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Math 371
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Homework #7: The Elementary Theory of Initial-Value Problems **April 5, 2020**

Q1
Show that the following function satisfies Lipschitz condition

$$f(t,y) = 1/t^2(\sin(2t) - 2ty),\ 1 \leq t \leq 2$$

Answer: Since f is defined and continuous on the convex set $D = \{(t,y)|0 \leq t \leq 1,\ -\infty \leq y \leq \infty\}$. f satisfies a Lipschitz condition if there exists a constant L such that: $|\frac{\delta f}{\delta y}(t,y)| \leq L$. We have:

$$|\frac{\delta f}{\delta y}(t,y)| = |1/t^2(0-2t)| = |-2/t| \leq 2 = L$$

Therefore, f satisfies the Lipschitz condition.

Q2
Let $f(t,y) = 1 + ty^2$ and $D = \{(t,y)|0 \leq t \leq 2,\ -1 \leq y \leq 1\}$. Does f satisfy a Lipschitz condition on D? If so, find its Lipschitz constant.

Answer: Since f is defined and continuous on the convex set $D = \{(t,y)|0 \leq t \leq 1,\ -1 \leq y \leq 1\}$. f satisfies a Lipschitz condition if there exists a constant L such that: $|\frac{\delta f}{\delta y}(t,y)| \leq L$. We have:

$$|\frac{\delta f}{\delta y}(t,y)| = |2ty| \leq 4 = L;\ \text{since } t \leq 2, y \leq 1$$

Therefore, f satisfies the Lipschitz condition.