

Pumping Lemma (For Regular Languages)

- >> Pumping Lemma is used to prove that a Language is NOT REGULAR
- >> It cannot be used to prove that a Language is Regular

If A is a Regular Language, then A has a Pumping Length ' P ' such that any string ' S ' where $|S| \geq P$ may be divided into 3 parts $S = x y z$ such that the following conditions must be true:

- (1) $x y^i z \in A$ for every $i \geq 0$
- (2) $|y| > 0$
- (3) $|xy| \leq P$

To prove that a language is not Regular using PUMPING LEMMA, follow the below steps:

(We prove using Contradiction)

- > Assume that A is Regular
- > It has to have a Pumping Length (say P)
- > All strings longer than P can be pumped $|S| \geq P$
- > Now find a string 'S' in A such that $|S| \geq P$
- > Divide S into x y z
- > Show that $x y^i z \notin A$ for some i
- > Then consider all ways that S can be divided into x y z
- > Show that none of these can satisfy all the 3 pumping conditions at the same time
- > S cannot be Pumped == CONTRADICTION

Using Pumping Lemma prove that the language $A = \{a^n b^n \mid n \geq 0\}$ is Not Regular

Proof:

Assume that A is Regular

Pumping length = P

$$S = a^P b^P$$

$$\Rightarrow S = aaaaaa bbb bbb$$

$\begin{array}{c} \text{-----} \\ \diagup \quad \diagdown \\ x \quad y \quad z \end{array}$

$$P = 7$$

$$p=7$$

Case 1: The γ is in the 'a' part

aaaaaaabbbbbb
x y z

Case 2: The γ is in the 'b' part

aaaaaabbbbbbb
x y z

Case 3: The γ is in the 'a' and 'b' part

aaaaaaabbbbbb
x y z

$$xy^iz \Rightarrow xy^2z \quad \times$$

aa aaaaaa a bbbbbb
11 \neq 7

$$xy^iz \Rightarrow xy^2z \quad \times$$

aaaaaa bb bbbb bbbb b
7 \neq 11

$$xy^iz \Rightarrow xy^2z \quad \times$$

aaaaa aabbaabb bbbb

$a^n b^n$

$$|xy| \leq p \quad p=7$$

Using Pumping Lemma prove that the language $A = \{yy \mid y \in \{0,1\}^*\}$ is Not Regular

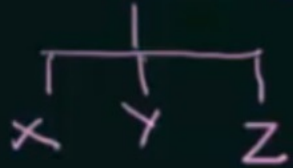
Proof:

0101

Assume that A is Regular

then it must have a Pumping Length = P

$$S = 0^P 1 0^P 1$$



$$P = 7$$

0000000100000001

Using Pumping Lemma prove that the language $A = \{yy \mid y \in \{0,1\}^*\}$ is Not Regular

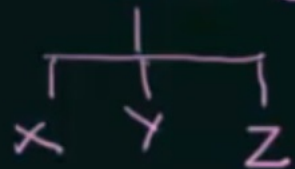
Proof:

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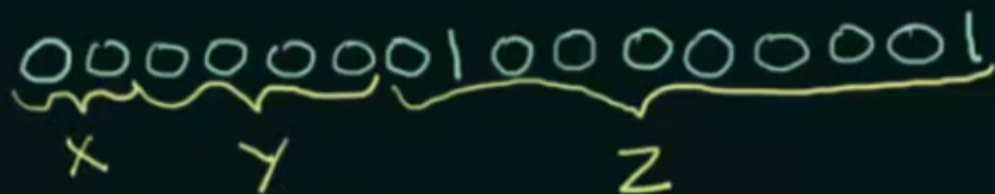
Assume that A is Regular

then it must have a Pumping Length = P

$$S = 0^P 1 0^P 1$$



$$P = 7$$



$$xy^iz \Rightarrow xy^2z$$

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$\notin A$

$$|y| > 0$$

$$|xy| \leq P = 7$$