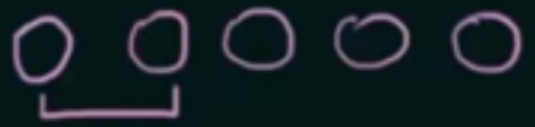
Minimization of DFA

Minimization of DFA is required to obtain the minimal version of any DFA which consist of the minimum number of states possible

DFA 5 states 4 states



Equivalent

Two states 'A' and 'B' are said to be equivalent if

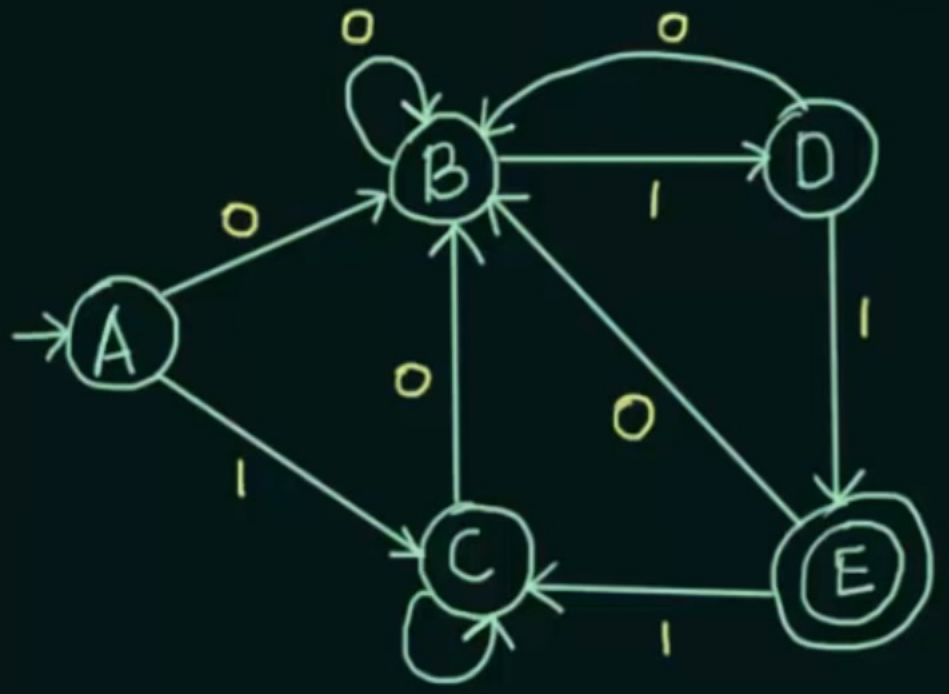
$\delta(A, X) \rightarrow F$ and $\delta(B, X) \rightarrow F$

OR

$\delta(A, X) \not\rightarrow F$ and $\delta(B, X) \not\rightarrow F$

where 'X' is any input String

Minimization of DFA - Examples (Part-1)



	0	1
→ A	B	C
B	B	D
C	B	C
D	B	E
(E)	B	C

0 Equivalence : {A, B, C, D} {E}

1 Equivalence : {A, B, C} {D} {E}

2 Equivalence : {A, C} {B} {D} {E}

A, B ✓

A, C ✓

C, D ✗

	0	1
→ A	B	C
B	B	D
C	B	C
D	B	E
(E)	B	C

0 Equivalence : {A, B, C, D} {E}

1 Equivalence : {A, B, C} {D} {E}

2 Equivalence : {A, C} {B} {D} {E}

3 Equivalance : {A, C} {B} {D} {E}

A, B ✓

A, C ✓

C, D ✗

0 Equivalence : {A, B, C, D} {E}

1 Equivalence : {A, B, C} {D} {E}

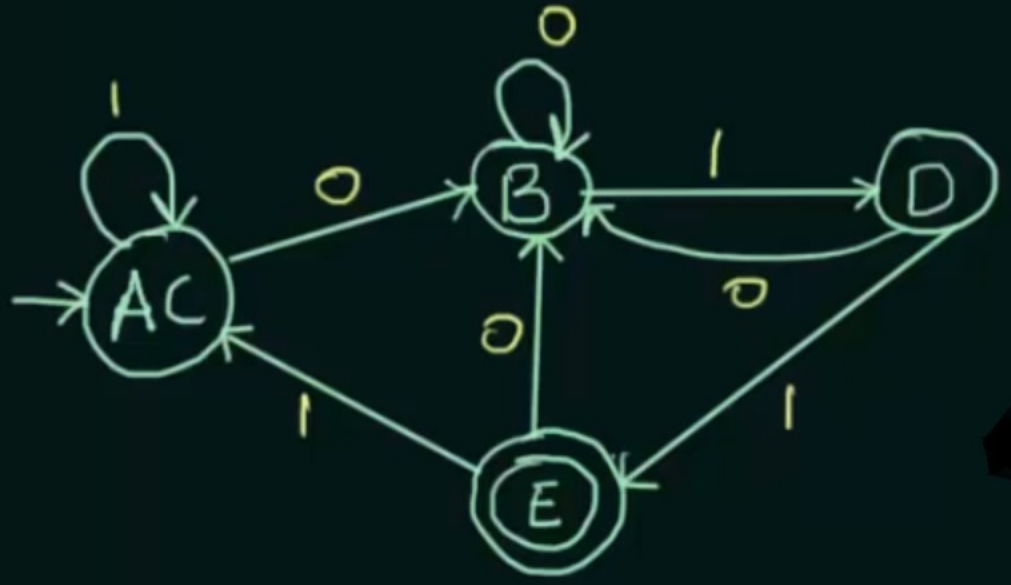
2 Equivalence : {A, C} {B} {D} {E}

3 Equivalance : {A, C} {B} {D} {E}

A, B ✓

A, C ✓

C, D ✗



	0	1
→ A	B	C
B	B	D
C	B	C
D	B	E
(E)	B	C

Minimization of DFA - Examples (Part-2)

Construct a minimum DFA equivalent to the DFA described by

	0	1
→ q_0	q_1	q_5
q_1	q_6	q_2
(q_2)	q_0	q_2
q_3	q_2	q_6
q_4	q_7	q_5
q_5	q_2	q_6
q_6	q_6	q_4
q_7	q_6	q_2

0 Equivalence

$\{q_0, q_1, q_3, q_4, q_5, q_6, q_7\}$ $\{q_2\}$

1- Equivalence

$\{q_0, q_4, q_6\}$

$\{q_1, q_7\}$

$\{q_3, q_5\}$ $\{q_2\}$

2- Equivalence

$\{q_0, q_4\}$ $\{q_6\}$ $\{q_1, q_7\}$ $\{q_3, q_5\}$ $\{q_2\}$

→ q_0	q_1	q_5
q_1	q_6	q_2
q_2	q_0	q_2
q_3	q_2	q_6
q_4	q_7	q_5
q_5	q_2	q_6
q_6	q_6	q_4
q_7	q_6	q_2

$\{q_0, q_1, q_3, q_4, q_5, q_6, q_7\}$ $\{q_2\}$

1- Equivalence

$\{q_0, q_4, q_6\}$

$\{q_1, q_7\}$

$\{q_3, q_5\}$ $\{q_2\}$

2- Equivalence

$\{q_0, q_4\}$ $\{q_6\}$ $\{q_1, q_7\}$ $\{q_3, q_5\}$ $\{q_2\}$

3- Equivalence

$\{q_0, q_4\}$ $\{q_6\}$ $\{q_1, q_7\}$ $\{q_3, q_5\}$ $\{q_2\}$

q_7	q_6	q_2

$\{q_0, q_4\}$ $\{q_6\}$ $\{q_1, q_7\}$ $\{q_3, q_5\}$ $\{q_2\}$

3. Equivalence

$\{q_0, q_4\}$ $\{q_6\}$ $\{q_1, q_7\}$ $\{q_3, q_5\}$ $\{q_2\}$

	0	1
→ $\{q_0, q_4\}$	$\{q_1, q_7\}$	$\{q_3, q_5\}$
$\{q_6\}$	$\{q_6\}$	$\{q_0, q_4\}$
$\{q_1, q_7\}$	$\{q_6\}$	$\{q_2\}$
$\{q_3, q_5\}$	$\{q_2\}$	$\{q_6\}$
($\{q_2\}$)	$\{q_0, q_4\}$	$\{q_2\}$

	0	1
→ q_0	q_1	q_5
q_1	q_6	q_2
(q_2)	q_0	q_2
q_3	q_2	q_6
q_4	q_7	q_5
q_5	q_2	q_6
q_6	q_6	q_4
q_7	q_6	q_2

Minimization of DFA - Examples (Part-3)

When there are more than one Final States involved

Minimize the following DFA:

	0	1
→A	B	C
B	A	D
C	E	F
D	E	F
E	E	F
F	F	F

0-Equivalence - {A, B, F} {C, D, E}

1-Equivalence - {A, B} {F} {C, D, E}

2-Equivalence - {A, B} {F} {C, D, E}

	0	1
→A	B	C
B	A	D
C	E	F
D	E	F
E	E	F
F	F	F

0-Equivalence - {A, B, F} {C, D, E}

1-Equivalence - {A, B} {F} {C, D, E}

2-Equivalence - {A, B} {F} {C, D, E}

	0	1
→{A, B}	{A, B}	{C, D, E}
{F}	{F}	{F}
{C, D, E}	{C, D, E}	{F}

Minimization of DFA - Examples (Part-4)

When there are Unreachable States involved

A state is said to be Unreachable if there is no way it can be reached from the Initial State

	0	1
→A	B	C
B	D	E
C	E	D
D	G	G
E	G	G
G	G	G

Minimization of DFA - Examples (Part-4)

When there are Unreachable States involved

A state is said to be Unreachable if there is no way it can be reached from the Initial State

	0	1
→A	B	C
B	D	E
C	E	D
D	G	G
E	G	G
G	G	G

0-Equivalence : {A, D, E} {B, C, G}

1-Equivalence : {A, D, E} {B, C} {G}

2-Equivalence : {A} {D, E} {B, C} {G}

3-Equivalence : {A} {D, E} {B, C} {G}

	0	1
→A	B	C
B	D	E
C	E	D
D	G	G
E	G	G
G	G	G

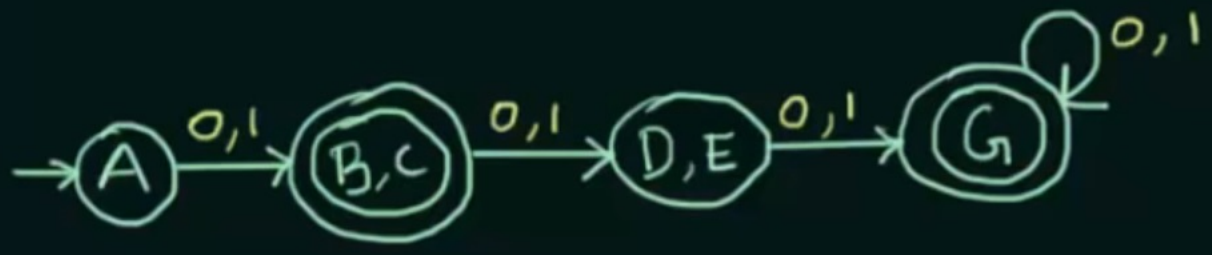
Initial State

0-Equivalence : {A, D, E} {B, C, G}

1-Equivalence : {A, D, E} {B, C} {G}

2-Equivalence : {A} {D, E} {B, C} {G}

3-Equivalence : {A} {D, E} {B, C} {G}



	0	1
→{A}	{B, C}	{B, C}
{D, E}	{G}	{G}
{B, C}	{D, E}	{D, E}
{G}	{G}	{G}