Pumping Lemma (For Regular Languages)

>> Pumping Lemma is used to prove that a Language is NOT REGULAR

>> It cannot be used to prove that a Language is Regular

If A is a Regular Language, then A has a Pumping Length 'P' such that any string 'S' where $|S| \geq P$ may be divided into 3 parts $S = x\,y\,z$ such that the following conditions must be true:

(1) $x y^i z \in A$ for every $i \geq 0$

(2) $|y| > 0$

(3) $|xy| \leq P$

To prove that a language is not Regular using PUMPING LEMMA, follow the below steps:

(We prove using Contradiction)

-> Assume that A is Regular

-> It has to have a Pumping Length (say P)

-> All strings longer than P can be pumped $|S| \geqslant P$

-> Now find a string 'S' in A such that $|S| \geqslant P$

-> Divide S into x y z

-> Show that $x y^i z \notin A$ for some i

-> Then consider all ways that S can be divided into x y z

-> Show that none of these can satisfy all the 3 pumping conditions at the same time

-> S cannot be Pumped == CONTRADICTION

Using Pumping Lemma prove that the language $A = \{a^n b^n \mid n \geq 0\}$ is Not Regular

Proof:

Assume that A is Regular

Pumping length = P

$S = a^P b^P \Rightarrow S = aaaaaaabbbbbbb$

$x \quad y \quad z$

$P = 7$

$p = 7$

Case 1: The y is in the 'a' part

$\underbrace{aa}_{x}\underbrace{aaaa}_{y}\underbrace{a\,bbbbbbb}_{z}$

$xy^1z \Rightarrow xy^2z$ ✗

$aa\ aaaaaaaa\ a\ bbbbbbb$

$11 \neq 7$

Case 2: The y is in the 'b' part

$\underbrace{aaaaaaa\,bb}_{x}\underbrace{bbbb}_{y}\underbrace{b}_{z}$

$xy^1z \Rightarrow xy^2z$ ✗

$aaaaaaa\ bb\ bbbb\ bbbb\ b$

$7 \neq 11$

Case 3: The y is in the 'a' and 'b' part

$\underbrace{aaaaa}_{x}\underbrace{aabb}_{y}\underbrace{bbbbb}_{z}$

$xy^1z \Rightarrow xy^2z$ ✗

$a^n b^n$

$aaaaa\ aabbaabb\ bbbbb$

$|xy| \leq p$ $\quad p=7$

Using Pumping Lemma prove that the language $A = \{yy \mid y \in \{0,1\}^*\}$ is Not Regular

$\underbrace{01}\underbrace{01}$

Proof:

Assume that A is Regular

then it must have a Pumping Length = P

$S = 0^P 1 0^P 1$

x y z

$P = 7$

0000000100000001

Using Pumping Lemma prove that the language $A = \{yy \mid y \in \{0,1\}^*\}$ is Not Regular

Proof :

$\underbrace{01}\underbrace{01}$

Assume that A is Regular

Then it must have a Pumping Length = P

$$S = 0^P 1 0^P 1$$

$$\underbrace{x \quad y \quad z}$$

$p = 7$

$$\underbrace{0}_{x}\underbrace{000}_{y}\underbrace{000}\underbrace{01000000001}_{z}$$

$xy^iz \Rightarrow xy^2z$

$00000000000010000000 1$

$\notin A$

$|y| > 0$

$|xy| \leq p = 7$