Lecture 2 – Lab2

FORMAL DEFINITIONS: FINDING CONSTANTS c AND n_0

O(g(n)) - THE ADDITIVE METHOD

$T(n) = c_1 * n^2 + c_2$

Find c such that $T(n) \leq c * n^2$

$T(n) \leq |c_1|n^2 + |c_2|n^2$

↓

$T(n) \leq (|c_1| + |c_2|)n^2$

↓

$\boxed{c = c_1 + c_2 \quad n_0 = 1}$

Inflate the terms to find the Ceiling.

Omega(g(n)) - THE SUBTRACTIVE METHOD

$T(n) = c_1 * n^2 + c_2$

Find c such that $T(n) \geq c * n^2$

Drop non-dominant term $\cancel{c_2}$

↓

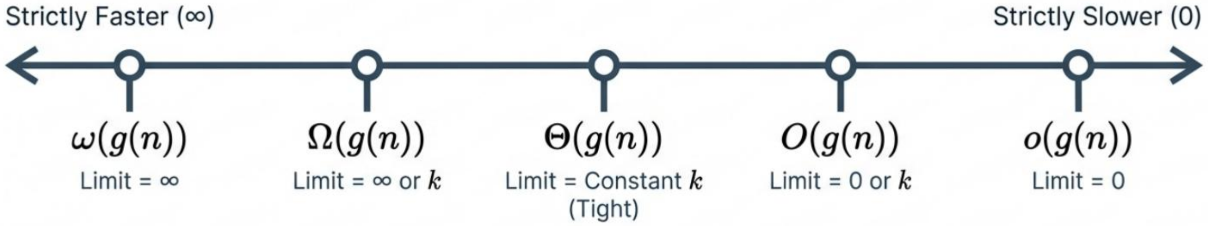
$T(n) \geq c_1 * n^2$

↓

$\boxed{c = c_1 \quad n_0 = 1}$

Reduce the terms to find the Floor.

Summary of Asymptotic Dominance



Limit Result	Meaning	o (Little)	O (Big)	Θ (Theta)	Ω (Big)	ω (Little)
0	$f < g$ (Strict)	✓	✓	✗	✗	✗
Constant (k)	$f = g$ (Tight)	✗	✓	✓	✓	✗
∞	$f > g$ (Strict)	✗	✗	✗	✓	✓

Logic Chain: $o \Rightarrow O$ but $O \neq o$. $\omega \Rightarrow \Omega$ but $\Omega \neq \omega$.

Questions

Identifying Asymptotic Dominance

Determine the complexity order by isolating the dominant term.

Function t(n)	Order
$t(n) = C_1 n^2 + C_2$	$O(\ldots\ldots)$
$t(n) = n^3 + n^4 + \sqrt{n}$	$O(\ldots\ldots)$
$t(n) = 10$	$O(\ldots\ldots)$
$t(n) = \sqrt{91}$	$O(\ldots\ldots)$
$t(n) = 2^n + n^2 + 3$	$O(\ldots\ldots)$
$t(n) = 2^{\log_3 n}$	$O(\ldots\ldots)$

Establishing Formal Bounds

$$t(n) = C_1 n^2 + C_2$$

Upper Bound (Big-O)

Find constants c, n_0 such that $t(n) \leq c \cdot n^2$

$$c = [\quad]$$

$$n_0 = [\quad]$$

Lower Bound (Big-Omega)

Find constants c, n_0 such that $t(n) \geq c \cdot n^2$

$$c = [\quad]$$

$$n_0 = [\quad]$$

Exponential Growth & Limit Theory

Is $2^{n+1} = O(2^n)$?

Evaluate the limit:

$$\lim_{n \to \infty} \frac{2^{n+1}}{2^n}$$

Proof Workspace

Is $2^{2n} = O(2^n)$?

Evaluate the limit:

$$\lim_{n \to \infty} \frac{2^{2n}}{2^n}$$

Proof Workspace

Conceptual Verification

Verify the following relationships.

Functions	Proposed Relationship	Verdict
$F(n) = \log n^2, \quad g(n) = \log n + 5$	$F(n) = \Theta(g(n))$	☐ TRUE ☐ FALSE
$F(n) = n, \quad g(n) = \log n^2$	$F(n) = \Omega(g(n))$	☐ TRUE ☐ FALSE
$F(n) = n, \quad g(n) = 10$	$F(n) = O(g(n))$	☐ TRUE ☐ FALSE
$F(n) = 10, \quad g(n) = \log 10$	$F(n) = \Theta(g(n))$	☐ TRUE ☐ FALSE
$F(n) = 2^n, \quad g(n) = 3^n$	$F(n) = O(g(n))$	☐ TRUE ☐ FALSE
$F(n) = 2^n, \quad g(n) = 10n^2$	$F(n) = \Omega(g(n))$	☐ TRUE ☐ FALSE

$F(n) = 2^n < g(n) = 10n^2 < F(n) = \Omega(g(n))$?

$F(n) = n^2 + n < g(n) = n^2 < F(n) = O(g(n))$?

Asymptotic Correctness Check

Determine whether the following statements are **True** or **False**.

$$\boxed{f(n) = 2^n + n^2}$$

Upper Bounds (O, o)

$$2^n + n^2 = O(2^n)$$

$$2^n + n^2 = O(3^n)$$

$$2^n + n^2 = o(2^n)$$

$$2^n + n^2 = o(3^n)$$

Lower Bounds (Ω, ω)

$$2^n + n^2 = \Omega(2^n)$$

$$2^n + n^2 = \Omega(n^2)$$

$$2^n + n^2 = \omega(2^n)$$

$$2^n + n^2 = \omega(n^2)$$

$$2^n + n^2 = \omega(\sqrt{n})$$

Proving Asymptotic Bounds

Establish the following relationships using the **Limit Definition**.

Method: $\lim_{n \to \infty} \frac{f(n)}{g(n)}$

Problem A: Little-o (Strict Upper Bound)

$$3n^2 + n = o(n^3)$$

Problem B: Little-omega (Strict Lower Bound)

$$n^3 + n^2 = \omega(n^2)$$