Homework 5

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6. Use the most accurate three-point formula to determine each missing entry in the following tables.

a.

x	$f(x)$	$f'(x)$
−0.3	−0.27652	
−0.2	−0.25074	
−0.1	−0.16134	
0	0	

b.

x	$f(x)$	$f'(x)$
7.4	−68.3193	
7.6	−71.6982	
7.8	−75.1576	
8.0	−78.6974	

c.

x	$f(x)$	$f'(x)$
1.1	1.52918	
1.2	1.64024	
1.3	1.70470	
1.4	1.71277	

d.

x	$f(x)$	$f'(x)$
−2.7	0.054797	
−2.5	0.11342	
−2.3	0.65536	
−2.1	0.98472	

Answer:

a) For -0.3 and 0, we use the end-point formula (4.4) with $h \in \{-0.1, 0.1\}$ which gives
$f'(-0.3) \approx 1/0.2 \cdot [-3f(-0.3) + 4f(-0.2) - f(-0.1)] = (1/0.2)(-3(-0.27652) + 4(-0.25074) - (-0.16134)) = -0.06030000000000049$

$f'(0) \approx -1/0.2 \cdot [-3f(0) + 4f(-0.1) - f(-0.2)] = (-1/0.2)(-3(0) + 4(-0.16134) - (-0.25074)) = 1.9731$

For -0.2 and -0.1 we use the mid-point formula (4.5) with $h = 0.1$ which gives
$f(-0.2) \approx (1/0.2) \cdot [f(-0.1) - f(-0.3)] = (1/0.2) \cdot (-0.16134 - (-0.27652)) = 0.5759$
$f(-0.1) \approx (1/0.2) \cdot [f(0) - f(-0.2)] = (1/0.2) \cdot (0 - (-0.25074)) = 1.2537$

b) For 7.4 and 8.0, we use the end-point formula (4.4) with $h \in \{-0.2, 0.2\}$ which gives
$f'(7.4) \approx 1/0.4 \cdot [-3f(7.4) + 4f(7.6) - f(7.8)] = -16.693250$

$f'(8.0) \approx -1/0.4 \cdot [-3f(8.0) + 4f(7.8) - f(7.6)] = -17.899$

For 7.6 and 7.8 we use the mid-point formula (4.5) with $h = 0.2$ which gives
$f(7.6) \approx (1/0.4) \cdot [f(7.8) - f(7.4)] = -17.09575$
$f(7.8) \approx (1/0.4) \cdot [f(8.0) - f(7.6)] = -17.498$

c) For 1.1 and 1.4, we use the end-point formula (4.4) with $h \in \{-0.1, 0.1\}$ which gives
$f'(1.1) \approx 1/0.2 \cdot [-3f(1.1) + 4f(1.2) - f(1.3)] = 1.3435999999999992$
$f'(1.4) \approx -1/0.2 \cdot [-3f(1.4) + 4f(1.3) - f(1.2)] = -0.20125000000000393$

For 1.2 and 1.3 we use the mid-point formula (4.5) with $h = 0.1$ which gives
$f(1.2) \approx (1/0.2) \cdot [f(1.3) - f(1.1)] = 0.8776$
$f(1.3) \approx (1/0.2) \cdot [f(1.4) - f(1.2)] = 0.36265$

d) For -2.7 and -2.1, we use the end-point formula (4.4) with $h \in \{-0.2, 0.2\}$ which gives
$f'(-2.7) \approx 1/0.4 \cdot [-3f(-2.7) + 4f(-2.5) - f(-2.3)] = -0.9151775$
$f'(-2.1) \approx -1/0.4 \cdot [-3f(-2.1) + 4f(-2.3) - f(-2.5)] = 1.11534999$

For -2.5 and -2.3 we use the mid-point formula (4.5) with $h = 0.2$ which gives
$f(-2.5) \approx (1/0.4) \cdot [f(-2.3) - f(-2.7)] = 1.5014075$
$f(-2.3) \approx (1/0.4) \cdot [f(-2.1) - f(-2.5)] = 2.17825$

8. The data in Exercise 6 were taken from the following functions. Compute the actual errors in Exercise 6 and find error bounds using the error formulas.

a. $f(x) = e^{2x} - \cos 2x$ **b.** $f(x) = \ln(x+2) - (x+1)^2$
c. $f(x) = x \sin x + x^2 \cos x$ **d.** $f(x) = (\cos 3x)^2 - e^{2x}$

Answer:
a) $f'(x) = 2e^{2x} + 2sin(2x)$; $f''(x) = 4(cos(2x) + e^{2x})$; $f'''(x) = 8\mathrm{e}^{2x} - 8\sin(2x)$

- $E_a(-0.3) = |f'(-0.3) - f'_a(-0.3)| = 0.028638325397982257$
- $E_a(-0.2) = |f'(-0.2) - f'_a(-0.2)| = 0.014096592546022357$
- $E_a(-0.1) = |f'(-0.1) - f'_a(-0.1)| = 0.013577155434158827$
- $E_a(0) = |f'(0) - f'_a(0)| = 0.026899999999999924$
- Since ϵ is between -0.3 and -0.1, max$(|f'''(\epsilon)|$ occurs at $x = -0.1$ since its increasing at that period hence: Error Bound $= E_b(-0.3) \leq 0.1^2/3(|f'''(-0.1)|) \leq 0.029692109586374985$
- Since ϵ is between -0.3 and -0.1, max$(|f'''(\epsilon)|$ occurs at $x = -0.1$ since its increasing at that period hence: Error Bound $= E_b(-0.2) \leq 0.1^2/6(|f'''(-0.1)|) \leq 0.014846054793187492$

- Since ϵ is between -0.2 and 0.0, $\max(|f'''(\epsilon)|$ occurs at $x = 0.0$ since its increasing at that period hence: Error Bound $= E_b(-0.1) \leq 0.1^2/6(|f'''(0.0)|) \leq 0.014129845177923869$

- Since ϵ is between 0 and 0.2, $\max(|f'''(\epsilon)|$ occurs at $x = 0.2$ since its increasing at that period hence: Error Bound $= E_b(0) \leq 0.1^2/3(|f'''(0.2)|) \leq 0.028259690355847737$

b) $f'(x) = \dfrac{1}{x+2} - 2(x+1)$; $f''(x) = 4(cos(2x) + e^{2x})$; $f'''(x) - \dfrac{1}{(x+2)^2} - 2$

- $E_a(7.4) = |f'(7.4) - f'_a(7.4)| = 0.00036702127659893335$
- $E_a(7.6) = |f'(7.6) - f'_a(7.6)| = 8.3333333332547 \times 10^{-5}$
- $E_a(7.8) = |f'(7.8) - f'_a(7.8)| = 4.0816326531967206 \times 10^{-5}$
- $E_a(8) = |f'(8) - f'_a(8)| = 0.0009999999999976694$
- Since ϵ is between 7.4 and 7.8, $\max(|f'''(\epsilon)|$ occurs at $x = 7.8$ since its increasing at that period hence: Error Bound $= E_b(7.4) \leq 0.2^2/3(|f'''(7.8)|) \leq 3.210592386401215 \times 10^{-5}$

- Since ϵ is between 7.4 and 7.8, $\max(|f'''(\epsilon)|$ occurs at $x = 7.8$ since its increasing at that period hence: Error Bound $= E_b(7.6) \leq 0.2^2/6(|f'''(7.8)|) \leq 1.6052961932006087 \times 10^{-5}$

- Since ϵ is between 7.6 and 8.0, $\max(|f'''(\epsilon)|$ occurs at $x = 8.0$ since its increasing at that period hence: Error Bound $= E_b(7.8) \leq 0.2^2/6(|f'''(8.0)|) \leq 1.507040895061729 \times 10^{-5}$

- Since ϵ is between 8 and 8.4, $\max(|f'''(\epsilon)|$ occurs at $x = 8.4$ since its increasing at that period hence: Error Bound $= E_b(8) \leq 0.2^2/3(|f'''(8.4)|) \leq 3.014081790123458 \times 10^{-5}$

c) $f'(x) = -x^2 \sin(x) + \sin(x) + 3x\cos(x)$; $f''(x) = -5x\sin(x) - x^2\cos(x) + 4\cos(x)$; $f'''(x) = x^2\sin(x) - 9\sin(x) - 7x\cos(x)$

- $E_a(1.1) = |f'(1.1) - f'_a(1.1)| = 0.033886344908495625$
- $E_a(1.2) = |f'(1.2) - f'_a(1.2)| = 0.01679071829044554$
- $E_a(1.3) = |f'(1.3) - f'_a(1.3)| = 0.015740283698027435$
- $E_a(1.4) = |f'(1.4) - f'_a(1.4)| = 0.030919740607905255$
- Since ϵ is between 1.1 and 1.3, $\max(|f'''(\epsilon)|$ occurs at $x = 1.3$ since its increasing at that period hence: Error Bound $= E_b(1.1) \leq 0.1^2/3(|f'''(1.3)|) \leq 0.034783984899518436$

- Since ϵ is between 1.1 and 1.3, $\max(|f'''(\epsilon)|$ occurs at $x = 1.3$ since its increasing at that period hence: Error Bound $= E_b(1.2) \leq 0.1^2/6(|f'''(1.3)|) \leq 0.017391992449759214$

- Since ϵ is between 1.2 and 1.4, $\max(|f'''(\epsilon)|$ occurs at $x = 1.4$ since its increasing at that period hence: Error Bound $= E_b(1.3) \leq 0.1^2/6(|f'''(1.4)|) \leq 0.016816701045860485$

- Since ϵ is between 1.4 and 1.6, $\max(|f'''(\epsilon)|$ occurs at $x = 1.6$ since its increasing at that period hence: Error Bound $= E_b(1.4) \leq 0.1^2/3(|f'''(1.6)|) \leq 0.03363340209172097$

d) $f'(x) = -6\cos(3x)\sin(3x) - 2e^{2x}$;
$f''(x) = 18\sin^2(3x) - 18\cos^2(3x) - 4e^{2x}$;
$f'''(x) = 216\cos(3x)\sin(3x) - 8e^{2x}$

- $E_a(-2.7) = |f'(-2.7) - f'_a(-2.7)| = 0.5111216210806331$
- $E_a(-2.5) = |f'(-2.5) - f'_a(-2.5)| = 0.43598012647317974$
- $E_a(-2.3) = |f'(-2.3) - f'_a(-2.3)| = 0.6327333368430459$
- $E_a(-2.1) = |f'(-2.1) - f'_a(-2.1)| = 1.1862279780224598$
- Since ϵ is between -2.7 and -2.5, $\max(|f'''(\epsilon)|$ occurs at $x = -2.5$ since its increasing at that period hence: Error Bound $= E_b(-2.7) \leq 0.1^2/3(|f'''(-2.5)|) \leq 0.2342833010432044$

- Since ϵ is between -2.6 and -2.4, $\max(|f'''(\epsilon)|$ occurs at $x = -2.4$ since its increasing at that period hence: Error Bound $= E_b(-2.5) \leq 0.1^2/6(|f'''(-2.4)|) \leq 0.17392812973952365$

- Since ϵ is between -2.4 and -2.2, $\max(|f'''(\epsilon)|$ occurs at $x = -2.2$ since its increasing at that period hence: Error Bound $= E_b(-2.3) \leq 0.1^2/6(|f'''(-2.2)|) \leq 0.17392812973952365$

- Since ϵ is between -2.1 and -1.9, $\max(|f'''(\epsilon)|$ occurs at $x = -1.9$ since its increasing at that period hence: Error Bound $= E_b(-2.1) \leq 0.1^2/3(|f'''(-1.9)|) \leq 0.3399984899530679$