Homework 6

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Use Simpson's Rule to approximate $\int_1^{1.5} x^2 lnx \ dx$ and find a bound for the error.

Answer: $\int_1^{1.5} x^2 lnx \ dx \approx \frac{1.5-1}{3\cdot 2}(f(1) + 4f(1.25) + f(1.5)) = 0.08333(0 + 4 \cdot 0.348661 + 0.912296) =$ $\boxed{0.19224530741309842}$

$E(h) = (0.5^5/90) \cdot f^{(4)}(\zeta)$ Because $f^{(4)}(x) = -2/x^2$. We have $|E(1.25)| \leq (0.5^5/90) \cdot f^{(4)}(1) = 0.0006944$ since it is increasing as x decreases.

Consider the following quadrature formula: $\int_1^{-1} f(x)dx = af(-1) + bf(1) + cf'(-1) + df'(1)$ Determine the constants a,b,c and d such that this formula has degree of precision 3.

Answer

degree	RHS	LHS
0	$\int_1^{-1} x^0 dx = x\|_1^{-1} = 2$	$a + b$
1	$\int_1^{-1} x^1 dx = 0.5x^2\|_1^{-1} = 0$	$b - a + c + d$
2	$\int_1^{-1} x^2 dx = 0.333x^3\|_1^{-1} = 0.666$	$a + b - 2c + 2d$
3	$\int_1^{-1} x^3 dx = 0.25x^4\|_1^{-1} = 0$	$b - a + 3c + 3d$

Gaussian elimination gives values as $\boxed{\text{a=1,b=1, c=1/3, d=-1/3}}$