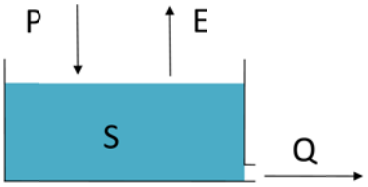
Exercise: Reservoir water balance

Introduction



The behaviour of the reservoir is defined by the mass balance equation:

$$\frac{dS}{dt} = -Q + P - E$$

The discharge Q in the mass balance equation can be express as a function of stored water in the reservoir:

$$Q = f(S)$$

P and E represent precipitation and actual evaporation respectively.

Linear reservoir without external drivers

If we neglect the external drivers (P and E). and assume $Q = kS^{\alpha}$ and $\alpha = 1$ (linear reservoir), the system can be solved analytically and therefore storage and discharge can be calculated at any time knowing the initial condition and k value.

Part 1

Derive the analytical solution for this reservoir in case α=1 (linear reservoir):

However, for hydrological systems it is not realistic to neglect the external drivers. When taking into account these drivers, the equation cannot be solved analytically, but has to be solved numerically.

An example of a numerical solution is the explicit forward Euler method. This method calculates the state of a system at each time step considering state of the system at previous time step and rate of changes:

$$S(t + \Delta t) = F\big(S(t)\big)$$

Considering the linear reservoir problem (α=1) without external drivers:

$$\frac{dS}{dt} = -kS^{\alpha}$$

The forward Euler method yields:

$$\frac{S(t + \Delta t) - S(t)}{\Delta t} = -kS^{\alpha}$$

which yields:

$$S(t + \Delta t) = S(t) - k\Delta t S^{\alpha}$$

Hence, if $S(t)$ is known, $S(t + \Delta t)$ can be calculated in one iteration.

Part 2

Write a matlab script to calculate the analytical and numerical solution of the linear reservoir problem without external drivers. The following (starting) values can be used, during the exercise you can change k and t_{max}:

- S_0 = 75 mm
- k = 0.01 d^{-1}
- t_0 = 0 days
- t_{max} = 200 days;

The following questions might help writing the script:

- What do you want to calculate?
- Which variables need to be defined?
- For how many time steps do you want to calculate the output?
- How can a *for loop* help you to calculate the output for multiple time steps (search in help)?
- How can you use a *function* for this exercise?

Part 3

Use the matlab script to plot the analytical and numerical solution. Investigate also the influence of the timestep and the value for k.

Linear reservoir with external drivers

Hydrological models tend to mimic the interactions of system with external drivers, therefore, as mentioned earlier, neglecting the external drivers is not realistic. Considering external derivers, the water balance equation cannot be solved analytically because precipitation and evaporation cannot be formulated, therefore, we rely on numerical analysis.

Part 4

Write a matlab script to calculate the numerical solution of the linear reservoir problem including the external drivers (see the model structure in Fig. 1). The values for precipitation and evaporation are given in P_PE_date.txt (search for *load* in help). You can use the same values as in the previous assignment for the other variables.

The following questions might help you to systematically set up your matlab script.

1. What do you want to calculate?
2. Which part of the previous script can you reuse?
3. Which (additional) variables need to be defined?
4. Which column of **P_PE.txt** represents precipitation and potential evaporation? Why?.
5. What should be the time step?
6. Which formulas do you need?
7. How can you take into account the different fluxes?

Bonus question

The average runoff coefficient $\bar{Q}/\bar{P}$ of this catchment is between 0.3 and 0.35. Try to change the values of the parameters k and α in such a way that your modelled runoff coefficient is between 0.3 and 0.35 as well. But be careful to have a more or less reasonable hydrograph as well!!!