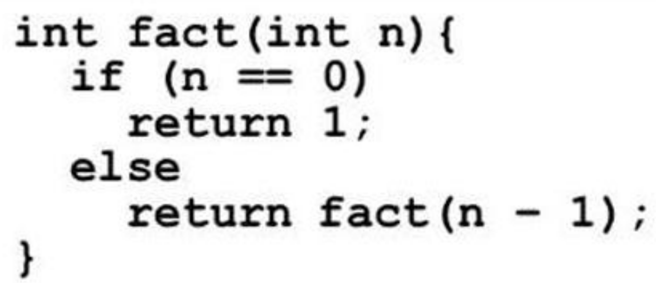
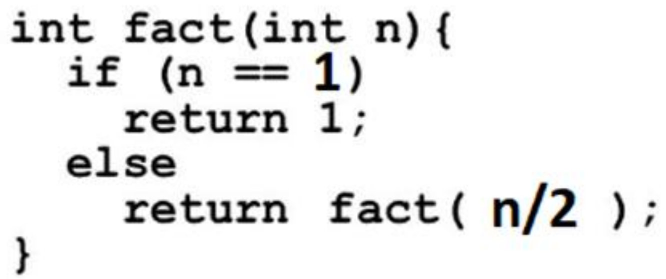
Lecture 3 – Lab 3

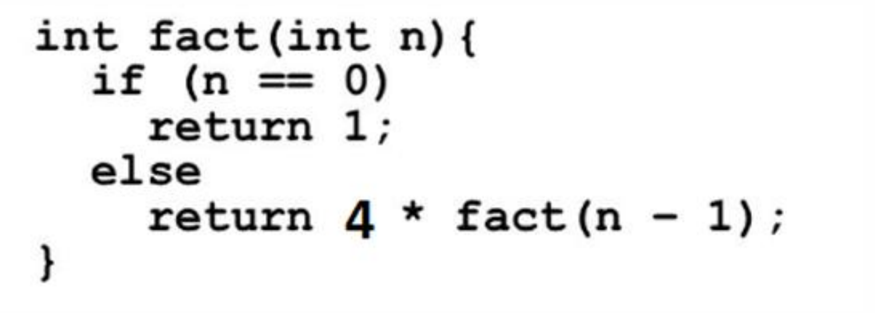
Find $T(n)$



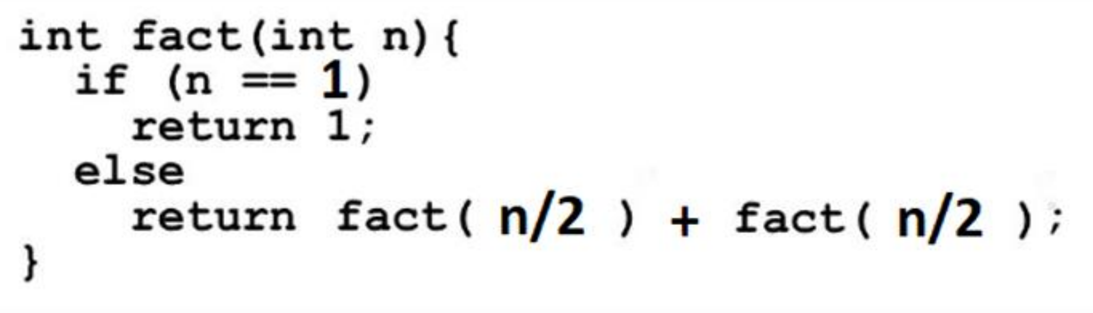
```
int fact(int n){
  if (n == 0)
    return 1;
  else
    return fact(n - 1);
}
```



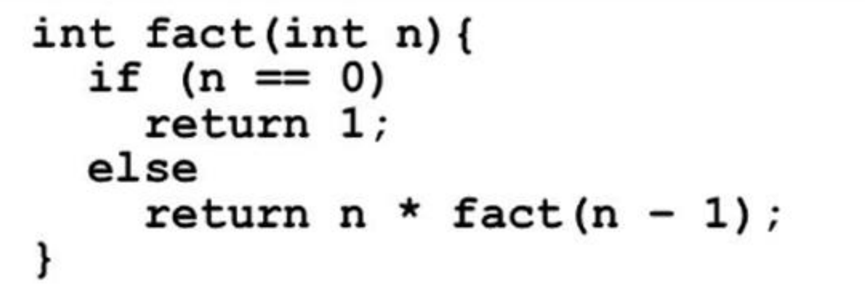
```
int fact(int n){
  if (n == 1)
    return 1;
  else
    return fact( n/2 );
}
```



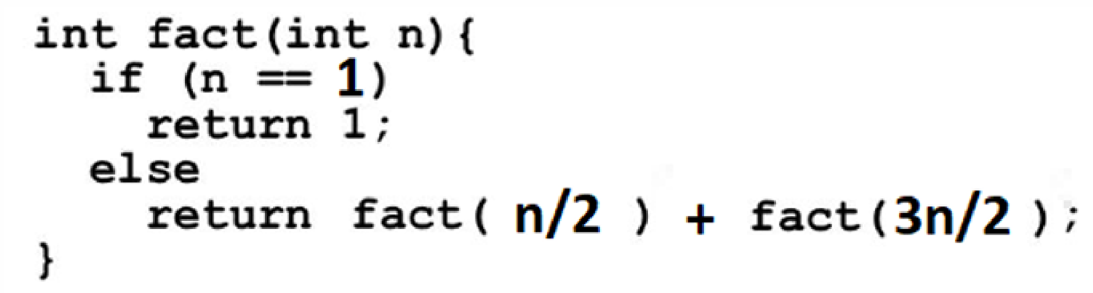
```
int fact(int n){
  if (n == 0)
    return 1;
  else
    return 4 * fact(n - 1);
}
```



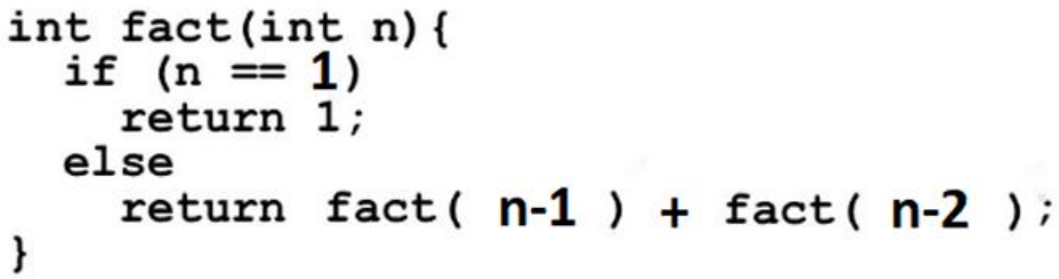
```
int fact(int n){
  if (n == 1)
    return 1;
  else
    return fact( n/2 ) + fact( n/2 );
}
```



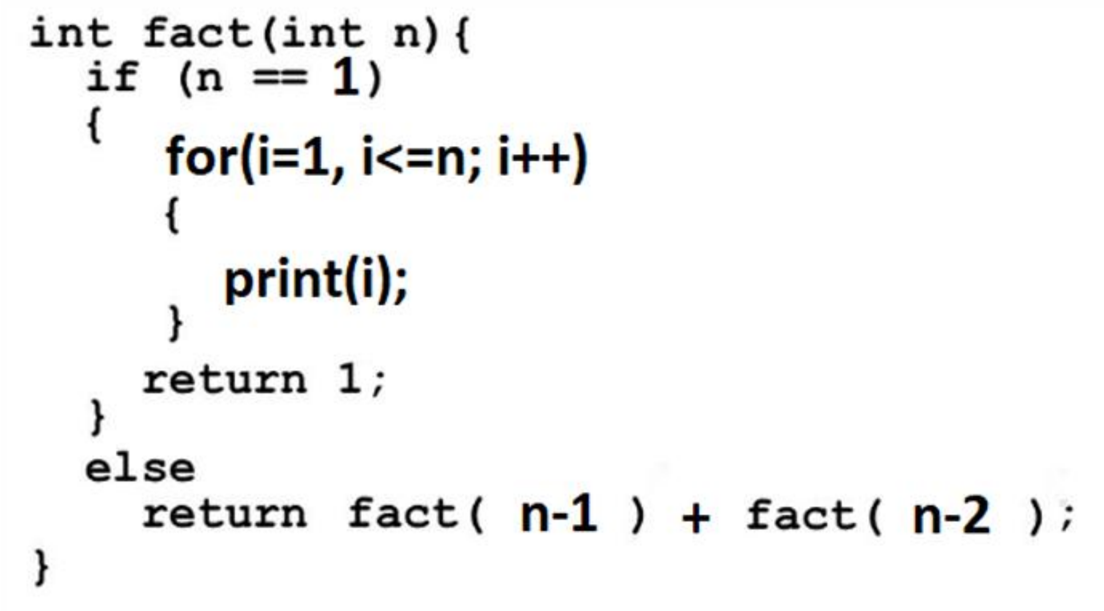
```
int fact(int n){
  if (n == 0)
    return 1;
  else
    return n * fact(n - 1);
}
```



```
int fact(int n){
  if (n == 1)
    return 1;
  else
    return fact( n/2 ) + fact(3n/2 );
}
```

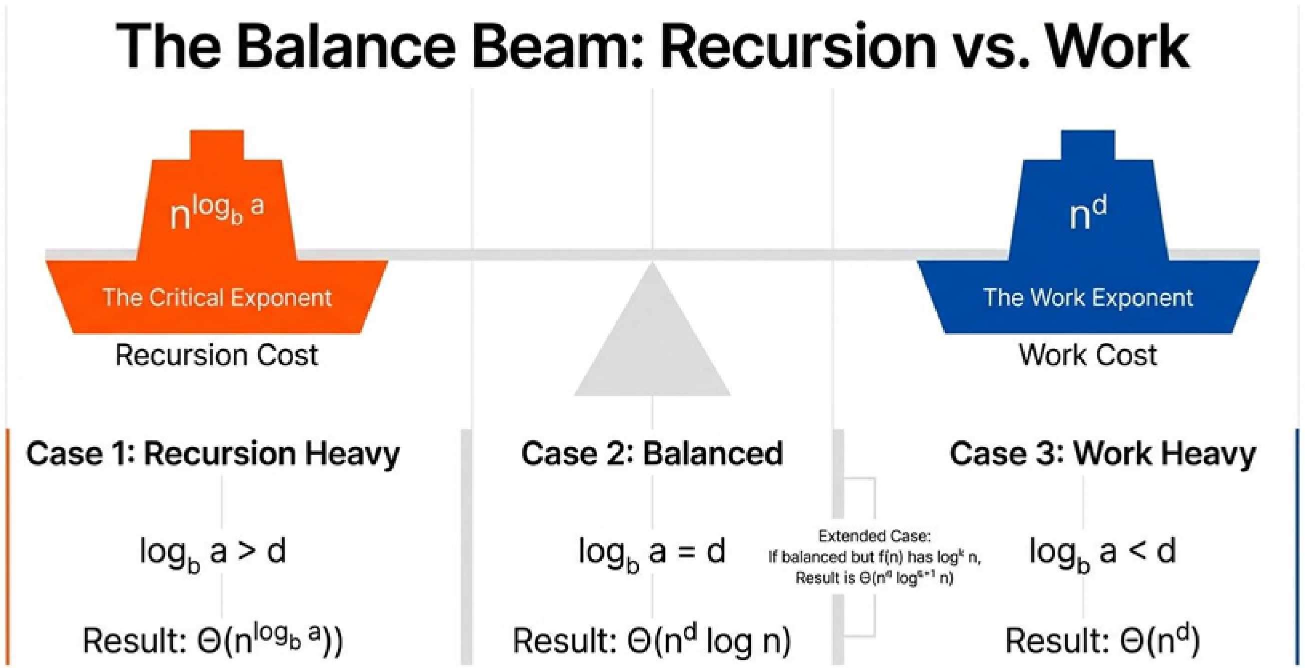


```
int fact(int n){
  if (n == 1)
    return 1;
  else
    return fact( n-1 ) + fact( n-2 );
}
```



```
int fact(int n){
  if (n == 1)
  {
     for(i=1, i<=n; i++)
     {
        print(i);
     }
    return 1;
  }
  else
    return fact( n-1 ) + fact( n-2 );
}
```

Solving Recurrence Using Master Method



The Balance Beam: Recursion vs. Work

Case 1: Recursion Heavy

$\log_b a > d$

Result: $\Theta(n^{\log_b a})$

Case 2: Balanced

$\log_b a = d$

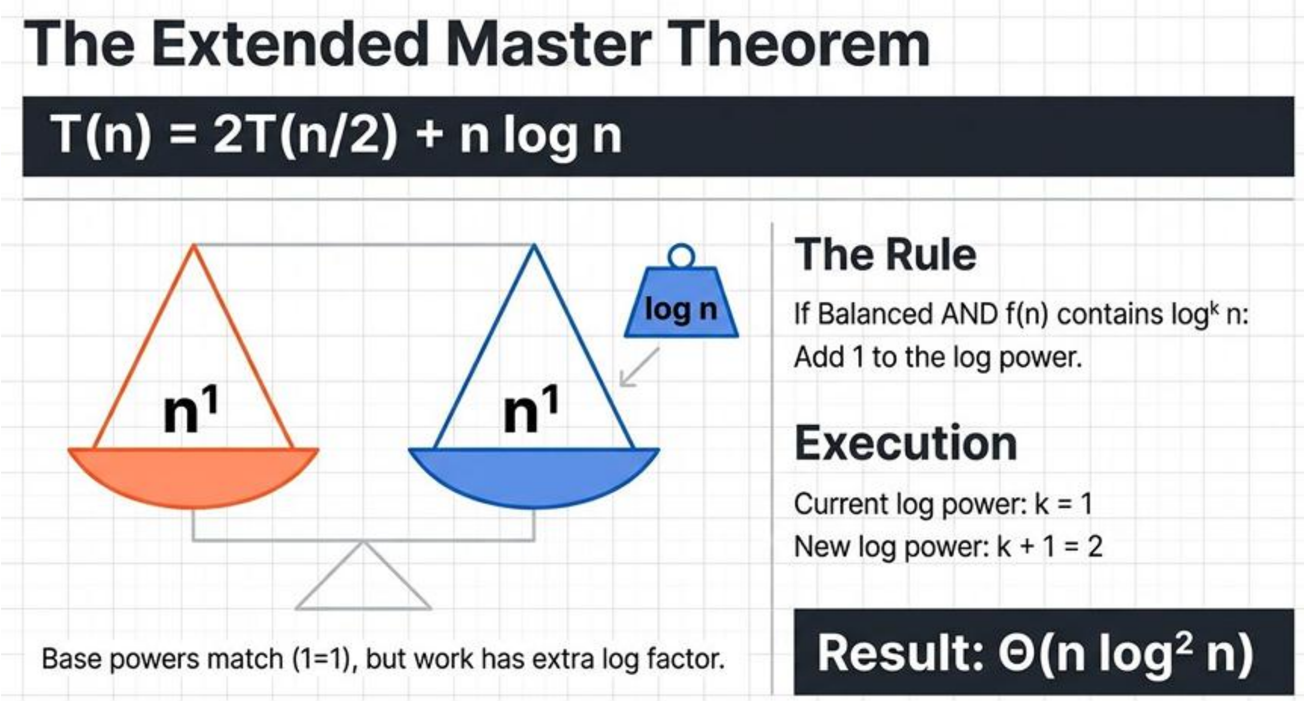
Result: $\Theta(n^d \log n)$

Extended Case:
If balanced but f(n) has $\log^k n$,
Result is $\Theta(n^d \log^{k+1} n)$

Case 3: Work Heavy

$\log_b a < d$

Result: $\Theta(n^d)$



The Extended Master Theorem

$$T(n) = 2T(n/2) + n \log n$$

Base powers match (1=1), but work has extra log factor.

The Rule

If Balanced AND f(n) contains $\log^k n$:
Add 1 to the log power.

Execution

Current log power: $k = 1$
New log power: $k + 1 = 2$

Result: $\Theta(n \log^2 n)$

$$T(n) = 4T(n/2) + n$$

$$T(n) = 4T(n/2) + n^2$$

$$T(n) = 4\,T(n/2) + n^3$$

$$T(n) = 2T(n/8) + \sqrt[3]{n}$$

$$T(n) = 4T(n/2) + n^2\sqrt{n}$$

$$T(n)=9T(n/3)+n$$

$$T(n)=T(2n/3)+1$$

$$T(n)=2T(n/2)+n \lg n$$

$$T(n)=8T(n/2)+cn^2$$

$$T(n) = 3T(n/2) + n$$

$$T(n)=2T(n/2)+n \lg n$$

$$T(n) = 4\,T(n/2) + n^2/\lg n$$

$$T(n) = 4T(n/2) + n$$

$$T(n) = 2\,T\left(\frac{n}{2}\right) + \log n$$

$$T(n) = T\left(\frac{n}{3}\right) + n \log n$$

$$T(n) = 4\,T(n/2) + n^2/\lg n$$