Variational Formulation of Dissipative Dynamics within the GENERIC Formalism: An Experimental Investigation of the Entropic Action Principle in Diffusive Fluids

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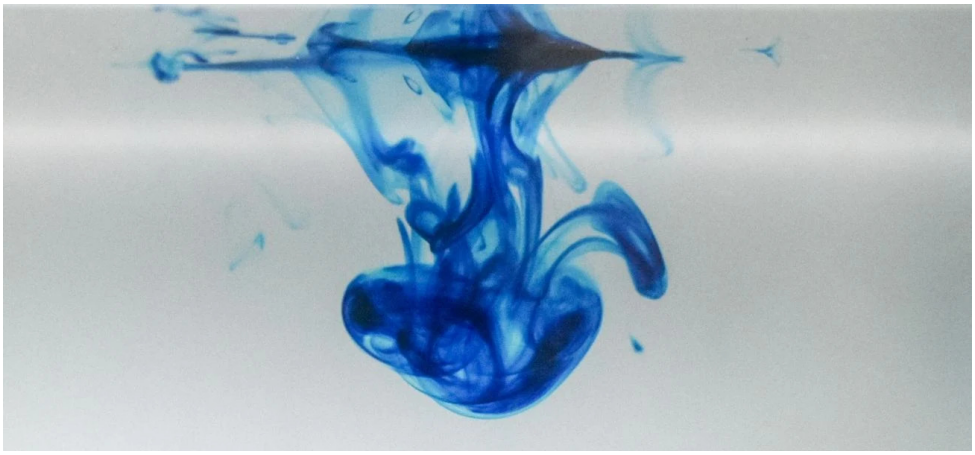

Figure 1: visualization of a methylene blue droplet diffusing in a water solution

Abstract

Context and Motivation. The modeling of complex fluid dynamics faces the challenge of identifying a universal set of state variables and evolution equations. The GENERIC (*General Equation for the Nonequilibrium Reversible-Irreversible Coupling*) formalism addresses this by decomposing the time evolution of a state variable x into reversible and irreversible contributions:

$$\frac{dx}{dt} = L\frac{\delta E}{\delta x} + M\frac{\delta S}{\delta x} \tag{1}$$

where L and M are operators representing Poisson and dissipative kinematics, respectively. While the reversible dynamics governed by energy E is deeply rooted in Hamiltonian mechanics and variational principles, the dissipative dynamics driven by the entropy potential S is typically described by relaxation equations. The formulation of the irreversible contribution as a trajectory minimizing a global action functional—an "Entropic Action Principle"—remains an open question in the geometric formulation of nonequilibrium thermodynamics.

Research Objective. The primary objective of this project is to investigate whether the hydrodynamic evolution of a scalar concentration field satisfies a variational principle derived from the GENERIC structure. Specifically, I hypothesize that the physical trajectory of a diffusing species minimizes a functional $\mathcal{A}[x]$ constructed from the nonequilibrium entropy defined in the GENERIC formalism as:

$$S(x) = -k_B \int d^3r \int d^3v \, f(\mathbf{r}, \mathbf{v}) \ln f(\mathbf{r}, \mathbf{v}) \tag{2}$$

By adapting this definition to the hydrodynamic limit (where x represents concentration $c(\mathbf{r}, t)$), I aim to verify if the stationary points of the action functional correspond to the observed diffusion equations governed by the operator M.

Methodology. The investigation will adopt a tripartite theoretical-experimental approach:

- **Theoretical Formulation:** I will derive the explicit form of the action functional for a scalar field using the Onsager-Machlup Lagrangian formulation compatible with the GENERIC structure. This involves performing a coarse-graining of the Boltzmann entropy (Eq. 2) to obtain an effective potential for the concentration field.
- **Experimental Acquisition:** I will set up a controlled diffusion experiment involving a methylene blue tracer in a quiescent water solution. Using high-resolution spectrophotometry, I will reconstruct the scalar field $c_{exp}(\mathbf{r}, t)$ over time, providing the empirical trajectory in the state space $\mathcal{M}$.
- **Variational Analysis:** The experimental data will be numerically substituted into the theoretical entropic functional. I will compute the expansion of the functional to test for stationarity, verifying if the experimental data minimizes the action compared to perturbed trajectories.

Possible Results and Implications. This study seeks to demonstrate that macroscopic dissipative phenomena, such as diffusion, can be described not merely as local differential laws but as global minimization problems of an entropic action. Validating this hypothesis would provide a tangible experimental foundation for the abstract geometric structures of GENERIC, effectively bridging the gap between variational calculus and the thermodynamics of complex fluids.