Mansoura University
Faculty of Computers and Information Sciences
Department of Computer Science
First Semester- 2020-2021

[CS212P/IT212] Computer Organization and Architecture

Grade : 2nd General / 3rd Programs

Eng. Esraa Salah

NUMBER SYSTEMS

Many number systems are in use in digital technology.

The most common are :

- Decimal
- Binary
- Octal
- Hexadecimal

DECIMAL SYSTEM

- Composed of 10 numerals or symbols
- Using these symbols as digits of a number, can express any quantity.
- Called the base-10 system because it has 10 digits.
- 0, 1, 2, 3, 4, 5, 6, 7, 8, 9

Ex:

- 3.14_{10}
- 532_{10}
- 10824_{10}
- 649000_{10}

BINARY SYSTEM

- There are only two symbols or possible digit values, 0 and 1.
- This base-2 system can be used to represent any quantity that can be represented in decimal or other base system .

Ex:

- 1110_2
- 1011110_2
- 1111011100_2

OCTAL SYSTEM

- The octal number system has a base of eight .
- Eight possible digits: 0,1,2,3,4,5,6,7.

Ex:

- 5410_8
- 765421_8
- $1047664\ _8$
- 4123170137_8

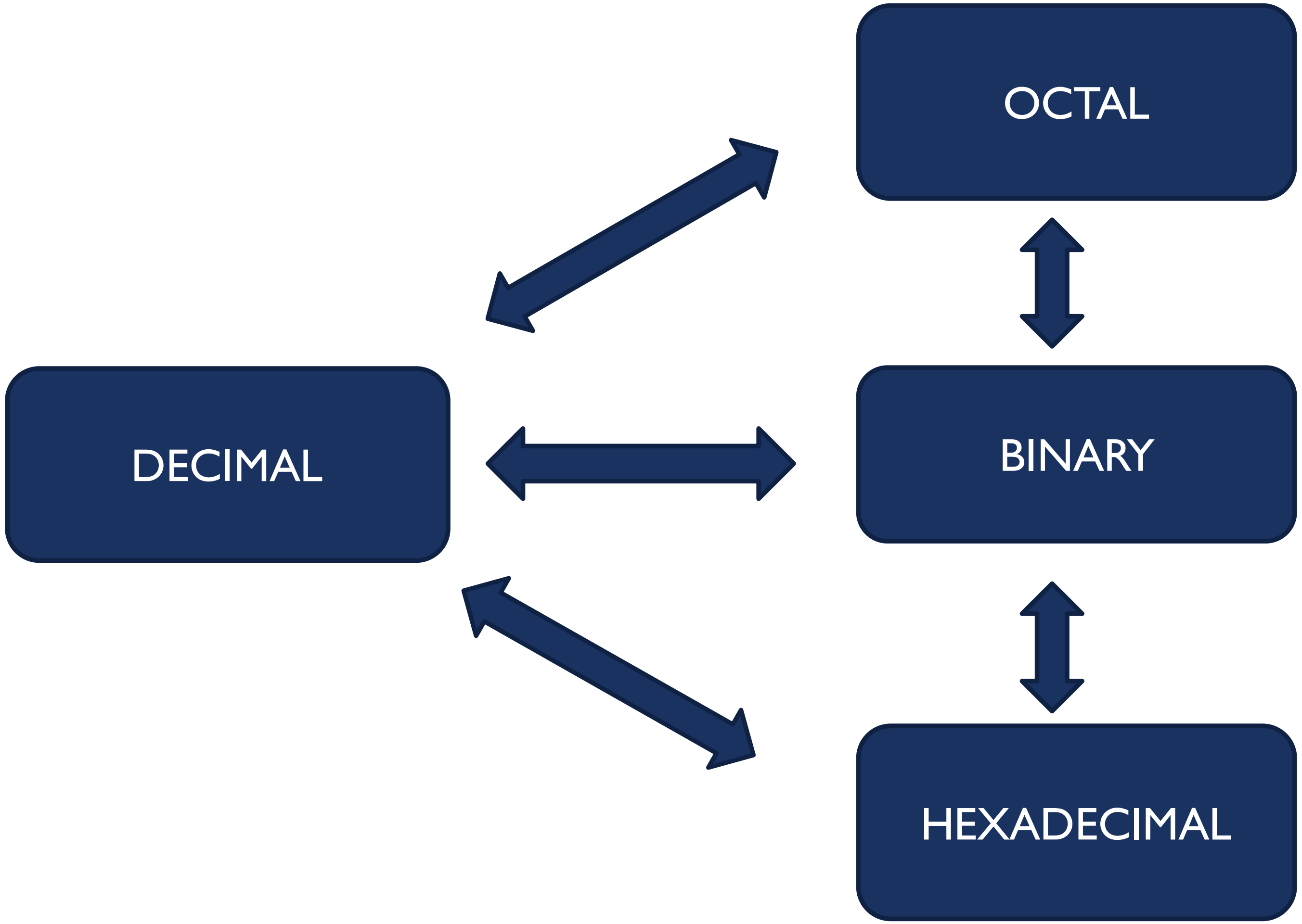
HEXADECIMAL SYSTEM

- The hexadecimal system uses base 16.
- It uses the digits 0 through 9 plus the letters A, B, C, D, E, and F as the 16 digit symbols.
- 0,1,2,3,4,5,6,7,8,9,A,B,C,D,E,F

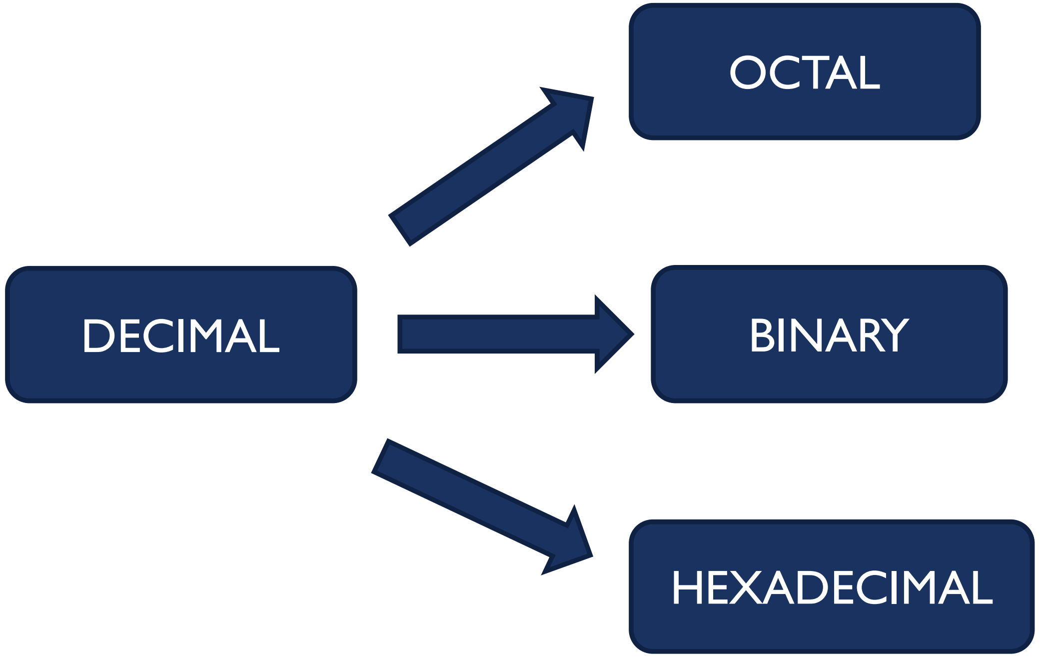
Ex:

- BD_{16}
- $452EA_{16}$
- $E451B2CD3_{16}$
- $35412BABE_{16}$

NUMBERING CONVERSION



NUMBERING CONVERSION



DECIMAL TO BINARY CONVERSION

Ex : 27_{10}

27/2 = 13 balance 1

13/2 = 6 balance 1

6/2 = 3 balance 0

3/2 = 1 balance 1

1/2 = 0 balance 1

Result : $27_{10} = 11011_2$

Ex : 181_{10}

181/2 = 90 balance 1

90/2 = 45 balance 0

45/2 = 22 balance 1

22/2 = 11 balance 0

11/2 = 5 balance 1

5/2 = 2 balance 1

2/2 = 1 balance 0

1/2 = 0 balance 1

Result : $181_{10} = 10110101_2$

DECIMAL TO OCTAL CONVERSION

Ex : 177_{10}

177/8 = 22 balance 1

22/8 = 2 balance 6

2/8 = 0 balance 2

Result : $177_{10} = 261_8$

Ex : 3985_{10}

3985/8 = 498 balance 1

498/8 = 62 balance 2

62/8 = 7 balance 6

7/8 = 0 balance 7

Result: $3985_{10} = 7621_8$

DECIMAL TO HEXADECIMAL CONVERSION

Ex : 378_{10}

378/16 = 23 balance 10 = (A)

23/16 = 1 balance 7

1/16 = 0 balance 1

Result: $378_{10} = 17A_{16}$

Ex : 6942_{10}

6942/16 = 433 balance 14 = (E)

433/16 = 27 balance 1

27/16 = 1 balance 11 = (B)

1/16 = 0 balance 1

Result: $378_{10} = 1B1E_{16}$

DECIMAL TO UNKNOWN BASE CONVERSION

Ex : 52_{10} to Base 3

$52/3 = 17$ balance 1

$17/3 = 5$ balance 2

$5/3 = 1$ balance 2

$1/3 = 0$ balance 1

Result: $52_{10} = 1221_3$

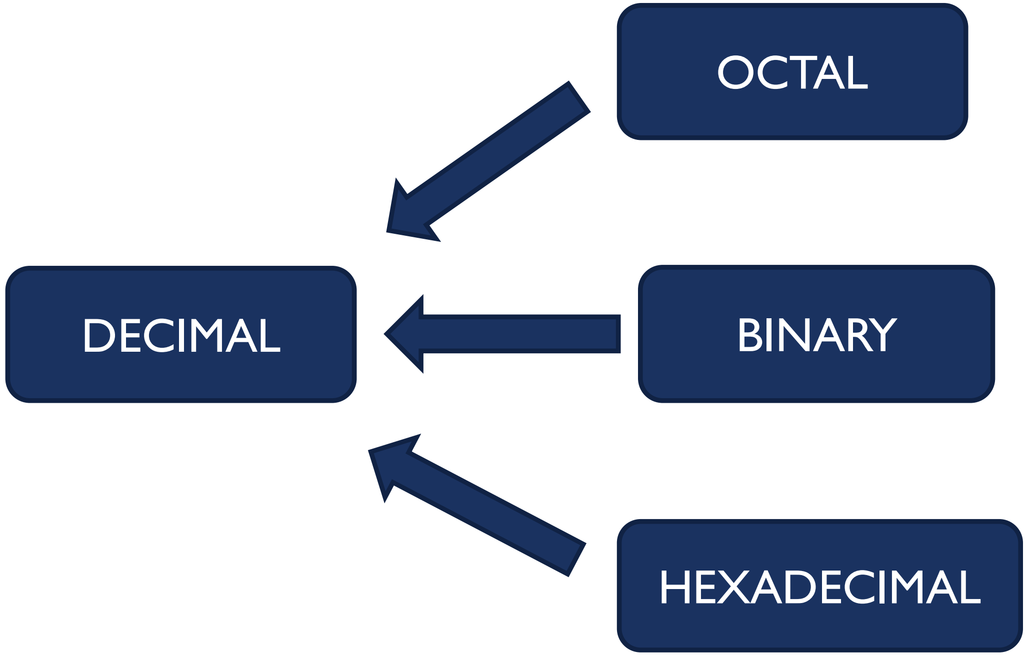
NUMBERING CONVERSION

- Technique :

– Multiply each bit by x^n , where x is the "Base" and n is the "weight" of the bit .

– The weight is the position of the bit, starting from 0 on the right .

– Add the results .



BINARY TO DECIMAL CONVERSION

$$
\begin{array}{llcccr}
101011_2 & \Rightarrow & 1 \times 2^0 & = & 1 \\
& & 1 \times 2^1 & = & 2 \\
& & 0 \times 2^2 & = & 0 \\
& & 1 \times 2^3 & = & 8 \\
& & 0 \times 2^4 & = & 0 \\
& & 1 \times 2^5 & = & \underline{32} \\
& & & & 43_{10}
\end{array}
$$

OCTAL TO DECIMAL CONVERSION

$$724_8 \Rightarrow \begin{array}{rcrcr} 4 & \times & 8^0 & = & 4 \\ 2 & \times & 8^1 & = & 16 \\ 7 & \times & 8^2 & = & 448 \\ \hline & & & & 468_{10} \end{array}$$

HEXADECIMAL TO DECIMAL CONVERSION

$$
\begin{array}{llllllllllll}
ABC_{16} & \Rightarrow & C & \times & 16^0 & = & 12 & \times & 1 & = & 12 \\
 & & B & \times & 16^1 & = & 11 & \times & 16 & = & 176 \\
 & & A & \times & 16^2 & = & 10 & \times & 256 & = & \underline{2560} \\
 & & & & & & & & & & 2748_{10}
\end{array}
$$

NUMBERING CONVERSION

OCTAL

↕

BINARY

↕

HEXADECIMAL

BINARY TO OCTAL CONVERSION

0	1	2	3	4	5	6	7
000	001	010	011	100	101	110	111

Example:

- $100111010_2 = (100)\ (111)\ (010)_2 = 4\ 7\ 2_8$
- $1101010_2 = (001)\ (101)\ (010)_2 = 1\ 5\ 2_8$

Hint :
$010_2 = 2_{10} = 2_8$

BINARY TO HEXADECIMAL CONVERSION

EXAMPLE :

- $1011101_2 = (0101)\ (1101)_2 = 5\ D_{16}$
- $1110011011_2 = (0011)\ (1001)\ (1011)_2 = 3\ 9\ B_{16}$
- $101100101111_2 = (1011)\ (0010)\ (1111)_2 = B\ 2\ F_{16}$

0	0000
1	0001
2	0010
3	0011
4	0100
5	0101
6	0110
7	0111
8	1000
9	1001
A	1010
B	1011
C	1100
D	1101
E	1110
F	1111

OCTAL TO BINARY CONVERSION

0	1	2	3	4	5	6	7
000	001	010	011	100	101	110	111

Example:

- $472_8 = (100)\ (111)\ (010)_2 = 100111010_2$
- $152_8 = (001)\ (101)\ (010)_2 = 1101010_2$

Hint :

$2^3 \quad 2^2 \quad 2^1 \quad 2^0$

8 4 2 1

HEXADECIMAL TO BINARY CONVERSION

- $5\ D_{16} = (101)\ (1101)_2 = 1011101_2$
- $3\ 9\ B_{16} = (11)\ (1001)\ (1011)_2 = 1110011011_2$
- $B\ 2\ F_{16} = (1011)\ (0010)\ (1111)_2 = 101100101111_2$

0	0000
1	0001
2	0010
3	0011
4	0100
5	0101
6	0110
7	0111
8	1000
9	1001
A	1010
B	1011
C	1100
D	1101
E	1110
F	1111

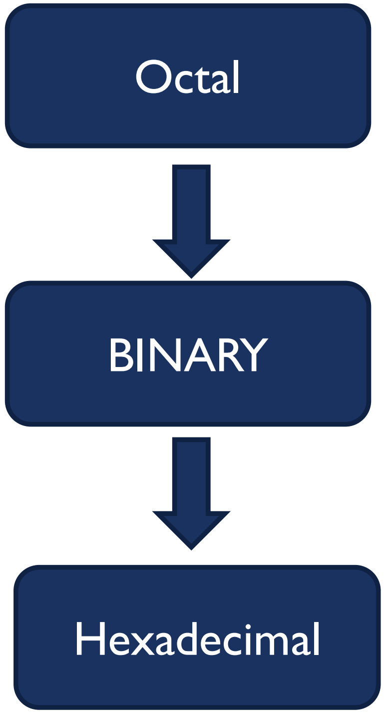
OCTAL TO HEXADECIMAL CONVERSION

- Technique :
  – Use binary as an intermediary .

Example: $1076_8=(?)_{16}$

1	0	7	6
001	000	111	110
0010	0011	1110	
2	3	E	

Result: $1076_8=23E_{16}$



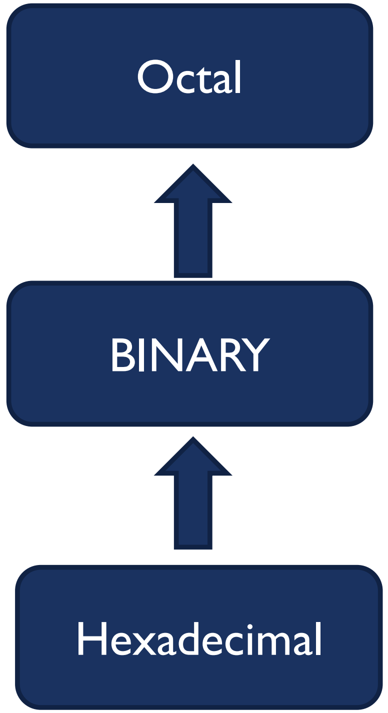
HEXADECIMAL TO OCTAL CONVERSION

- Technique :
  – Use binary as an intermediary .

Example: $1F0C_{16}=(?)_8$

	1	F	0	C	
	0001	1111	0000	1100	
0	001	111	100	001	100
0	1	7	4	1	4

Result: $1F0C_{16}=17414_8$



FRACTIONS

- Binary to decimal :

$$10.1011 \Rightarrow \quad \begin{array}{llll} 1 & \times & 2^{-4} & = 0.0625 \\ 1 & \times & 2^{-3} & = 0.125 \\ 0 & \times & 2^{-2} & = 0.0 \\ 1 & \times & 2^{-1} & = 0.5 \\ 0 & \times & 2^{0} & = 0.0 \\ 1 & \times & 2^{1} & = \underline{2.0} \\ & & & 2.6875 \end{array}$$

FRACTIONS

- Decimal to binary : 3.703125_{10}
  - Result : $3_{10} = 11_2$
  - $0.703125_{10} = ?_2$

0.703125*2=1.40625	1
0.40625*2=0.8125	0
0.8125*2 =1.625	1
0.625 *2=1.25	1
0.25*2= 0.5	0
0.5*2=1.0	1
0.0*2=0	

Result : $0.703125_{10} = .101101_2$

Result : $3.703125_{10} = 11.101101_2$

FRACTIONS

- Decimal to binary : 263.3_{10}
  - Result : $263_{10} = 100000111_2$
  - $0.3_{10} = ?_2$

0.3*2=0.6	0
0.6*2=1.2	1
0.2*2=0.4	0
0.4*2=0.8	0
0.8*2=1.6	1
0.6*2=1.2	1

  - Result : $0.3_{10} = .010011\ldots_2$

Result : $263.3_{10} = 100000111.010011..{}_2$

EXAMPLE

- 312_4 to base 7

1) 312_4 to DECIMAL : $312_4 = 3*4^2 + 1*4^1 + 2*4^0 = 54_{10}$
2) 54_{10} to base 7 : $54_{10} = ?_7$

54/7 = 7 balance 5

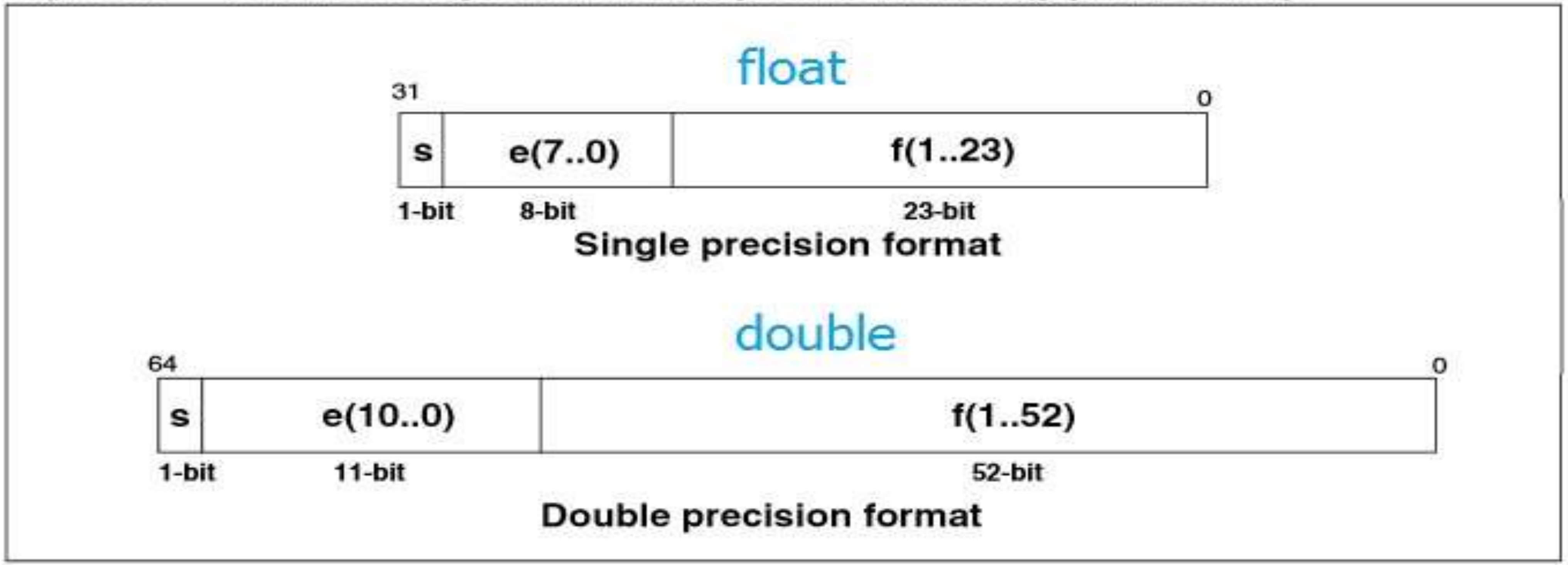
7/7= 1 balance 0

1/7= 0 balance 1 $54_{10} = 105_7$

Result : $312_4 = 105_7$

FLOATING POINT REPRESENTATION

Figure 1. IEEE.754 single and double precision floating-point coding



FLOATING POINT REPRESENTATION

- Express the following numbers in IEEE 32-bit floating point format
  - 21.75_{10}

    1. Convert to binary: $21.75_{10} = 10101.11_2$

    2. Normalize: $= 1.010111 \times 2^4$, Mantissa = 1

    3. Change exponent to biased exponent $4 + 127 = 131_{10} = 10000011_2$

    4. Format :

0	**10000011**	**01011100000000000000000**

In hex: = 41AE0000 H

FLOATING POINT REPRESENTATION

- Express the following numbers in IEEE 32-bit floating point format
  - -0.4375_{10}

    1. Convert to binary: $-0.4375_{10} = -0.0111_2$

    2. Normalize: $= 1.11 \times 2^{-2}$, Mantissa = 1

    3. Change exponent to biased exponent $-2 + 127 = 125_{10} = 1111101_2$

    4. Format :

1	**01111101**	**11000000000000000000000**

In hex: = BEE00000 H

THANKS ♥