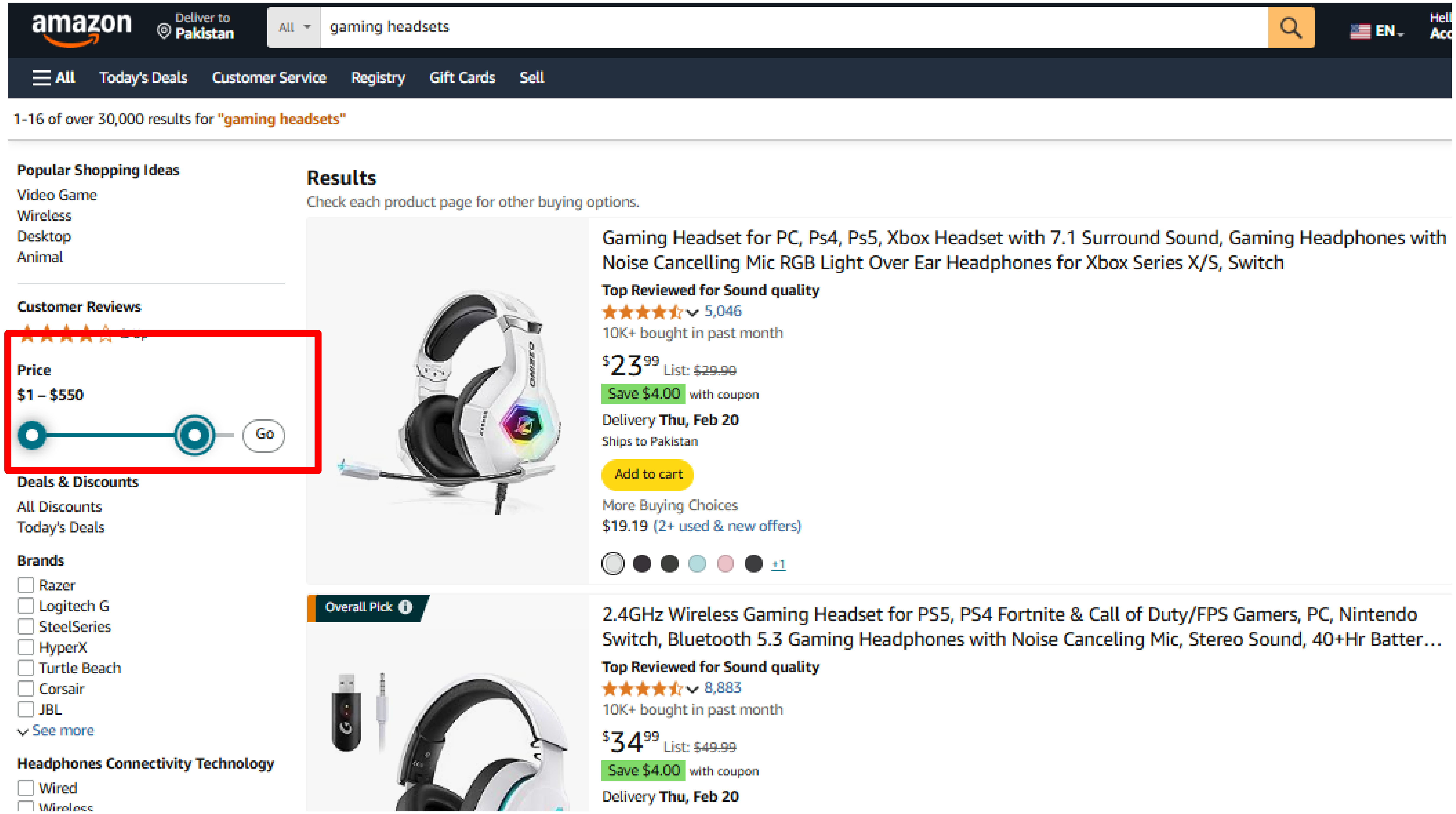


Natural way of writing expression

Local Search and Binary Trees

Price Ranges

- Retrieve headsets matching the specified price criteria



Local Search – Definition

- A local search data structure stores elements each with a key coming from an ordered set. It supports operations:
  - RangeSearch(x, y): Returns all elements with keys between x and y (including x and y).
  - NearestNeighbors(z): Returns the element with keys on either side of z.

Dynamic Updates: Insertions and Deletions

- We would also like to be able to modify the data structure as we go
  - Insert(x): Adds an element with key x
  - Delete(x): Removes the element with key x

Possible Design Choices so far

- How good are structures we have learnt so far for implementing local search?
  - Arrays
  - Sorted Arrays
  - Linked Lists

Array

- RangeSearch(1,15) O(n) X
- NearestNeighbors(15) O(n) X
- Insert(12) O(1) ✓
- Delete(11) O(n) X

7	10	4	13	1	6	15	

RangeSearch(5, 12)

NearestNeighbors(3)

2 – Sorted Array

- RangeSearch(1,15) O(log n) ✓
- NearestNeighbors(15) O(log n) ✓
- Insert(12) O(n) X
- Delete(11) O(n) X

1	4	6	7	10	13	15	

RangeSearch(5, 12)

NearestNeighbors(3)

3 – Linked List

- RangeSearch(1,15) O(n) X
- NearestNeighbors(15) O(n) X
- Insert(12) O(1) ✓
- Delete(11) O(n) X

RangeSearch(5, 12)

NearestNeighbors(3)

7 → 10 → 4 → 13 → 1 → 6 → 15

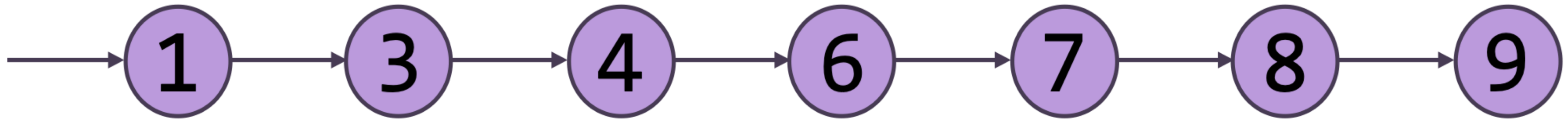
Need something new

- Sorted arrays can search efficiently but are inefficient with insertion/deletion
  - Therefore, none of the existing data structures work
- We need a new data structure for local search

	Array	Sorted Arrays	Linked Lists
RangeSearch:	O(n)	O(logn)	O(n)
NearestNeighbors:	O(n)	O(logn)	O(n)
Insert:	O(1)	O(n)	O(1)
Delete:	O(n)	O(n)	O(n)

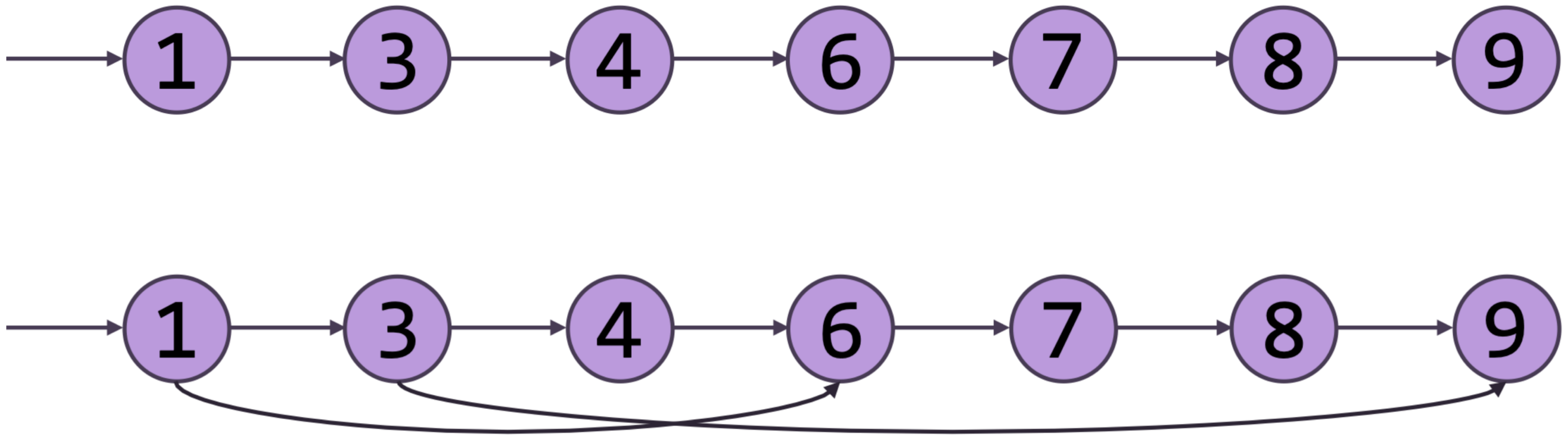
Let's consider Sorted Linked List

- Fundamental problem: slow search even though it is in order
- How can we speed up search?



Speeding Up Search in Linked Lists

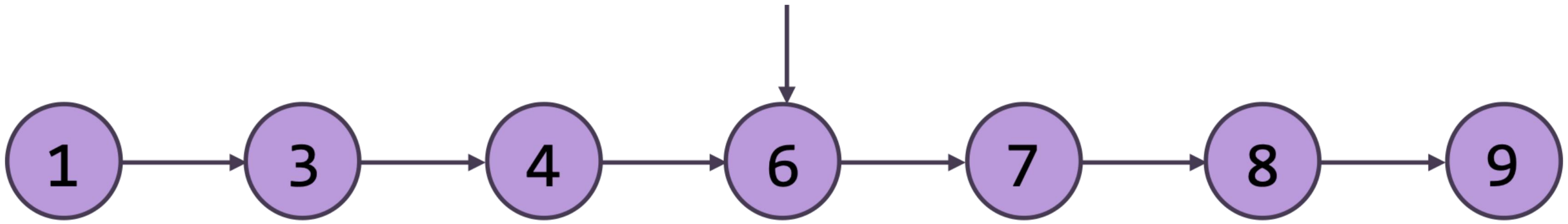
- Fundamental problem: slow search even though it is in order
- How can we speed up search?



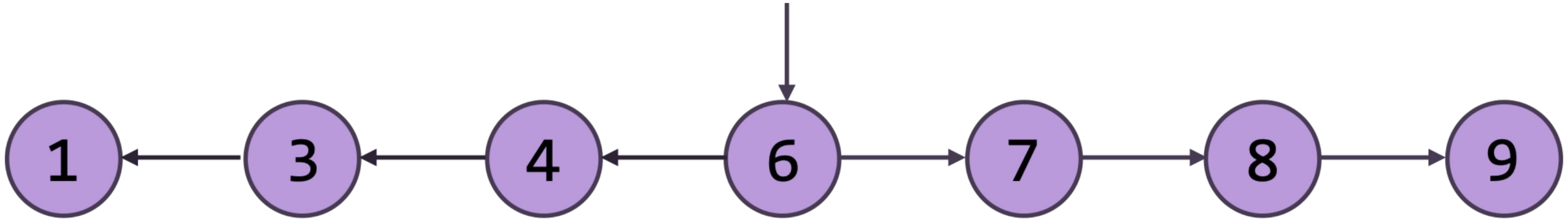
One idea: add (random) **express lanes** → Skip List

Optimizing Search in Sorted Linked Lists: Changing the Entry Point

- Move pointer to middle

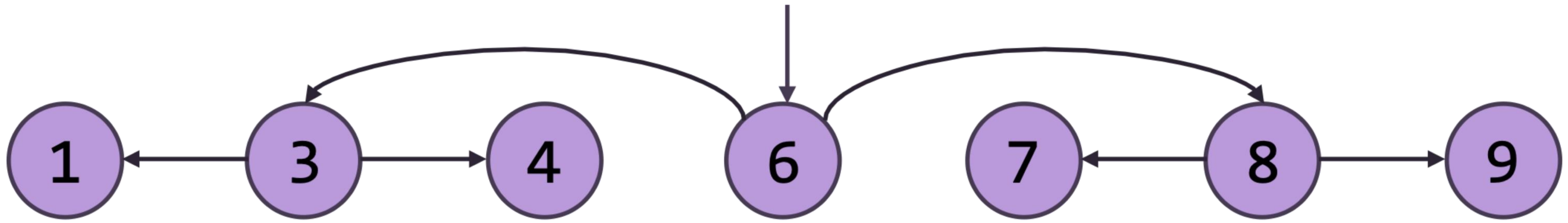


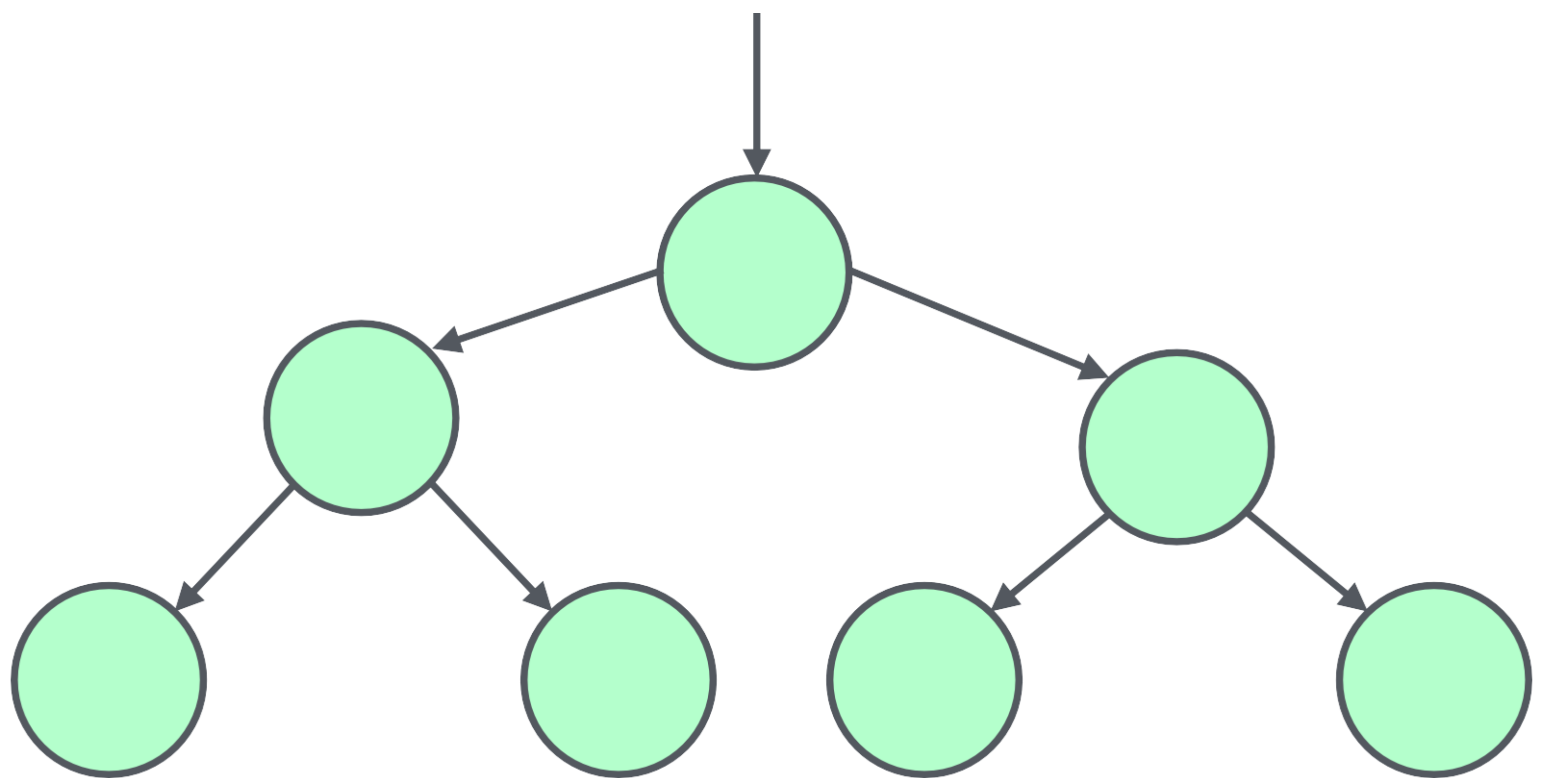
- Flip left links. Halves search time!



Can we do even Better?

- Change entry points, flip links, allow big jumps

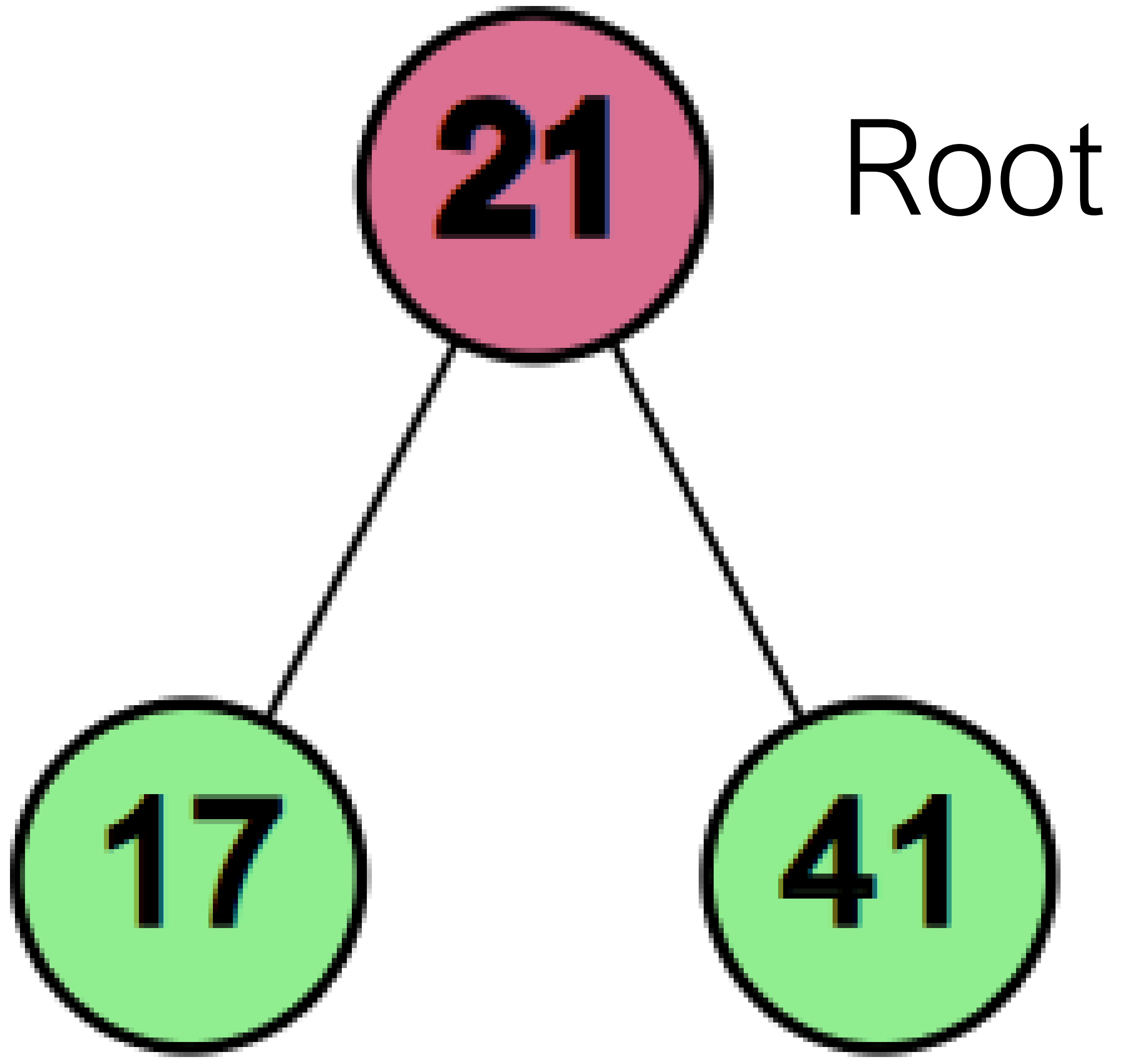




So, we can adapt our linked list to support array-style binary search

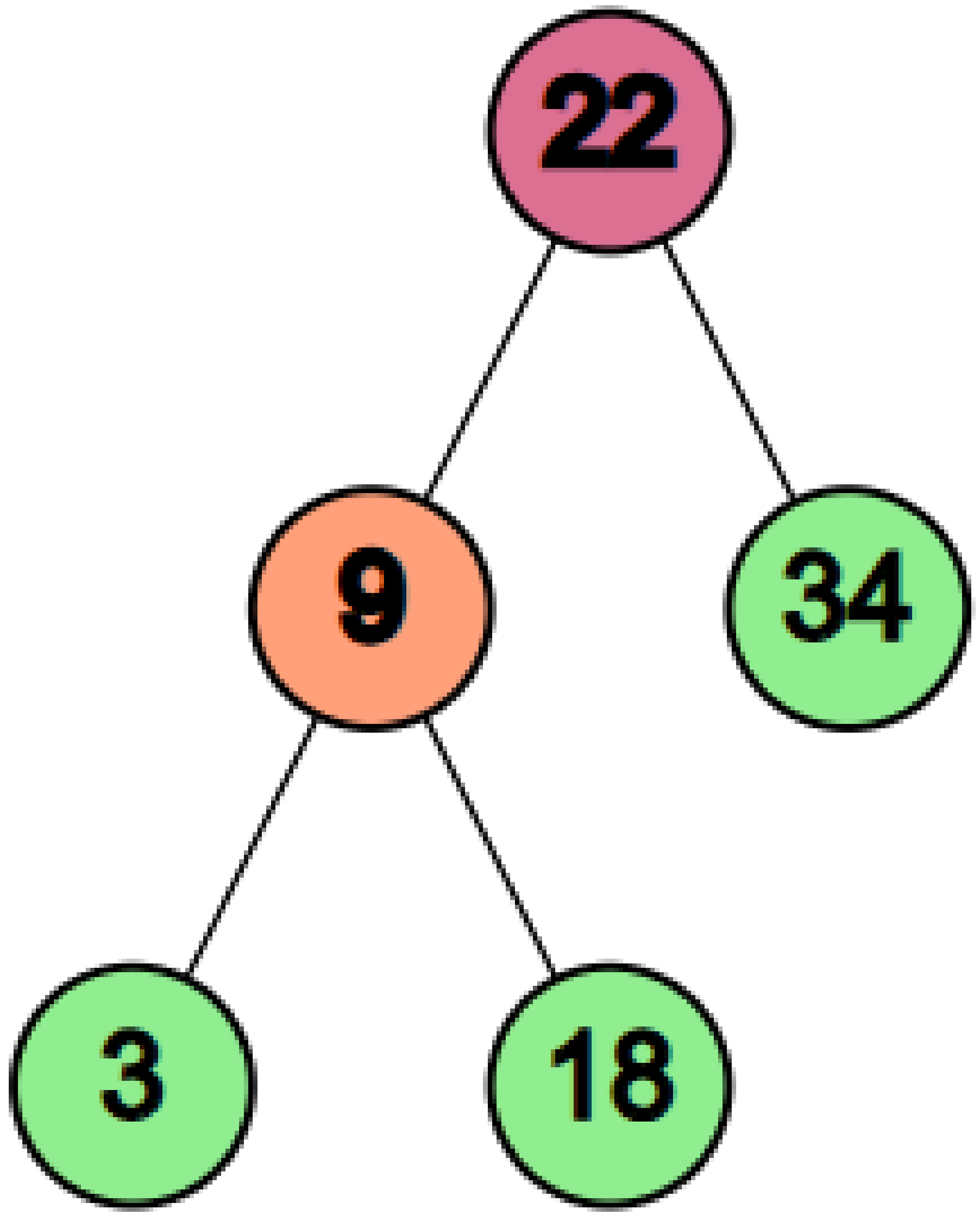
Binary Search Tree (BST)

- A binary search tree (BST) is a binary tree where (for all nodes):
  - Key of the left child is smaller than the parent's key
  - Key of the right child is greater than the parent's key
  - Each node stores a unique key



Can we have different BSTs for the same keys?

- Store keys: 22, 9, 34, 18, 3



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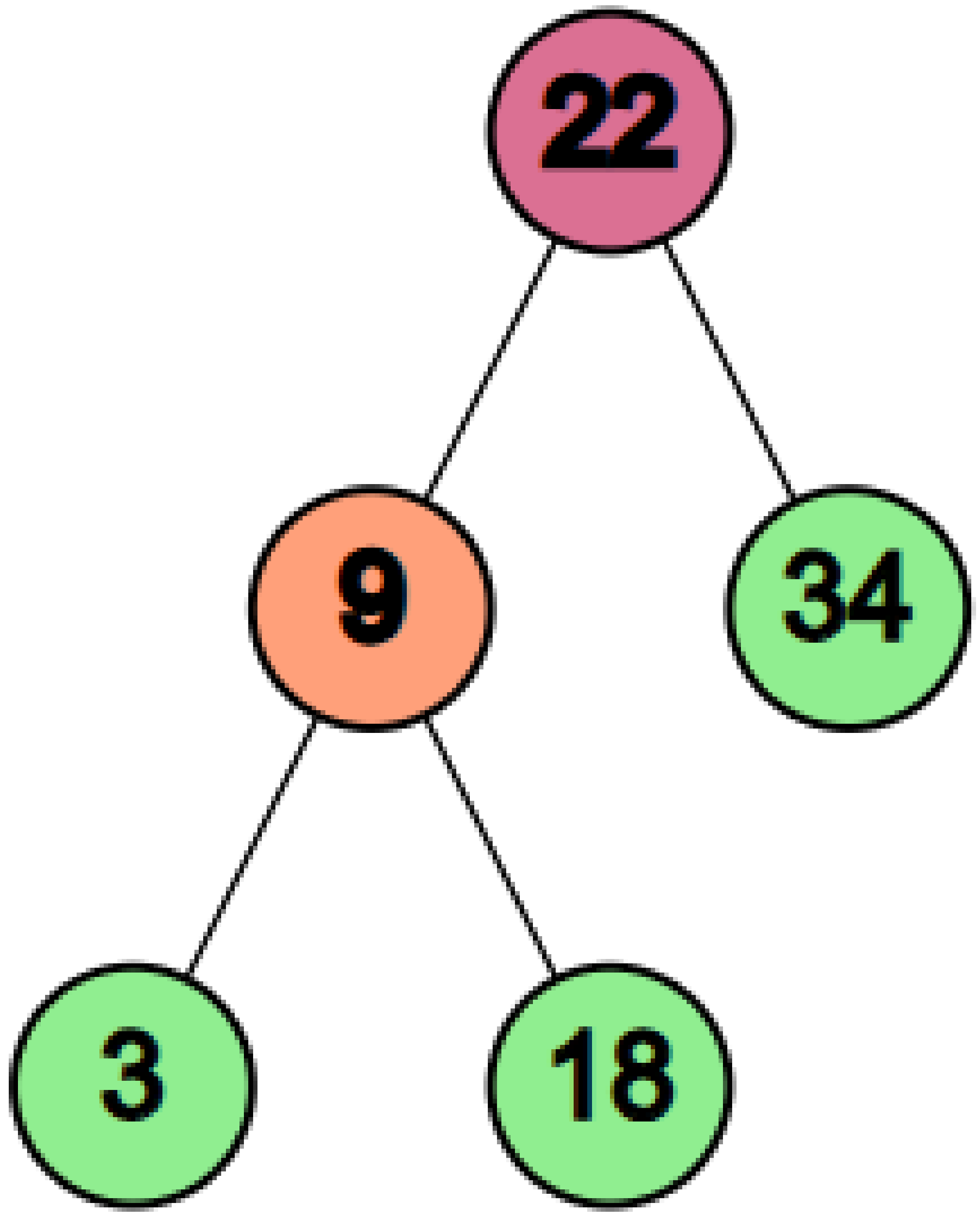
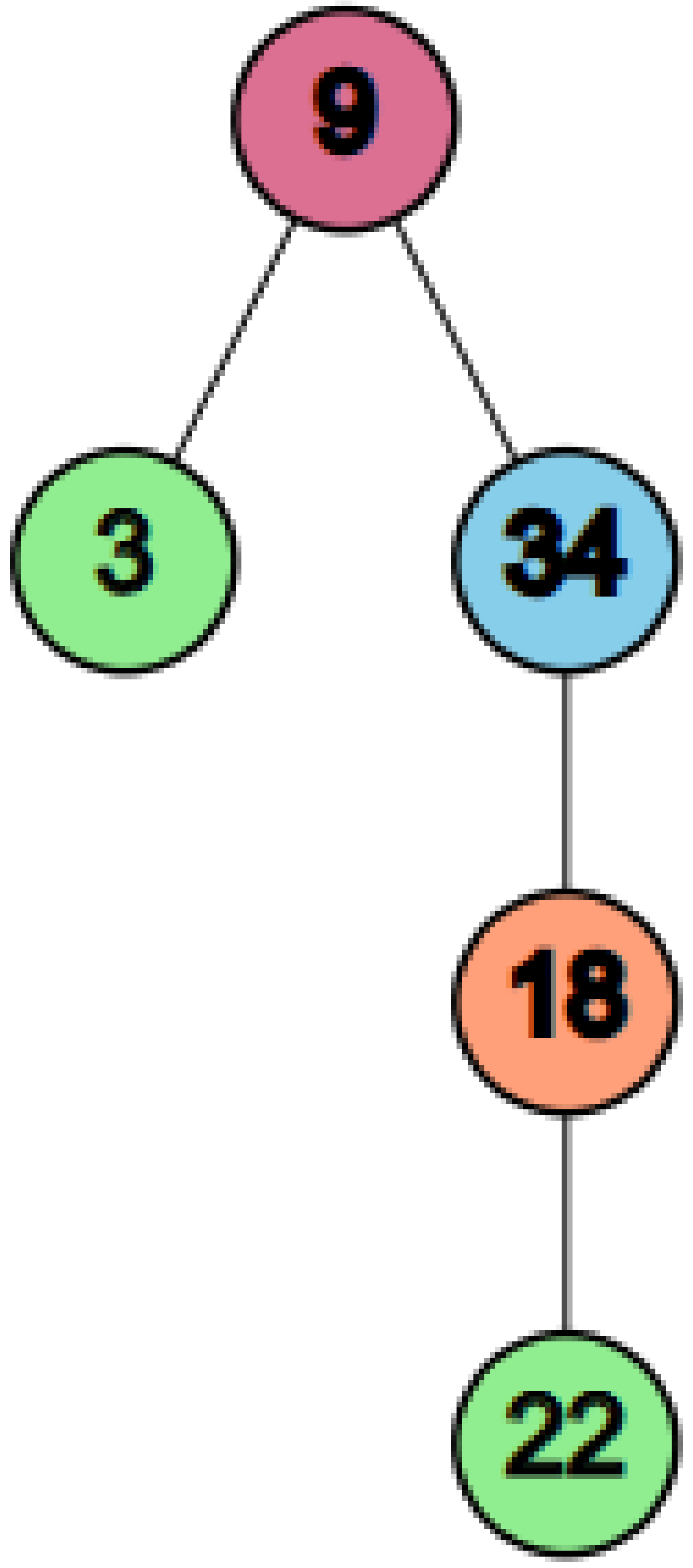
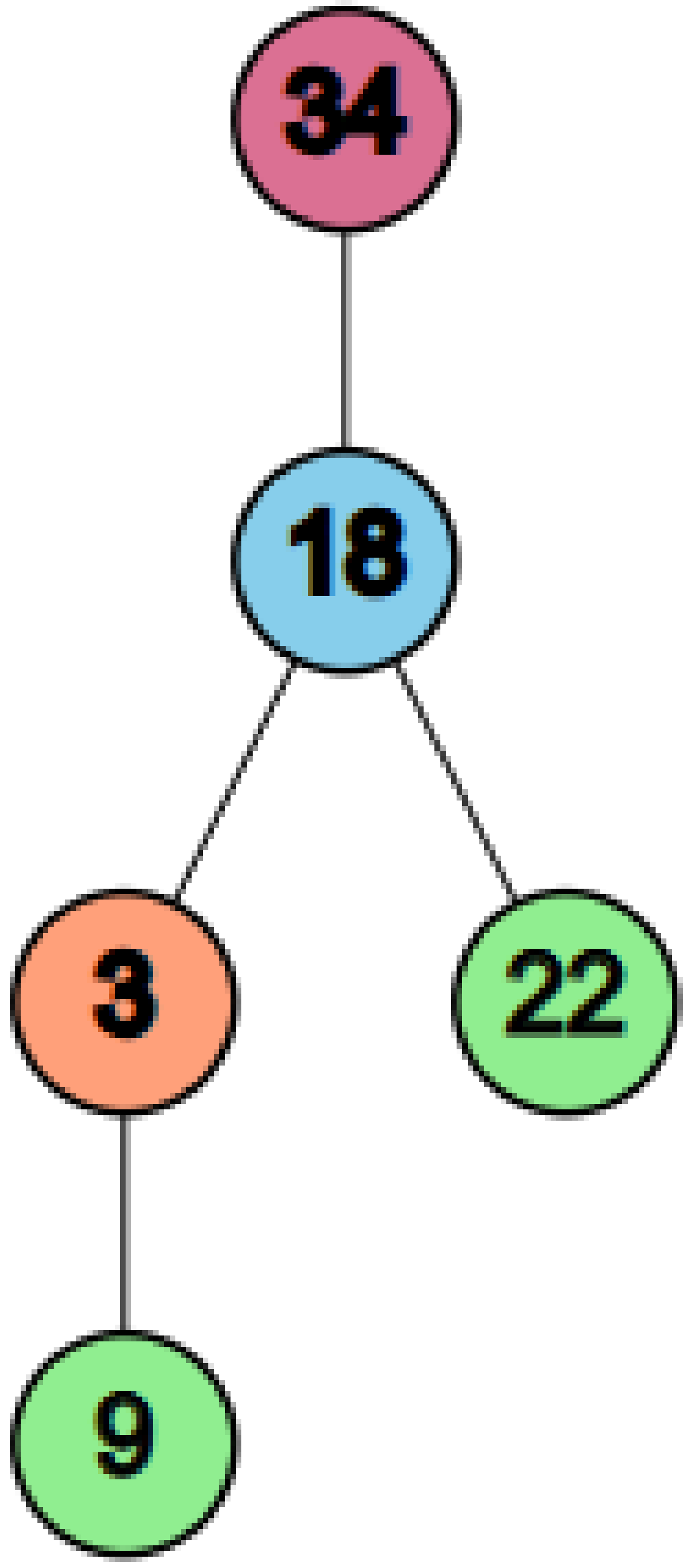


22, 9, 34, 18,3



9, 34, 18, 22,3



34, 18, 22,3,9

Exercise

- Write iterative solution of tree traversals.
  - Pre-order, post-order, in-order, level-order

- Hint: you can use stacks (one or more), queues etc.

Questions