Homework 4: Cubic Spline Interpolation

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Q1
Answer

$$S_0'(x) = b + 2c(x-1) + 3d(x-1)^2$$
$$S_0''(x) = 2c + 6d(x-1)$$
$$S_1'(x) = 3(x-1)^2 + 2ex$$
$$S_1''(x) = 6(x-1) + 2e$$

$$S_0(1) = S_1(1) \implies a + \cancel{b(1-1)} + \cancel{c(1-1)^2} + \cancel{d(1-1)^3} = \cancel{(1-1)^3} + e - 1$$
$$a = e - 1 \tag{1}$$

$$S_0'(1) = S_1'(1) \implies b + 0 + 0 = 0 + 2e$$
$$b = 2e \tag{2}$$

$$S_0''(1) = S_1''(1) \implies 2c + 0 = 6(0) + 2e$$
$$c = e \tag{3}$$

Because we have a natural cubic spline, then $S_0''(0) = S_1''(2) = 0$

$$S_0''(0) = 2c - 6d = 0$$
$$2c = 6d \tag{4}$$
$$S_0''(2) = 6(1) + 2e = 0 \implies e = -3$$

Hence from (1) $a = -3 - 1 = -4$, and from (2) $b = 2 \cdot -3 = -6$, from (3) $c = -3$ and from (3) and (4) $-6 = 6 \cdot d \implies d = -1$.

Q2

$$S_0'(x) = 2x + 3x^3$$
$$S_0''(x) = 2x + 6x$$
$$S_0'''(x) = 6 \neq 12(?)$$
$$S_1'(x) = b + 2cx + 3dx^2$$
$$S_1''(x) = 2c + 6dx$$
$$S_1'''(x) = 6d = 12 \implies d = 2$$

$$S_0''(1) = S_1''(1) \implies 2c + 12 = 8 \implies c = -2$$
$$S_0'(1) = S_1'(1) \implies 2 + 3 = b + 2c + 3(2) \implies 2c + b = -1 \implies b = 3$$
$$S_0(1) = S_1(1) \implies 1^2 + 1^3 = a + 3(1) + -2(1)^2 + 2(1)^3 \implies a = -1$$