Fluid-gravity correspondence, many sides and consequences

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Abstract

The essay analyzes the fluid-gravity correspondence, highlighting how physically diverse systems share common non-linear dynamics. By comparing the Plateau-Rayleigh instability in viscoelastic jets with the Gregory-Laflamme instability in black strings, the author illustrates the universal phenomenon of 'beads-on-a-string' formations. The work explores the evolution of the holographic duality (AdS/CFT), providing evidence of gravitational turbulence through Einstein's equations. Finally, it examines how non-linear interactions influence black hole quasi-normal modes and the cosmological constant, suggesting that gravity's intrinsic complexity can regularize emerging strong-field phenomena.

1 Dynamic analogies between fluid dynamics and gravity

In this section, the parallelism between fluid mechanics and gravitational physics is explored, focusing on the mechanisms of topological rupture known as pinch-off. Through the analysis of the Plateau–Rayleigh instability in viscoelastic jets and the Gregory–Laflamme instability in black strings, it is highlighted how physically distant systems share a common dynamic based on self-similar cascades and "beads-on-a-string" formations.

1.1 Plateau–Rayleigh instability and pinch-off dynamics

The Plateau–Rayleigh instability represents the fluid-dynamic phenomenon that explains the tendency of a cylindrical liquid jet to fragment into discrete spherical droplets rather than maintaining itself as a continuous flow.

The driving force of this instability is surface tension, which pushes the fluid to minimize its potential energy by reducing its surface area.

The process begins due to small perturbations always present in the flow, which can be described as sinusoidal waves. Some of these dampen over time, while others grow depending on their wavenumber k and the jet radius R_0.Based on the Young–Laplace equation, internal pressure depends on two opposing effects: the curvature of the jet and the curvature of the perturbation along the axis. In the wave necks, the smaller radius would tend to increase the pressure, but this effect can be countered by the curvature of the wave itself.The instability develops only when the effect of the jet radius prevails, i.e., for:

$$kR_0 < 1$$

In this case, the liquid flows from the necks toward the bulges, and the perturbation with the maximum growth rate, occurring at:

$$kR_0 \simeq 0.697$$

eventually dominates the evolution.The process finally leads to pinch-off, the phase in which the filament thins until it ruptures in finite time, characterized by divergences in pressure and velocity. Consider, for example, the instability in saliva: the phenomenon acquires additional complexity due to the presence of polymeric chains such as mucin, which impart viscoelastic properties to the fluid. Unlike water, where pinch-off occurs almost instantaneously once the perturbation dominates, saliva resists rupture; while surface tension seeks to pinch the filament, the polymer molecules are stretched, exerting an elastic restorative force.

This balance of forces delays the final singularity, leading to the formation of a structure defined as "beads-on-a-string." In this context, pinch-off is no longer a point-like event but a prolonged process, in which the central filament can undergo further secondary instabilities before final separation. This demonstrates how the microscopic structure of the fluid can regularize the dynamics of the singularity. In Figure 1, snapshots of an animation from Wagner's experiments [1] show how the pinch-off dynamics follow a "pearl necklace" pattern, in contrast to Newtonian fluids.

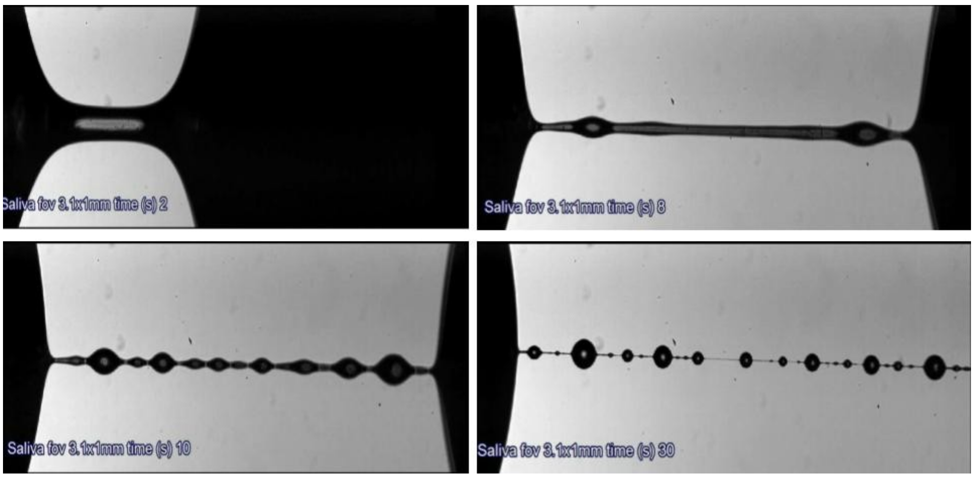

Figure 1: Temporal evolution of the instability in saliva showing the formation of the "beads-on-a-string" structure. The sequences (from $t = 1$ s to $t = 30$ s) highlight how the elasticity of the polymeric chains counteracts surface tension, delaying the final pinch-off.

1.2 Gregory–Laflamme instability and the pinch-off phenomenon

The Gregory–Laflamme instability represents a crucial phenomenon in general relativity in dimensions higher than four ($D > 4$), describing how extended gravitational objects such as black strings become unstable despite being valid solutions to the Einstein field equations.This instability manifests when a black string has a length significantly greater than its Schwarzschild radius.In this regime, small perturbations trigger a vibration of the system that generates zones of varying density, causing the horizon to evolve toward a "beads-on-a-string" conformation, characterized by massive bulges separated by thin necks. This dynamics culminates in pinch-off, a process in which the radius of the necks progressively thins until it vanishes in finite time. The dynamics of this process were elucidated by the numerical simulations of Luis Lehner and Frans Pretorius [2], which demonstrated that the instability does not lead to an immediate rupture of the black string, but rather to a self-similar cascade. In this evolution, the residual filaments become unstable in turn, generating a sequence of increasingly thin structures and a population of black holes on progressively smaller scales.

The cascade proceeds until a finite-time pinch-off occurs, where the radius of the string tends toward zero and the spacetime curvature diverges. At this point, the event horizon loses its continuity, leading to the formation of a naked singularity. This result constitutes an explicit violation of Penrose's weak cosmic censorship conjecture, according to which singularities should always be shielded by an horizon to ensure the causality and predictability of general relativity.

Since the singularity comes into direct contact with the external spacetime during this phase, the description provided by classical general relativity ceases to be valid. The ultimate fate of the system cannot, therefore, be determined within this theoretical framework.

In Figure 2, snapshots are presented from an animation simulating a black string subject to the Gregory–Laflamme instability.

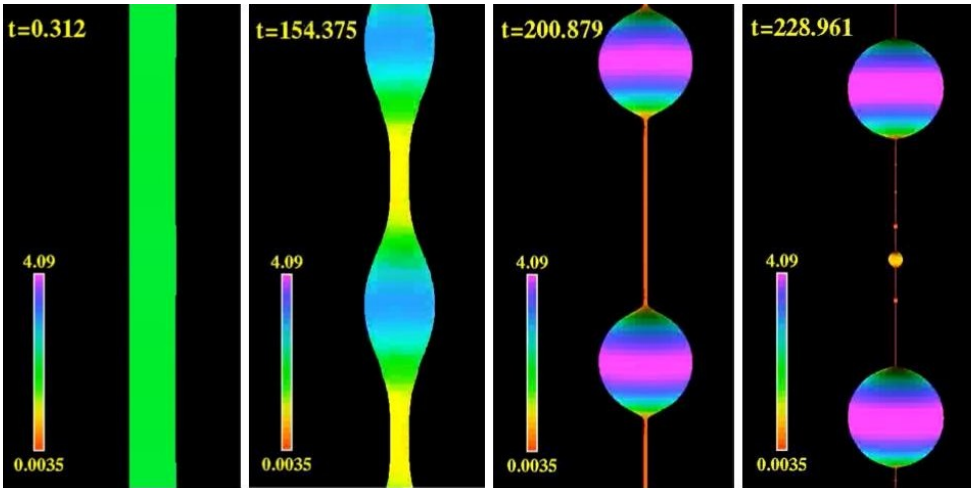

Figure 2: Temporal evolution of the Gregory–Laflamme instability in a black string. The images show the transition from a uniform cylindrical horizon to a "beads-on-a-string" structure, where spherical bulges are connected by thin necks. The self-similar cascade described by Lehner and Pretorius is clearly observed, with the formation of successive generations of smaller black holes along the residual filaments prior to the final pinch-off.

1.3 Qualitative comparison between gravitational and hydrodynamic instabilities

The qualitative analogy between the Gregory–Laflamme instability and the Plateau–Rayleigh instability is based on the description of the event horizon of a black string as a membrane endowed with mathematical properties similar to those of a fluid. In this context, the fluid-gravity correspondence allows for the mapping of gravitational dynamics onto those of fluid mechanics. From a morphological perspective, both systems react to external perturbations by deviating from their initial cylindrical symmetry to assume a "beads-on-a-string" configuration.

In the case of saliva, the presence of polymeric chains generates an elastic resistance that leads to the formation of droplets connected by thin filaments. Similarly, the numerical simulations of Lehner and Pretorius [2] show that a black string in five dimensions evolves in an analogous manner, developing a sequence of spherical black holes joined by increasingly thin string segments.

A further point of qualitative contact concerns the self-similar nature of the fragmentation process. In both the saliva filament and the black string, the thin necks connecting the main masses become unstable themselves, triggering a cascade that repeats the same structure on progressively smaller spatial scales. This evolution leads both systems toward the phenomenon of pinch-off: the point of topological rupture where the filament radius tends toward zero, marking the definitive transition toward a series of separate objects.

2 Gravity and fluids in the nonlinear regime

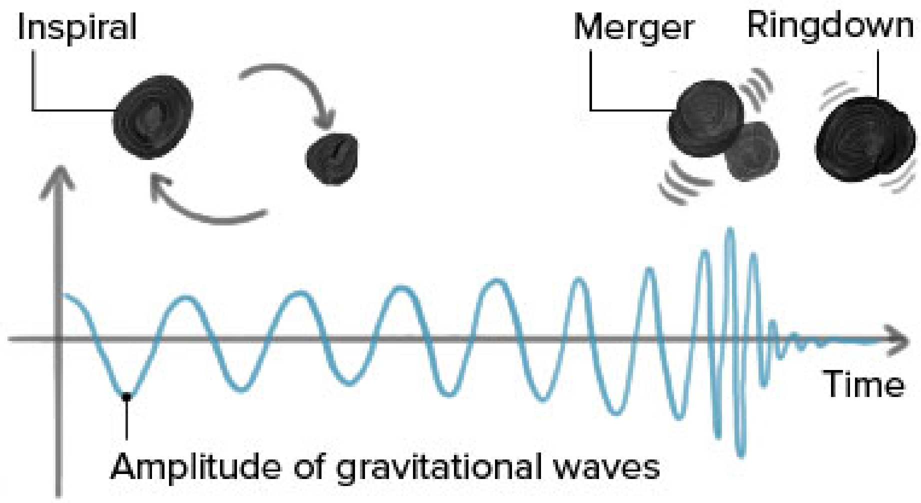

Figure 3: Waveform of a gravitational wave produced by the merger of a binary black hole system, eventually leading to the formation of a larger black hole that resonates like a struck bell. (Image: RIKEN Interdisciplinary Theoretical and Mathematical Sciences Program)

Figure 3 illustrates the temporal evolution of the amplitude of a gravitational wave generated by the coalescence of two compact objects, typically black holes, allowing for the distinction of three fundamental physical phases.

- The initial Inspiral phase shows a sinusoidal signal whose amplitude and frequency grow steadily as the bodies orbit one another, losing energy in the process.

- This is followed by the Merger, the moment of maximum energy emission in which the event horizons merge, visible in the graph as the peak of the signal's maximum amplitude.

- Finally, the Ringdown phase describes the exponential decay of the wave as the newly formed black hole stabilizes toward a stationary state.

This process raises a fundamental problem regarding the nature of gravity in its most extreme regime: the merger of two black holes represents one of the most nonlinear events observable in nature, with masses colliding at significant fractions of the speed of light; yet, the resulting waveform appears surprisingly regular and orderly, devoid of the structural chaos one might expect from such spacetime turbulence.

The reason for this unexpected simplicity in such a complex regime is not yet fully understood; however, hydrodynamics offers a concrete example where systems characterized by extremely complicated dynamics produce elementary phenomenological outcomes through mechanisms of damping and self-organization.

Consider, for example, the formation of hurricanes, which emerges as a macroscopic phenomenon of order within two-dimensional turbulence regimes ($2+1$ dimensions).In such systems, the physical reason for stability lies in the existence of dynamic constraints that invert the normal energy flow typical of three-dimensional fluids.While turbulence in a purely three-dimensional fluid is characterized by the fragmentation of energy into increasingly smaller vortices toward ultraviolet (UV) scales until viscous dissipation occurs, in two dimensions, the quasi-conservation of enstrophy intervenes. Defined as the integral of the square of the flow's vorticity:

$$\mathcal{E} = \frac{1}{2} \int_V |\boldsymbol{\omega}(\mathbf{x}, t)|^2 \, \mathrm{d}V$$

enstrophy acts as a constraint that physically prevents energy from fragmenting toward small scales. Consequently, the injected energy is forced to move toward large spatial scales (IR) through a process known as the inverse energy cascade 4.In this dynamics, small turbulent vortices tend to merge with one another instead of dissipating, shifting energy toward the widest dimensional scales permitted by the boundary conditions.The final result is the condensation of disordered turbulence into a single, massive, stable structure: the hurricane.

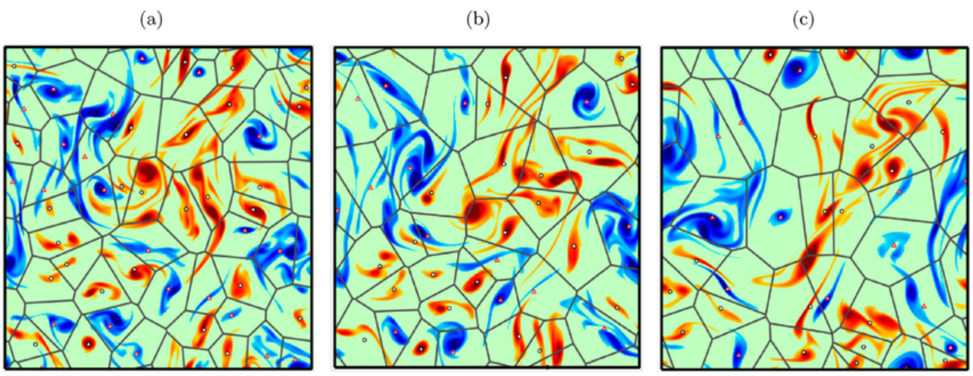

Figure 4: Three moments of the inverse energy cascade during the decay of a two-dimensional turbulence box. Source: Chaos, coherence, and turbulence - Scientific Figure on ResearchGate. Available from: `https://www.researchgate.net/figure/Three-moments-of-the-inverse-energy-cascade-during-the-decay-of-a-two-dimensional_fig1_396960022`

2.1 The dilemma of linearity: quasi-normal modes and nonlinear regimes

The understanding of the transition toward stability faces a crucial methodological tension regarding the applicability of linear perturbation theory.

Although the ringdown signal can be modeled through quasi-normal modes (oscillations characterized by a real frequency and an imaginary decay rate uniquely determined by the mass and spin of the black hole according to General Relativity) the problem arises of establishing when an intrinsically nonlinear system can effectively be considered linear.

The analysis of these modes offers a rigorous test of the theory, as the measurement of two or more frequencies would allow for the verification of the system's parameter consistency. However, to do so, one would have to wait until the signal is purely linear, which means reducing its intensity until it becomes nearly imperceptible; conversely, advancing the analysis toward the merger regime exposes the researcher to nonlinear interferences.

The risk is that such nonlinear characteristics could be misinterpreted as evidence of new physics or deviations from General Relativity. This fuels a heated debate between those who claim to have already identified multiple modes and those who, on the contrary, believe that such conclusions are the result of an analytical overreach in the attempt to extract linear behavior from a regime still dominated by nonlinear complexity.

3 The Fluid-Gravity Duality: Historical Framework and Perturbative Criticality

Over the last 50 years, various attempts have been made to establish a mapping (or correspondence) between the Einstein equations (EE), which describe general relativity and gravitation, and the equations of hydrodynamics, which govern fluid behavior. This correspondence, known as "Fluid-Gravity duality", aims to interpret certain aspects of gravity in terms of fluid dynamics and vice versa. Three fundamental stages can be identified in this evolution, each characterized by a different approach to projecting the Einstein equations onto specific geometric structures:

- **The Membrane Paradigm** (1980s): Inspired by the pioneering work of Thierry Damour, this approach projects the Einstein equations onto a time-like surface positioned just outside the event horizon of a black hole. This method describes the horizon as a "viscous bubble"—a fluid membrane with properties similar to a dense, sticky fluid—whose dynamics are governed by the classical Navier-Stokes equations for non-relativistic viscous flows.

- **Holography and AdS/CFT** (2000s): Developed extensively by researchers such as Veronika Hubeny, this phase involves projecting the Einstein equations onto the asymptotic boundary of an Anti-de Sitter (AdS) space. The AdS/CFT correspondence connects gravity in an AdS space to a Conformal Field Theory (CFT) on its boundary. In this context, the result is relativistic hydrodynamics, where the "fluid" lives on the holographic boundary of spacetime.

- **Carrollian Hydrodynamics** (2020s): The most recent frontiers, explored in works by Véronique Donnay and Olivier Marteau, focus on projections performed directly onto null (light-like) surfaces, such as the event horizon itself. This approach leads to the emergence of "Carrollian" hydrodynamics, based on the Carroll symmetry (a contraction of the Poincaré group where the speed of light tends to zero). This is characterized by ultralocal dynamics, offering new perspectives on how entropy and information are encoded on the horizon.

3.1 Conceptual Tension: Amplitude vs. Gradient Expansion

We can formalize the tension between these models by comparing two common perturbative approaches. A perturbative approach is an analytical method for solving complex equations by approximating them with a series of small corrections relative to a base solution, regulated by a small parameter ϵ.

The General Relativist's Approach (Amplitude Expansion): Based on the perturbative decomposition of the spacetime metric:

$$g_{\mu\nu} = g_{\text{Kerr}} + \epsilon h_1 + \epsilon^2 h_2 + \dots$$

where g_{Kerr} is the exact Kerr solution (stationary rotating black hole with mass M and angular momentum a). Here, a stationary base configuration is perturbed by linear and nonlinear fluctuations, with ϵ controlling the deviation amplitude.

The Field Theory Approach (Gradient Expansion): The physical parameters of the base solution are promoted to spacetime-dependent functions:

$$g_{\text{Kerr}}(M, a) \to g(M(x), a(x))$$

It is required that the gradients (spatial and temporal derivatives) are small relative to other scales (e.g., $\partial M \ll M/L$). This allows mass and spin to be treated as variables that evolve slowly across spacetime.

Significant risks exist in these approximations, as they can introduce biases that alter the underlying physics.

Consider the Navier-Stokes equation for incompressible fluids ($\nabla \cdot \mathbf{v} = 0$):

$$\partial_t \mathbf{v} + \mathbf{v} \cdot \nabla \mathbf{v} = -\nabla P + \nu \nabla^2 \mathbf{v}$$

where $\mathbf{v}$ is the velocity field, P is the pressure, and ν is the kinematic viscosity. The choice of "perturbative path" drastically conditions the result:

- **Gradient Path** (Expansion in Gradients, Neglecting High-Derivative Dissipative Terms): By assuming spatial derivatives are small and neglecting the viscous term $\nu \nabla^2 \mathbf{v}$, the equation reduces to the Euler equation for ideal fluids:

  $$\partial_t \mathbf{v} + \mathbf{v} \cdot \nabla \mathbf{v} = -\nabla P$$

  Here, the Reynolds number $Re = \frac{UL}{\nu}$ effectively diverges ($Re \to \infty$). Without the damping effect of $\nu \nabla^2 \mathbf{v}$, there is no energy dissipation at small scales. The system becomes "all turbulent", dominated by nonlinearities and energy cascades.

- **Amplitude Path** (Expansion in Amplitudes, Linearizing Velocity Terms): By assuming the amplitude of $\mathbf{v}$ is small and neglecting the nonlinear term $\mathbf{v} \cdot \nabla \mathbf{v}$, the equation reduces to the Stokes equation:

$$\partial_t \mathbf{v} = -\nabla P + \nu \nabla^2 \mathbf{v}$$

  In Fourier space, for a mode with wavenumber k, this implies $\partial_t v_k \approx -\nu k^2 v_k$, leading to exponential decay: $v_k(t) = v_k(0)e^{-\nu k^2 t}$. Here, Re is low, and the system is "all decay", dominated by regular dissipation without nonlinear coupling.

Perturbative theories can be "dangerous" because the final result is often implicitly contained within the initial choice of the scheme. If the model is overly simplified, one risks ignoring fundamental nonlinear physics—such as the inverse energy cascade in 2D fluids or mode coupling—making numerical simulations an indispensable tool for validating analytical predictions.

3.2 Validating the Duality: Comparative Analysis of Relativistic Hydrodynamics and Full Gravity

While classical hydrodynamics tends to naturally develop shocks—that is, abrupt discontinuities in fluid density, pressure, or velocity, in General Relativity (GR), perturbations always and invariably propagate at the speed of light, regardless of local conditions. This behavior is due to the relativistic nature of the theory, where causality is rigidly constrained by the speed of light.

However, the absence of structural shocks in GR (like classical hydrodynamic ones) does not exclude the possibility of turbulence. Although the underlying mechanisms differ (for instance, in GR, turbulence emerges from the nonlinear complexity of the equations describing spacetime curvature rather than from molecular interactions as in fluids) these equations can exhibit turbulent behaviors analogous to those observed in classical fluids, such as energy cascades.

The formal "bridge" connecting this gravitational turbulence to fluid dynamics is the fluid-gravity correspondence, valid in a spacetime with a negative cosmological constant, known as Anti-de Sitter (AdS) space. In this context, the Einstein equations, when projected onto the asymptotic boundary of the spacetime, translate into a gradient expansion of the energy-momentum tensor (T_{ab}). This expansion organizes the system's response into a hierarchy of terms, ordered by the number of spatial and temporal derivatives applied to the macroscopic fluid variables.

In practice, this is a power series expansion that approximates the system's behavior at increasing orders of complexity, similar to a perturbative expansion:

$$T_{ab} = \underbrace{(\rho + p)u_a u_b + p g_{ab}}_{\text{Zeroth Order}} + \underbrace{\Pi^{(1)}_{ab}}_{\text{First Order}} + \underbrace{\Pi^{(2)}_{ab}}_{\text{Second Order}} + \cdots$$

Specifically, the Zeroth Order describes a perfect fluid in local equilibrium, without energy dissipation (e.g., without friction or viscosity).

At the First Order, the shear tensor (σ_{ab}) emerges ad it measures volume-preserving fluid deformation and physically represents the cause of kinematic viscosity. In this regime, the projected Einstein equations are equivalent to the relativistic Navier-Stokes equations, which describe viscous fluids in GR.

Finally, the Second Order introduces second derivatives and nonlinear terms, which are essential to ensure the causality and stability of the model. Without these corrections—typical of the Müller-Israel-Stewart (1979) theory perturbations in the fluid could propagate faster than the speed of light, violating fundamental principles of GR.

Each order of this expansion represents an increasing level of precision with which the curved geometry of AdS space captures the dissipative and turbulent phenomenology of the dual fluid.

To demonstrate that turbulence is a universal property of gravity, independent of the specifics of AdS, it is necessary to isolate the dynamical essence of these phenomena beyond particular geometric features. By formalizing the problem through the equation

$$G_{ab} = \lambda \rho_{ab}$$

where (G_{ab}) is the Einstein tensor (encoding spacetime curvature), λ is a coupling constant, and represents the source term including matter, energy, and the cosmological constant can be shown that the key nonlinearities reside primarily in G_{ab} , rather than in $\lambda\rho_{ab}$. The latter only influences temporal and spatial scales but not the essence of the turbulence. Therefore, if the dominant nonlinearities are in G_{ab}, turbulence emerges as an intrinsic property of gravity itself.

Supporting this, two fundamental studies demonstrate that the turbulence observed in fluids is a faithful representation of nonlinear gravitational dynamics:

- **"Hydrodynamics-Only" Simulation** (Fig. 5a): The group of Adams, Chesler, and Liu simulated the relativistic hydrodynamic equations (Navier-Stokes) on the holographic boundary, starting from smooth initial conditions. They observed that the fluid develops turbulence and energy cascades, with energy transferring from large to small scales [3].
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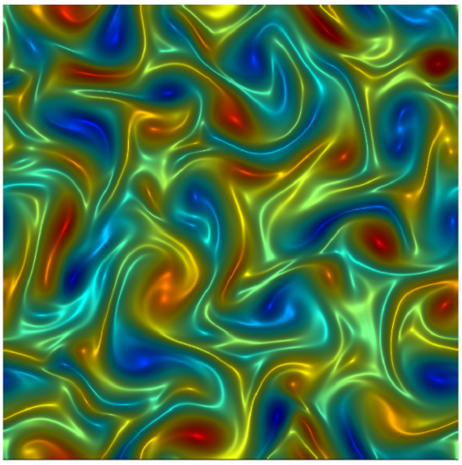
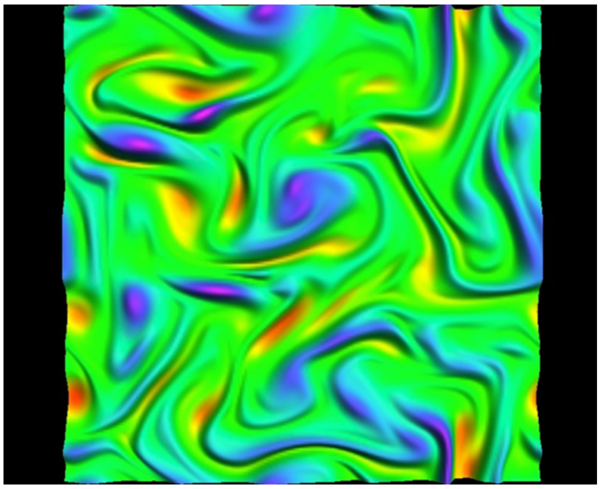
- **"Full Gravity" Simulation** (Fig. 5b): The group of Carrasco, Green, and Lehner solved the full Einstein equations in the AdS bulk spacetime, using the same initial conditions as the previous simulation. By projecting the results onto the boundary, they obtained a direct match with the fluid dynamics [4].

This comparison numerically demonstrated that gravitational turbulence exists and behaves analogously to fluid-dynamic turbulence, validating the fluid-gravity duality even beyond the linear regime. The evolution of the black hole geometry in the bulk corresponds almost perfectly to the fluid dynamics on the boundary.

Furthermore, by calculating the Ricci tensor (a measure of local spacetime curvature) in the full gravity simulation results, Green et al. (2014) showed that hydrodynamics captures the essence of gravity with extreme accuracy, exhibiting minimal deviations only at very small scales [4].



(a) Numerical simulation of a turbulent relativistic fluid on the holographic boundary, showing the development of energy cascades (from Adams et al., 2013 [3]).



(b) Full gravity simulation in AdS bulk, projected onto the boundary, reproducing fluid-dynamic turbulence with high fidelity (from Carrasco et al., 2014 [4]).

4 The Role of Non-linear Gravitational Dynamics in Hiding the Negative Cosmological Constant

The presence of a negative cosmological constant ($\Lambda < 0$) is a well-established premise in certain theoretical frameworks; therefore, it is natural to wonder why we have not observed it in certain contexts, such as in black holes within asymptotically flat spacetimes (without a cosmological constant).

The cosmological constant Λ is a term introduced by Einstein in the field equations of General Relativity to represent a constant energy density of empty space. Einstein's equations are:

$$G_{ab} = 8\pi T_{ab} + \Lambda g_{ab}$$

where:G_{ab} is the Einstein tensor, describing the curvature of spacetime,T_{ab} is the energy-momentum tensor, representing matter and energy, g_{ab} is the metric tensor, which defines the geometry of spacetime.

The existence of a negative cosmological constant necessarily leads us to think that in regimes where it is not apparent, it is due to the non-linearity of gravity. This non-linearity stems from the fact that gravity is self-interacting: curvature influences itself.

To demonstrate this, one must study a black hole in an asymptotically flat spacetime, where similar phenomena emerge from non-linear interactions between perturbations.

To explain the problem, let us consider a simple analogy with a lake (a fluid) and rocks thrown into it. Imagine a calm lake, representing the stable "background." Throwing a small rock generates waves (linear perturbations), similar to small oscillations on the surface.If the rock is small, the waves are weak and can be described as an independent superposition on the flat background. The velocity of the fluid surface v is approximated as $v = v_b + P_1$, where v_b is the background velocity (zero in the calm lake) and P_1 is the perturbation caused by the first rock.

Now, a second rock is thrown. If the viscosity of the fluid (water) is not too high, the second rock falls onto a surface already oscillating due to the first. Here, the key point emerges: the perturbations are not independent.Following a traditional linear approach, one would simply sum the perturbations: $v = v_b + P_1 + P_2$, assuming each acts on the original background without mutual influence.

However, since they do influence one another, one must follow the non-linear approach, which is the correct method to treat the phenomenon: the effective background for the second rock is the original background plus the oscillations of the first $(v_b + P_1)$. Therefore, P_2 must be calculated on this modified background. This captures the "interaction" between perturbations, similar to how viscosity or other non-linear properties of the fluid amplify or dampen waves.

This analogy highlights that in non-linear systems (such as real fluids or gravity), perturbations "talk" to each other: one influences the behavior of the other, leading to emergent effects such as amplification or instability.If one applies this idea to gravity, specifically to a Kerr black hole (a rotating black hole described by the Kerr metric), the general metric is perturbed:

$$g_{ab} = g_{ab}^{\text{Kerr}} + \delta g_{ab} \cdot h_1$$

where:g_{ab}^{Kerr} is the background metric of the Kerr black hole (including mass M and spin J, satisfying the vacuum Einstein equations: $G_{ab} = 0$),δg_{ab} is a small perturbation factor (often denoted as ϵ with $\epsilon \ll 1$ for perturbative approximations),h_1 is the metric perturbation (a tensor describing small deviations from the Kerr metric).

Let us consider that the perturbation h_1 evolves in time and space, representing "quasinormal modes" (QNMs); these are the damped oscillations of a black hole following a perturbation, similar to the vibrations of a bell that fade away. The black hole's mass and spin vary slowly due to this perturbation, making the spacetime dynamic.

To capture the non-linear interactions, a second perturbation is introduced: a scalar field ψ. The equation for ψ is:

$$\Box\psi = 0$$

where $\Box$ is the d'Alembertian (or box operator), defined as:

$$\Box = g^{ab}\nabla_a\nabla_b$$

This $\Box$ is not calculated on the pure Kerr metric, but on the perturbed metric $g_{ab} = g_{ab}^{\text{Kerr}} + \delta g_{ab} \cdot h_1$. Thus, the equation becomes coupled: the perturbation h_1 (the QNMs) influences ψ through the spatial and temporal dependence in the metric.By expanding the equation $\Box\psi = 0$, a "parametric coupling" emerges: terms that link the modes of ψ with those of h_1.

Mathematically, expanding $\Box$ in terms of background + perturbation yields equations of the type:

$$\Box^{\text{Kerr}}\psi + \delta\Box\psi = 0$$

where $\delta\Box$ includes products like $h_1 \cdot \partial\psi$, representing non-linear interactions. Depending on the initial amplitude of the perturbation h_1, some modes lose energy, "feeding" ψ. Consequently, for a brief period, ψ grows exponentially ($\psi \propto e^{\gamma t}$, with $\gamma > 0$) at the expense of the primary modes. Figure 6 graphically illustrates this effect.

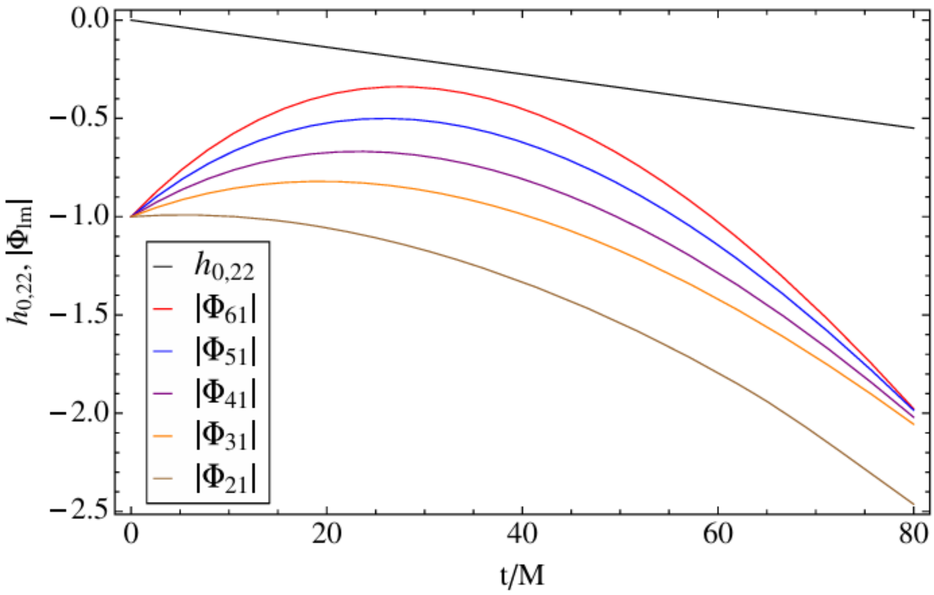


Figure 6: Growth of the scalar quasinormal mode amplitudes due to a perturbation, on a logarithmic scale

On the x-axis is time (in units of M, the black hole mass), and on the y-axis is the mode amplitude (logarithmic, from -0.5 to 0.6). Different curves represent different modes: the primary mode decays, while others temporarily grow (exponentially) before stabilizing.

This demonstrates that non-linearities in gravity lead to energy transfers between modes, similar to the lake analogy. The implications are broad: in contexts such as black hole mergers (detected by LIGO), these effects could influence the observed gravitational waves.

This analysis (pioneered by Huan Yang, Aaron Zimmerman, and Luis Lehner in [5]) allows us to argue that the negative cosmological constant is "hidden" by non-linearities but emerges in calculations that capture the interactions between perturbations.

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