HOMEWORK 3

Guidelines:

- Attempt all questions by yourself before discussing with peers, this is practice to strengthen your concepts.
- For coding questions, try writing clean, readable code on paper.
- Since this homework is ungraded, focus on learning rather than using LLMs to generate codes and get answers.
- If you get stuck, you are encouraged to:
  - Post your doubts on the course Slack channel.
  - Visit the TAs during office hours for guidance.

Section A: Multiple Choice Questions (MCQs)

Q1. Suppose you have an array H representing a max-heap of size n. What is the tightest upper bound on the number of leaves in the heap?

a) $\lfloor n/2 \rfloor + 1$
b) $\lceil n/2 \rceil$
c) $n - \lfloor n/2 \rfloor$
d) $\lceil n/2 \rceil + 1$
e) $\lfloor n/2 \rfloor$

Q2. What is the tightest worst-case time complexity to build a Max-Heap from an unsorted array of n elements?
a) O(logn)
b) O(nlogn)
c) O(n)
d) O(n2)

Q3**.** You have two valid Max-Heaps, H1 of size n and H2 of size m. What is the worst-case time complexity to **merge** the two heaps into a single Max-Heap of size n+m?
a) O(log(n+m))
b) O(n+m)
c) O((n+m)log(n+m))
d) O(min(n,m)log(n+m))

Q4. Consider a Max-Heap where you want to **increase the value** of a key at an arbitrary position i. Which heap maintenance operation is required after the increase?

a) Heapify-Down
b) Heapify-Up
c) Build-Heap
d) Level-Order Traversa

Q5. Given an array A = [10, 5, 8, 3, 2, 7] that is a valid min-heap, which of the following is a possible sequence of elements after deleting the minimum element twice?

a) `[7, 8, 10, 3, 2]`
b) `[7, 8, 10, 2]`
c) `[8, 7, 10, 3, 2]`
d) `[8, 7, 10]`
e) `[8, 10, 7]`

Section B: Short Questions

Q1. Starting with an empty array, insert the following numbers one by one to form a max-heap: [4, 1, 3, 2, 16, 9, 10, 14, 8, 7]. Show the final heap as a complete binary tree and as an array.

Q2. Given an array representing a binary tree, A = [10, 8, 9, 7, 5, 6, 3], determine whether it is a valid max-heap. If it is not, identify the first node (parent or child) that violates the max-heap property. Justify your answer by referencing the indices of the array.'

Q3. Design an algorithm(pseudo code) that merges k sorted lists into a single sorted list. Your solution should use a min-priority queue. Analyze the time and space complexity of your algorithm. Consider a scenario where the total number of elements across all lists is N and the number of lists is k.

Q4. Given a list of numbers [15, 30, 10, 25, 5, 20], demonstrate the step-by-step process of inserting each number into a min-priority queue and then show the order in which the elements would be extracted. You should draw the state of the priority queue (as a heap) after each insertion and deletion.

Q5.You have a stream of integers and need to find the **k-th smallest element** at any given time. Explain how you would use a priority queue to solve this problem and write the pseudo code for it. What type of priority queue (min or max) would you use, and what is the time complexity of the operation?

Q6. A priority queue is often implemented using a binary heap. While this is efficient, what is a potential drawback of using a standard binary heap to implement a priority queue if you frequently need to update the priority of an existing element? Describe a scenario where this limitation is significant and suggest an alternative or a modification to the standard heap structure that could improve performance for this specific operation.

Q7.Suppose we have a max heap represented as an array:

[90, 85, 80, 70, 60, 75, 65].

If we insert 95 into this heap, what will be the final heap structure after performing the necessary swaps?

Q8.A max heap is stored in an array H. If H[i] > H[j] for indices i < j, does this imply H[i] is the parent of H[j]? Why or why not?