Topology of Capillary Caustics and Analogy with Multipole Potential Fields: An Investigation based on Catastrophe Theory

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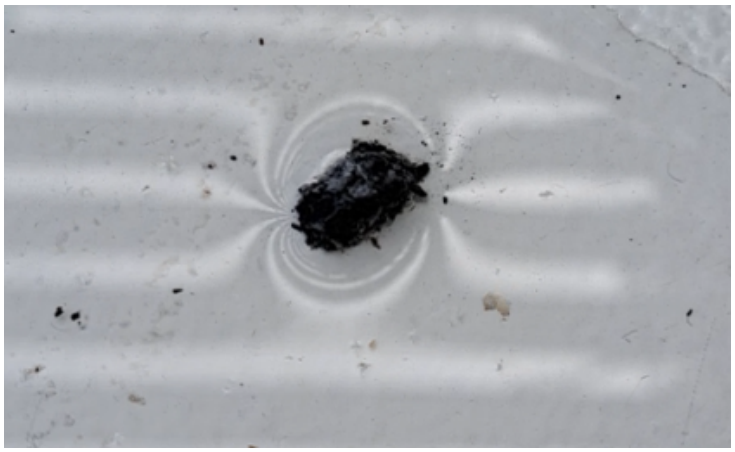

Figure 1: A piece of cigarette ash floating on a water surface on a table, illuminated by light passing through a striped cover.

Abstract

Context and Motivation. My empirical observation of floating irregular particles (e.g., combustion residues) on static fluid surfaces has revealed light refraction patterns (caustics) that show a surprising visual isomorphism with the field lines of magnetic fields generated by irregular dipoles or multipoles. Although these phenomena are governed by distinct physical laws — the Young-Laplace equation for fluid meniscus deformation and Maxwell's equations for the magnetostatic field — the morphological similarity I observed leads me to suggest the existence of a common underlying mathematical structure governing the distribution of singularities in both domains.

Research Objective. In this project, I would like to rigorously investigate the formal analogy between optical caustics generated by anisotropic capillary menisci and the topology of field lines in static magnetic systems. My central hypothesis is that the geometry of minimum energy level surfaces in the fluid (surface tension) and in the electromagnetic field shares the same catastrophe germs defined by René Thom's classification, specifically the "fold" and "cusp" catastrophes.

Methodology. I would like to adopt a tripartite theoretical-computational approach:

- **Hydrodynamic Modeling:** I will perform a perturbative solution of the Young-Laplace equation to obtain the curvature profile $z(x, y)$ generated by floating objects with non-trivial boundary geometries (simulating the irregularity of the particle I observed).
- **Optical Mapping (Ray-Tracing):** I will derive the caustic map resulting from refraction through the $z(x, y)$ profile, treating the surface as a variable-phase lens.
- **Comparative Topological Analysis:** I will compare the Lagrangian manifolds of the optical caustics with the equipotential lines of magnetic fields generated by arbitrary multipolar configurations.

Possible Results and Implications. Through this analysis, I seek to demonstrate that the observed luminous "lines" are not simple artifacts but represent the critical set of the projection map, topologically equivalent to domain separation lines in magnetic fields. My goal is to validate the use of free-surface fluid-dynamic systems as "analog computers" to visualize and study complex configurations of invisible force fields, providing a tangible bridge between geometric optics and potential theory.