Cairo University	Academic Year:	2023/2024	Final Exam	
	Course Code	PHY1251	Course Title:	Introduction to Modern Phys. & EM Fields
	Day:	Thursday	Date:	18/1/2024
	Time:	2 Hours	Full Mark	60

Faculty of Engineering

Attempt the following questions:

1-(A) A hemisphere of radius R_o is charged with surface charge density $\rho_{so}\cos\theta$, where ρ_{so} is a constant. Find: (i) the electric field at the center of the hemisphere, and (ii) the electric potential at the center of the hemisphere. [5 pts]

(B) A continuous symmetric distribution of electric flux density is given by

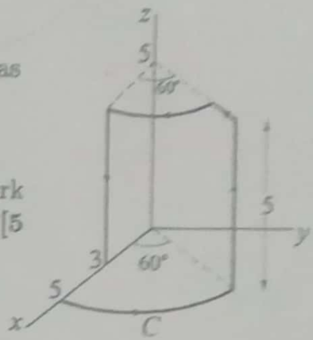
$$D = \begin{cases} a_r\, 5r^2/4 & \text{for } r \le 2 \\ a_r\, 20/r^2 & \text{for } r > 2 \end{cases}$$

(i) Find the charge density ρ_v in each region using the differential form of Gauss's law. (ii) Find the total charge for $r \le 2$. (iii) Verify divergence theorem for the spherical volume defined by $r \le 2$. [5 Marks]

(C) If a vector field $\boldsymbol{E}$ is given in the cylindrical coordinates system as

$$\boldsymbol{E} = 2\rho(z^2+1)\cos\phi\, \boldsymbol{a}_\rho - \rho(z^2+1)\sin\phi\, \boldsymbol{a}_\phi + 2\rho^2 z\cos\phi\, \boldsymbol{a}_z \ \ V/m$$

(i) prove that $\boldsymbol{E}$ is a genuine electric field. (ii) Evaluate the work done in moving a unit positive charge along the contour C. [5 Marks]

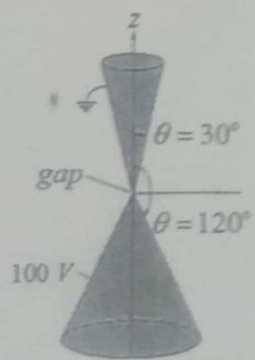


2-(A) State Maxwell's equations in point form. [4 Marks]

(B) The polarization in a dielectric cube of side L centered at the origin is given by $P = P_o(x\, a_x + y\, a_y + z\, a_z)$. Determine the surface and volume bound charge densities. (ii) Prove that that the total bound charge is zero. [4 Marks]

(C) The plane $z = 0$ separates region 1 ($z > 0$), which is a dielectric material with $\varepsilon_r = 2$ and region 2 ($z < 0$), which is a dielectric material with $\varepsilon_r = 3$. The electric field in the region above the dielectric interface is given by $\mathbf{E}_1 = 2y\, \mathbf{a}_x - 3x\, \mathbf{a}_y + (5+z)\, \mathbf{a}_z$ V/m. The surface free charge on the interface is given by $\rho_s = 4\varepsilon_o\ C/m^2$. Find: (i) the electric field $\mathbf{E}_2$ just below the interface, and (ii) the surface polarized charge density. [5 Marks]

3-(A) Two infinite conducting cones $\theta = 30^o$ and $\theta = 120^o$ are maintained at the two potentials $V = 0$ and $V = 100\,V$; respectively, as shown in figure. (i) Use Laplace's equation in the spherical coordinates to solve for the potential function and electric field vector in the space between the two cones. (ii) Determine the charge density on the grounded conductor at $r = 10\,cm$. [7 Marks]



(B) If $\mathbf{J} = \frac{100}{\rho^2}\, \mathbf{a}_\rho$ A/m^2, find: (i) the time rate of increase in the volume charge density, (ii) the total current passing through surface defined by $\rho = 2, 0 < z < 1, 0 < \phi < 2\pi$. [4 Marks]

(C) A sphere is made of magnetic material for which $\mu_r = 200$. The region outside sphere is air. The magnetic field intensity is given inside sphere as $H = 5\,a_r + 3\,a_\theta + 9\,a_\phi$ mA/m. Determine the magnetic field intensity just outside the sphere. **[4 Marks]**

4-(A) Suppose that a light of total intensity $1.0\,\mu$W/cm^2 falls on a clean iron sample $1.0\,cm^2$ in an area that has a work function of $4.7\,eV$. Assuming that all the photons have an effective wavelength of 250 nm, how many electrons will be emitted per second? And what is the maximum speed of these electrons? **[4 Marks]**

(B) In X-ray production, electrons are accelerated through a high voltage of 35 keV and then decelerated by striking a target. What is the shortest wavelength of the produced x-ray. **[3 Marks]**

(C) A line of wavelength $\lambda = 486.27$ nm is observed in the hydrogen spectrum. Identify the transition that leads to this line. **[6 Marks]**

(D) The speed of an electron is measured to be 8×10^4 m/s to an accuracy of 0.002 %. Find the minimum uncertainty in determining the position of this electron. **[4 Marks]**

(E) What are the quantum numbers (n, l, m_l, m_s) for the $4d$ subshell? (define range if necessary) **[3 Marks]**

Best Wishes

You may use the following constants and relations

$\varepsilon_o = 8.854 \times 10^{-12}$ **F/m**, $\mu_o = 4\pi \times 10^{-7}$ **H/m**, $h = 6.6261 \times 10^{-34}$ **Js**,
$|e| = 1.6022 \times 10^{-19}$ **C**, $m_e = 9.109 \times 10^{-31}$ **kg**, $m_p = 1.6725 \times 10^{-27}$ **kg**,
$hc = 1240$ **eV.nm**, $E_o(\text{electron}) = 0.511$ **M eV**, $\sigma = 5.67 \times 10^{-8} Wm^{-2}K^{-4}$
$c = 3 \times 10^8\, m/s$, $\lambda_c = 2.4\, pm$, $K = 1.38 \times 10^{-23}$ $m^2kg.s^{-2}K^{-1}$

$$\int \frac{dx}{(x^2+a^2)^{3/2}} = \frac{1}{a^2}\frac{x}{\sqrt{x^2+a^2}}, \quad \int \frac{dx}{x^2+a^2} = \frac{1}{a}\tan^{-1}(x/a), \quad \int \frac{x\,dx}{(x^2+a^2)^{3/2}} = -\frac{1}{\sqrt{x^2+a^2}}$$

$$\int \frac{dx}{\sqrt{x^2+a^2}} = \ln[(x/a) + \sqrt{1+(x/a)^2}] \quad , \int \frac{x^2dx}{(x^2+a^2)^{3/2}} = \ln[\frac{x+\sqrt{x^2+a^2}}{a}] - \frac{x}{\sqrt{x^2+a^2}}$$

$$\int \frac{x\,dx}{\sqrt{x^2+a^2}} = \sqrt{x^2+a^2}, \quad \int \sqrt{x^2+a^2}dx = \frac{1}{2}\left[x\sqrt{x^2+a^2} + a^2\ln(x+\sqrt{x^2+a^2})\right], \quad \int \frac{1}{\sin\theta}\,d\theta = \ln\left(\tan\frac{\theta}{2}\right)$$

In cylindrical coordinates:

$$d\ell = d\rho\; a_\rho + \rho d\phi\; a_\phi + dz\; a_z$$

$$\nabla A = \frac{\partial A}{\partial \rho}\,a_\rho + \frac{1}{\rho}\frac{\partial A}{\partial \phi}\,a_\phi + \frac{\partial A}{\partial z}\,a_z, \qquad \nabla \cdot \mathbf{A} = \frac{1}{\rho}\frac{\partial}{\partial \rho}(\rho A_\rho) + \frac{1}{\rho}\frac{\partial A_\phi}{\partial \phi} + \frac{\partial A_z}{\partial z}$$

$$\nabla \times \mathbf{A} = \left[\frac{1}{\rho}\frac{\partial A_z}{\partial \phi} - \frac{\partial A_\phi}{\partial z}\right] a_\rho + \left[\frac{\partial A_\rho}{\partial z} - \frac{\partial A_z}{\partial \rho}\right] a_\phi + \frac{1}{\rho}\left[\frac{\partial \rho A_\phi}{\partial \rho} - \frac{\partial A_\rho}{\partial \phi}\right] a_z$$

$$\nabla^2 A = \frac{1}{\rho}\frac{\partial}{\partial \rho}\left(\rho\frac{\partial A}{\partial \rho}\right) + \frac{1}{\rho^2}\frac{\partial^2 A}{\partial \phi^2} + \frac{\partial^2 A}{\partial z^2}$$

In spherical coordinates:

$$a_r = a_x \sin\theta\cos\phi + a_y \sin\theta\sin\phi + a_z \cos\theta, \quad d\ell = dr\, a_r + rd\theta\, a_\theta + r\sin\theta d\phi\, a_\phi$$

$$\nabla A = \frac{\partial A}{\partial r}\,a_r + \frac{1}{r}\frac{\partial A}{\partial \theta}\,a_\theta + \frac{1}{r\sin\theta}\frac{\partial A}{\partial \phi}\,a_\phi, \qquad \nabla \cdot A = \frac{1}{r^2}\frac{\partial(r^2 A_r)}{\partial r} + \frac{1}{r\sin\theta}\frac{\partial(A_\theta \sin\theta)}{\partial \theta} + \frac{1}{r\sin\theta}\frac{\partial A_\phi}{\partial \phi}$$

$$\nabla \times \mathbf{A} = \frac{1}{r\sin\theta}\left[\frac{\partial A_\phi \sin\theta}{\partial \theta} - \frac{\partial A_\theta}{\partial \phi}\right] a_r + \frac{1}{r}\left[\frac{1}{\sin\theta}\frac{\partial A_r}{\partial \phi} - \frac{\partial r A_\phi}{\partial r}\right] a_\theta + \frac{1}{r}\left[\frac{\partial r A_\theta}{\partial r} - \frac{\partial A_r}{\partial \theta}\right] a_\phi$$

$$\nabla^2 A = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial A}{\partial r}\right) + \frac{1}{r^2\sin\theta}\frac{\partial}{\partial \theta}\left(\sin\theta\frac{\partial A}{\partial \theta}\right) + \frac{1}{r^2\sin^2\theta}\frac{\partial^2 A}{\partial \phi^2}$$