

(a) The parse tree for the string $4z6+x$ would be as follows:

```

S
|
SS
|
num
|
4
SS
|
id
|
z
SS*
|

```

```

SS
|
num
|
6
SS+
|

```

```

SS
|
id
|
x
SS*
|

```

(b) To eliminate left recursion in the grammar, we can rewrite the production rule $S \rightarrow SS^*$ as follows:

$S \rightarrow \text{num } S^* \mid \text{id } S^*$

and the production rule $S \rightarrow SS^+$ as follows:

$S \rightarrow \text{num } S^+ \mid \text{id } S^+$

This results in the following grammar:

$S \rightarrow \text{num } S^* \mid \text{id } S^* \mid \text{num } S^+ \mid \text{id } S^+ \mid \text{num} \mid \text{id}$

(c) To do left factorization of the grammar produced in b, we can rewrite the production rule $S \rightarrow \text{num } S^*$ as follows:

$S \rightarrow \text{num } T$
 $T \rightarrow S^* \mid \text{epsilon}$

and the production rule $S \rightarrow \text{id } S^*$ as follows:

$S \rightarrow \text{id } T$
 $T \rightarrow S^* \mid \text{epsilon}$

We can also rewrite the production rule $S \rightarrow \text{num } S^+$ as follows:

$S \rightarrow \text{num } U$
 $U \rightarrow S^+ \mid \text{epsilon}$

and the production rule $S \rightarrow \text{id } S^+$ as follows:

$S \rightarrow \text{id } U$
 $U \rightarrow S^+ \mid \text{epsilon}$

This results in the following grammar:

$S \rightarrow \text{num } T \mid \text{id } T \mid \text{num } U \mid \text{id } U \mid \text{num} \mid \text{id}$
 $T \rightarrow S^* \mid \text{epsilon}$
 $U \rightarrow S^+ \mid \text{epsilon}$

(d) Now, we can calculate the Null able, FIRST, and FOLLOW sets for each production in the grammar:

Null able:

$S \rightarrow \text{num } T \mid \text{id } T \mid \text{num } U \mid \text{id } U \mid \text{num} \mid \text{id}: \{\text{epsilon}\}$
 $T \rightarrow S^* \mid \text{epsilon}: \{\text{epsilon}\}$
 $U \rightarrow S^+ \mid \text{epsilon}: \{\}$

FIRST:

$S \rightarrow \text{num } T \mid \text{id } T \mid \text{num } U \mid \text{id } U \mid \text{num} \mid \text{id}: \{\text{num}, \text{id}\}$
 $T \rightarrow S^* \mid \text{epsilon}: \{*, \text{epsilon}\}$
 $U \rightarrow S^+ \mid \text{epsilon}: \{+, \text{epsilon}\}$

FOLLOW:

$S: \{\$, +, *, \text{num}, \text{id}\}$
 $T: \{+, *, \$\}$

U: {+, *, \$}

(e) Using the Null able, FIRST, and FOLLOW sets, we can construct the following parse table for the grammar:

	num	id	+	*	\$
S	S	S	S	S	S
T				T	T
U			U		U

(f) To parse the string $3x5+y$ using the parse table, we can follow these steps:

1. Begin at the starting symbol S and the first token 3.
2. Look up the entry in the parse table for S and the token 3. The entry is S, so we replace