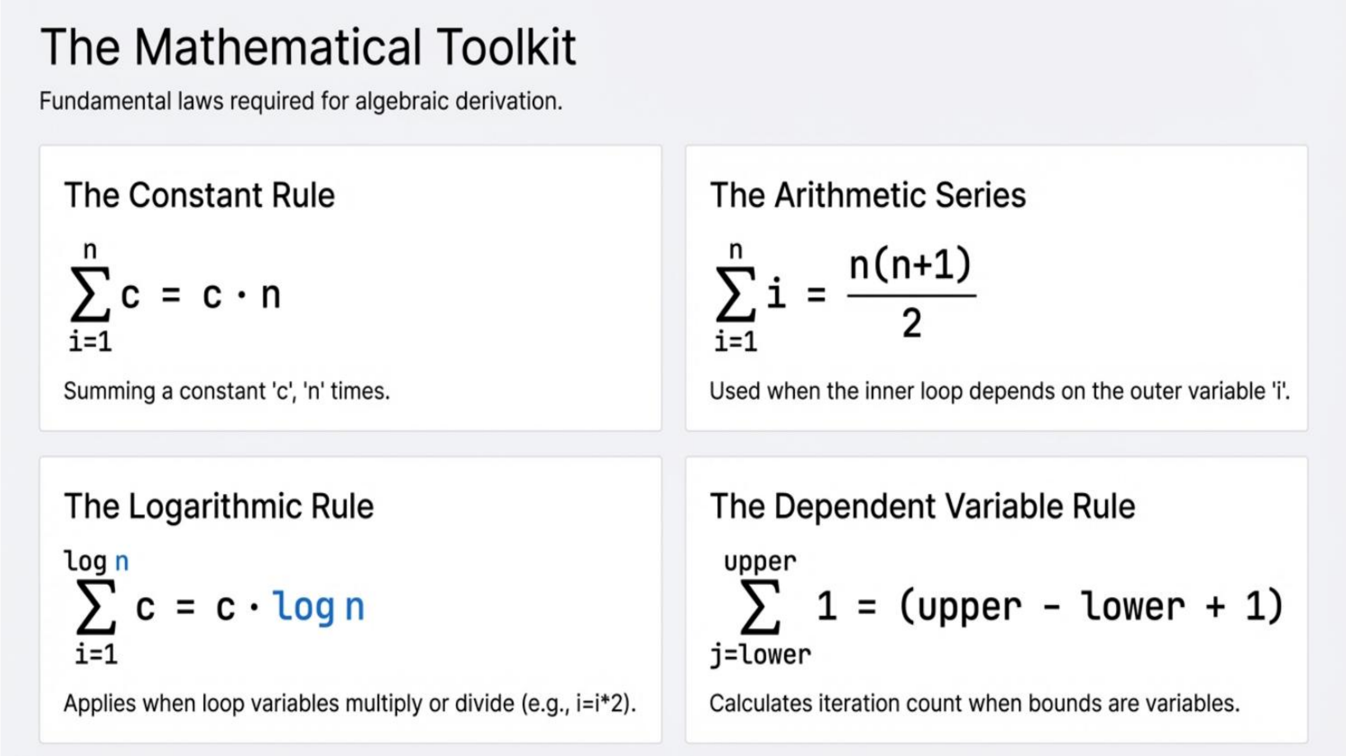
Lecture 1 – Lab1



The Mathematical Toolkit

Fundamental laws required for algebraic derivation.

The Constant Rule

$$\sum_{i=1}^{n} c = c \cdot n$$

Summing a constant 'c', 'n' times.

The Arithmetic Series

$$\sum_{i=1}^{n} i = \frac{n(n+1)}{2}$$

Used when the inner loop depends on the outer variable 'i'.

The Logarithmic Rule

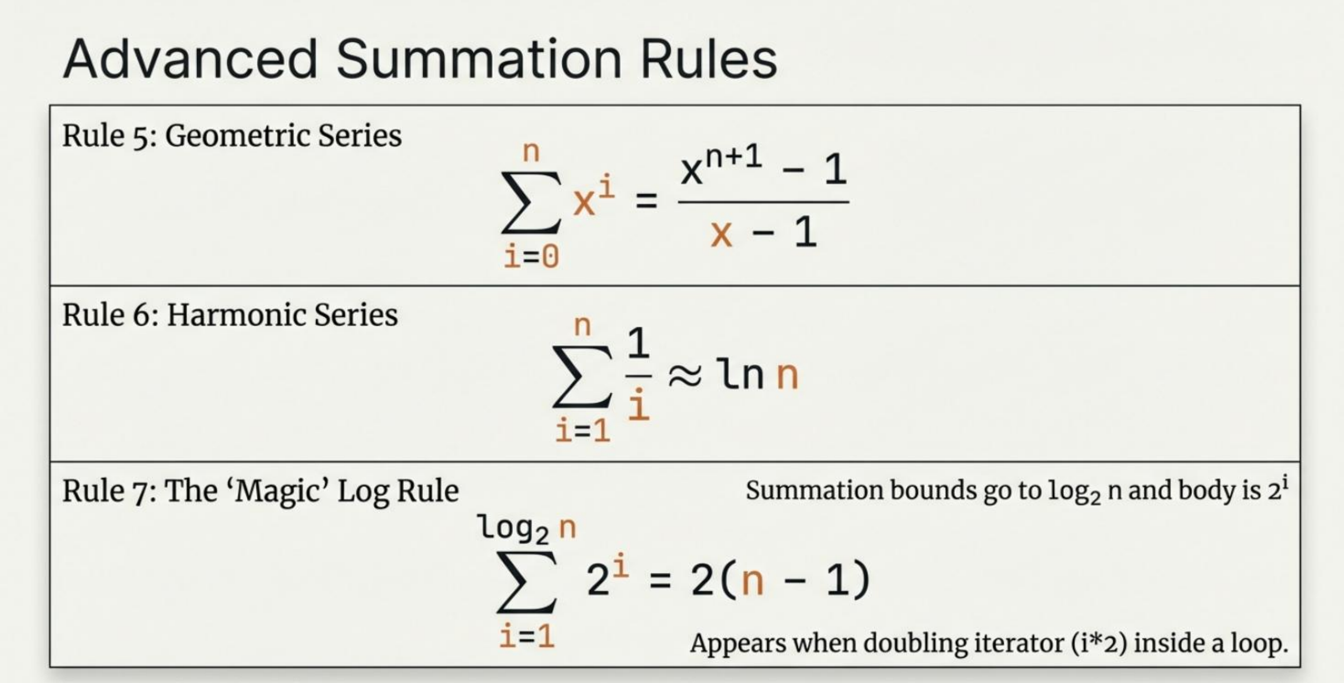
$$\sum_{i=1}^{\log n} c = c \cdot \log n$$

Applies when loop variables multiply or divide (e.g., i=i*2).

The Dependent Variable Rule

$$\sum_{j=lower}^{upper} 1 = (upper - lower + 1)$$

Calculates iteration count when bounds are variables.



Advanced Summation Rules

Rule 5: Geometric Series

$$\sum_{i=0}^{n} x^i = \frac{x^{n+1} - 1}{x - 1}$$

Rule 6: Harmonic Series

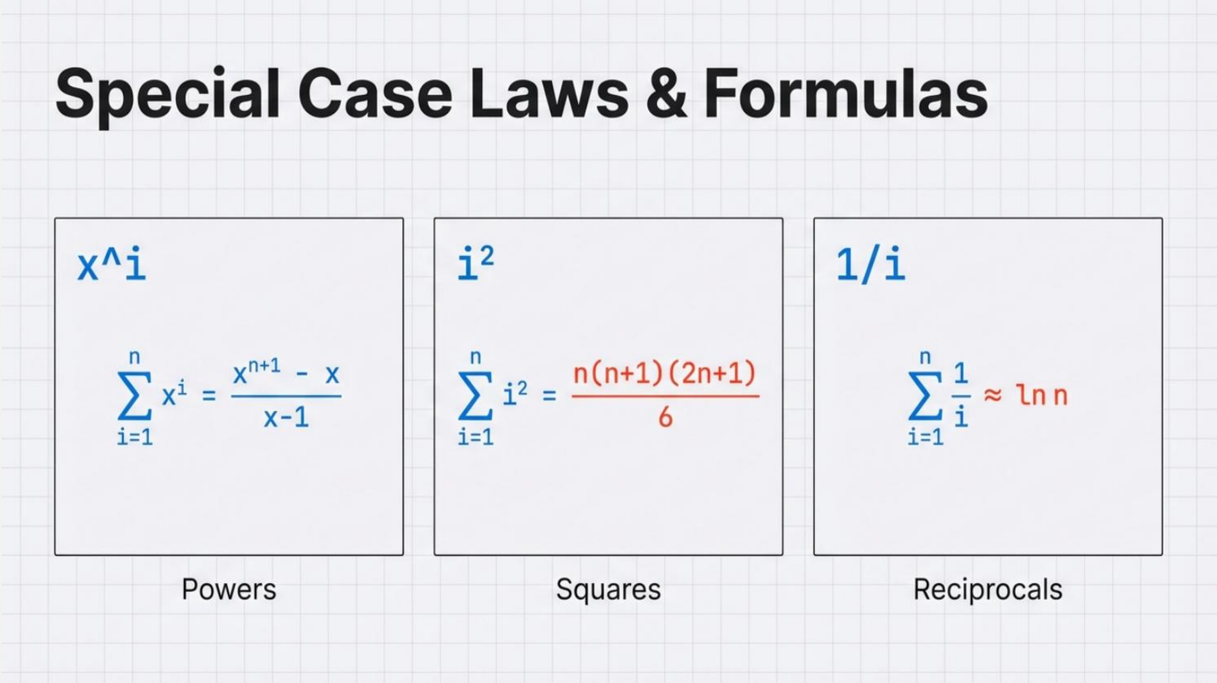
$$\sum_{i=1}^{n} \frac{1}{i} \approx \ln n$$

Rule 7: The 'Magic' Log Rule

Summation bounds go to $\log_2 n$ and body is 2^i

$$\sum_{i=1}^{\log_2 n} 2^i = 2(n - 1)$$

Appears when doubling iterator (i*2) inside a loop.



Special Case Laws & Formulas

x^i

$$\sum_{i=1}^{n} x^i = \frac{x^{n+1} - x}{x-1}$$

Powers

i²

$$\sum_{i=1}^{n} i^2 = \frac{n(n+1)(2n+1)}{6}$$

Squares

1/i

$$\sum_{i=1}^{n} \frac{1}{i} \approx \ln n$$

Reciprocals

Lab Practice

Find Time Equation T(n)

Using summation

```
int x = 1 + 3;
for(int i = 1; i ≤ n; i++)
    print i;
int y = x * 120;
print y;
```

```
for(int i = 1; i ≤ n; i *= 2)
    print i;
```

```
for(int i = n; i ≥ 1; i--)
    print i;
```

```
for(int i = n; i ≥ 1; i/= 2)
    print i;
```

```
$for(int\ i = 1; i \leq n; i++)$
    $for(int\ j = 1; j \leq n; j++)$
        $print\ "hello";$
```

```
$for(int\ i = 1; i \leq n; i *= 3)$
    $for(int\ j = 1; j \leq n; j++)$
        $print\ "hello";$
```

```
$for(int\ i = 1; i \leq n; i++)$
    $for(int\ j = 1; j \leq n; j++)$
        $print\ "hello";$
$for(int\ i = 1; i \leq n; i *= 3)$
    $print\ "hello";$
```

```
$for(int\ i = 1; i \leq n; i *= 3)$
    $for(int\ j = 1; j \leq n; j *= 3)$
        $print\ "hello";$
```

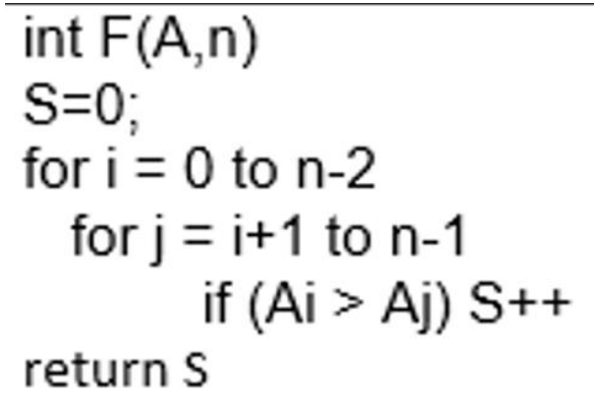
```
for(int i = 1; i ≤ n; i++)
    for(int j = 1; j ≤ i; j++)
        print "hello";
```

```
for(int i = 1; i ≤ n; i *= 2)
    print i;
```

```
for(int i = 1; i ≤ n; i++)
    print i;
```

```
for(int i = n; i ≥ 1; i /= 2)
    print i;
```

```
for(int i = n − 1; i ≥ 1; i−−)
    print i;
```



```
int F(A,n)
S=0;
for i = 0 to n-2
   for j = i+1 to n-1
         if (Ai > Aj) S++
return S
```

```
$for(int\ i = 1; i \leq n; i + +)$
    $for(int\ j = 1; j \leq n; j + +)$
        $print\ "hello";$
```

```
$for(int\ i = 1; i \leq n; i *= 2)$
    $for(int\ j = 1; j \leq i; j *= 2)$
        $print\ "hello";$
```

```
$for(int\ i = 1; i \leq n; i + +)$
    $for(int\ j = 1; j \leq i; j + +)$
        $print\ "hello";$
```

```
$for(int\ i = 1; i \leq n; i *= 2)$
    $for(int\ j = 1; j \leq i; j + +)$
        $print\ "hello";$
```

```
function mystery($n$)
    $r := 0$
    for $i := 1$ to $n - 1$ do
      for $j := i + 1$ to $n$ do
          for $k := 1$ to $j$ do
            $r := r + 1$
    return($r$)
```

```
function prestiferous($n$)
    $r := 0$
    for $i := 1$ to $n$ do
      for $j := 1$ to $i$ do
        for $k := j$ to $i + j$ do
          for $l := 1$ to $i + j - k$ do
            $r := r + 1$
    return($r$)
```

```
function pesky($n$)
    $r := 0$
    for $i := 1$ to $n$ do
      for $j := 1$ to $i$ do
        for $k := j$ to $i + j$ do
          $r := r + 1$
    return($r$)
```

```
function conundrum($n$)
    $r := 0$
    for $i := 1$ to $n$ do
      for $j := i + 1$ to $n$ do
        for $k := i + j - 1$ to $n$ do
          $r := r + 1$
    return($r$)
```