Types of Distribution

Discrete Distributions (for countable outcomes)

Bernoulli Distribution

Bernoulli Distribution is a type of discrete probability distribution where every experiment conducted asks a question that can be answered only in yes or no. In other words, the random variable can be 1 with a probability p or it can be 0 with a probability $(1-p)$. Such an experiment is called a Bernoulli trial. A pass or fail exam can be modeled by a Bernoulli Distribution.

Probability of Heads from One Coin Toss

Binomial Distribution

Binomial Distribution is a probability distribution used to model the number of successes in a fixed number of independent trials, where each trial has only two possible outcomes: success or failure. This distribution is useful for calculating the probability of a specific number of successes in scenarios like flipping coins, quality control, or survey predictions. Binomial Distribution is based on Bernoulli trials, where each trial has an independent and identical chance of success. The probability distribution for a Bernoulli trial is called the Bernoulli Distribution.

Poisson Distribution

The Poisson distribution is a type of discrete probability distribution that calculates the likelihood of a certain number of events happening in a fixed time or space, assuming the events occur independently and at a constant rate.
It is characterized by a single parameter, λ (lambda), which represents the average rate of occurrence of the event. The distribution is used when the events are rare and the number of occurrences is non-negative and can take on integer values (0, 1, 2, 3,...).

$$P (X = k) = e^{-\lambda}\lambda^k / k!$$

POISSON DISTRIBUTION

Continuous Distributions (for measurable outcomes)

Normal Distribution

Normal Distribution is the most common or normal form of distribution of Random Variables, hence the name "normal distribution." It is also called Gaussian Distribution in Statistics or Probability. We use this distribution to represent a large number of random variables. It serves as a foundation for statistics and probability theory.
It also describes many natural phenomena, forms the basis of the Central Limit Theorem, and also supports numerous statistical methods.

Chi-Squared Distribution

The Chi-Squared distribution (also chi-square or χ²-distribution) represents the distribution of the sum of the squares of k independent standard normal random variables. If Z1, Z2, . . ., Zk are independent standard normal random variables.
The Chi-Squared distribution is parameterized by the degrees of freedom (df), which corresponds to the number of independent random variables being summed.

Student's T Distribution

Student's t-distribution, also known as the t-distribution, is a probability distribution that is used in statistics for making inferences about the population mean when the sample size is small or when the population standard deviation is unknown. It is similar to the standard normal distribution (Z-distribution), but it has heavier tails. Theoretical work on t-distribution was done by W.S. Gosset; he has published his findings under the pen name "Student". That's why it is called a Student's t-test. The t-score represents the number of standard deviations the sample mean is away from the population mean.

Bayes' Theorem

Bayes' Theorem is a mathematical formula that helps determine the conditional probability of an event based on prior knowledge and new evidence.
It adjusts probabilities when new information comes in and helps make better decisions in uncertain situations.

Guessing the Pet
Is it a cat or a dog in the box?

P(Dog) = 0.5   P(Cat) = 0.5

Initial Belief: 50/50 chances

We have a CLUE
The Pet is very Quiet

→ (P(Quiet | Cat)) = 80 % or 0.8
→ (P(Quiet | Dog)) = 30 % or 0.3

Applying Bayes Theorem to Find Probability

$$P(Cat \mid Quiet) = \frac{P(Quiet\ Cat) \times P(Cat)}{P(Quiet)} = 72.7\%$$

$$P(Dog \mid Quiet) = \frac{P(Quiet\ Dog) \times P(Dog)}{P(Quiet)} = 27.3\%$$

Events

Dependent Events

Two events are dependent if the occurrence of one does affect the probability of the other.

Independent Events

Two events are independent if the occurrence of one does not affect the probability of the other occurring.

Difference between Dependent Events & Independent Events

Dependent Events:
- Events where the outcome of one event affects the probability of the other.
- Ex - Drawing cards without replacement.
- Formula : P(A and B)=P(A)×P(B|A)

Independent Events:
- Events where the outcome of one event does not affect the probability of the other.
- Ex - Tossing a coin and rolling a die.
- Formula : P(A and B) = P(A)×P(B)