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Math 371
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Homework #10

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Q1
Factor the following matrix into LU decomposition.

$$A = \begin{bmatrix} 2 & -1 & 1 \\ 3 & 3 & 9 \\ 3 & 3 & 5 \end{bmatrix}$$

Answer:

$$\begin{bmatrix} 2 & -1 & 1 \\ 3 & 3 & 9 \\ 3 & 3 & 5 \end{bmatrix} \xrightarrow[R_3 - 3/2 R_1]{R_2 - 3/2 R_1} \begin{bmatrix} 2 & -1 & 1 \\ 0 & 9/2 & 15/2 \\ 0 & 9/2 & 7/2 \end{bmatrix} \xrightarrow{R_3 - (1) R_2} \begin{bmatrix} 2 & -1 & 1 \\ 0 & 9/2 & 15/2 \\ 0 & 0 & -4 \end{bmatrix}$$

$$\text{Hence } L = \begin{bmatrix} 1 & 0 & 0 \\ 3/2 & 1 & 0 \\ 3/2 & 1 & 1 \end{bmatrix} \text{ and } U = \begin{bmatrix} 2 & -1 & 1 \\ 0 & 9/2 & 15/2 \\ 0 & 0 & -4 \end{bmatrix}$$

Q2
Use the above decomposition to solve the system $Ax = b$, where

$$b = \begin{bmatrix} 7 \\ 1 \\ 2 \end{bmatrix}$$

Answer: $A = LU \implies LUx = b \implies L(Ux) = b$, let $Y = Ux$ then:

$$\begin{aligned} &Y_1 = 7 \\ &3/2\, Y_1 + Y_2 = 1 \implies Y_2 = -19/2 \\ &3/2\, Y_1 + Y_2 + Y_3 = 2 \implies Y_3 = 1 \end{aligned}$$

We substitute for Y to get $Ux = \begin{bmatrix} 7 \\ -19/2 \\ 1 \end{bmatrix}$ and solve for the linear system

$$\begin{aligned} &-4x_3 = 1 \implies x_3 = -1/4 = -0.25 \\ &9/2\, x_2 + 15/2\, x_3 = -19/2 \implies x_2 = -61/36 \approx -1.694 \\ &2x_1 - x_2 + x_3 = 7 \implies x_3 = 100/36 \approx 2.78 \end{aligned}$$