

L.A PRACTICAL

(MCQ)

CH 1 : LINEAR EQUATIONS and MATRICES

Example of linear equation involving two variables is

- ☒ a) $7x+3y+4z = 20$
- ☒ b) $6x+2y = 10$
- ☒ c) $8x = 2+10$
- ☒ d) $7a+8b+9c = 10+5$

In linear equation ' $ax+by = c$ ' a , b and c are considered as

- a) variable
- ☒ b) constants
- c) zero
- d) real numbers

Number of ordered pair values (x,y) to satisfy linear equation $ax + by = c$ are

- ☒ a) finite
- ☒ b) infinite
- c) zero
- d) rational expression

Two equations that have no values to satisfy both equations then this is called no solution

- a) consistent system
- ☒ b) inconsistent system
- c) solution system
- d) constant system

Method in which both sides of equation are multiplied by nonzero constant is classified as

- ☒ a) Gaussian elimination method
- b) Gaussian inconsistent procedure
- c) Gaussian consistent procedure
- d) Gaussian substitute procedure

In Gaussian elimination method, original equations are transformed by using

- ☒ a) column operations
- ☒ b) row operations
- c) mathematical operations
- d) subset operations

In three dimensional coordinate systems, coordinates are

- ☒ a) perpendicular to each other
- b) parallel to each other
- c) same direction for each other
- d) opposite direction for each other

☒ **For solution set, set builder notation is $S =$**

- a) $\{(a,b,c) \mid ax+by = c\}$
- b) $\{(x,y,a) \mid abx+bcy = c\}$
- c) $\{(a,b,x,y) \mid ax+ bc = yc\}$
- ☒ d) $\{(x,y) \mid ax+by = c\}$

What form do you need to change your matrix into when using Gauss-Jordan elimination?

- ☐ row echelon form
- ☐ augmented form
- ☒ reduced row echelon form
- ☐ regular form
- ☐ none

What is the first row of our augmented matrix?

$$\begin{cases} x+z=3 \\ y=4 \\ 2z=6 \end{cases}$$

$$\left[\begin{array}{ccc|c} 1 & 0 & 1 & 3 \\ 0 & 1 & 0 & 4 \\ 0 & 0 & 2 & 6 \end{array} \right]$$

☐ 0, 0, 2, and 6

☐ 0, 1, 0, and 4

☒ 1, 1, 1, and 3

☒ 1, 0, 1, and 3 ←

☐ 0, 2, 0, 4



If a matrix is in reduced row echelon form, then it is also in row echelon form:

- ☒ a) False
- ☒ b) True
- ☐ c) May be
- ☐ d) None of the above

A homogeneous linear system always has the trivial solution, there are only two possibilities for its solutions:

- ☒ a) The system has only the trivial solution.
- ☒ b) The system has infinitely many solutions in addition to the trivial solution.
- ☒ c) Both (a) and (b)
- ☐ d) None of the above

A system of linear equations is said to be homogeneous if the constant terms are all:

- ☐ e) One
- ☒ f) Zero
- ☐ g) Both (a) and (b)
- ☐ h) None of the above

The system of equations $4x + 6y = 5$, $8x + 12y = 10$ has:

- ☐ a) No solution.
- ☒ b) Infinitely many solutions.
- ☐ c) A unique solution.
- ☐ d) None of the above

$$X = A^{-1}b$$

If $Ax = b$ is a system of n linear equations in n unknowns such that $\det(A) \neq 0$, then the system has:

- a) Infinitely many solutions.
- ☒ b) Unique solution.
- ☒ c) Both (a) and (b).
- d) None of the above

MCQ: A pair of equations to determine value of 2 variables is called

- ☒ a) simultaneous linear equations
- b) paired equations
- c) quadratic equations
- d) simple equations

MCQ: Methods to solve a pair of simultaneous linear equations are

- a) 3
- ☒ b) 2
- c) 4
- d) 5

In Gaussian reduction procedure, matrix A is augmented with an identity ($m \times m$) as:

- a) $(A | N)$
- ☒ b) $(A | I)$
- c) $(A | B)$
- d) None of the above A

After performing row operations on augmented matrix A in Gaussian reduction procedure then resulting matrix is:

- a) $(B^{-1} | I)$
- ☒ b) $(I | A^{-1})$
- c) $(M | B^{-1})$
- d) None of the above

Reduced echelon form of the matrix $\begin{pmatrix} 1 & 2 & 3 \\ 2 & 3 & 4 \end{pmatrix}$ is:

a) $\begin{pmatrix} 1 & 2 & 3 \\ 0 & 0 & 1 \end{pmatrix}$

☒ b) $\begin{pmatrix} 1 & 0 & -1 \\ 0 & 1 & 2 \end{pmatrix}$

c) $\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 1 \end{pmatrix}$

$$\begin{pmatrix} 1 & 2 & 3 \\ 2 & 3 & 4 \end{pmatrix} \rightarrow \begin{pmatrix} 1 & 0 & -1 \\ 0 & 1 & 2 \end{pmatrix}$$

A set of linear equations is represented by the matrix equation $Ax=b$. The necessary condition for the existence of a solution for this system is

- ✓ a) A must be invertible
- b) b must be linearly depended on the columns of A
- c) b must be linearly independent of the columns of A
- d) None of these

MCQ: If a matrix has equal number of columns and rows then it is said to be a

- A. row matrix
- B. identical matrix
- ✓ C. square matrix
- D. rectangular matrix

MCQ: If determinant of a matrix is equal to zero, then it is said to be

- A. square matrix
- ✓ B. singular matrix *non invertible*
- C. non-singular matrix
- D. identical matrix

MCQ: If number of columns and rows are not equal in a matrix, then it is said to be a

- ✓ A. rectangular matrix
- B. square matrix
- C. diagonal matrix
- D. null matrix

MCQ: Skew symmetric matrix is also called

- A. symmetric
- B. identical matrix
- C. square matrix
- ✓ D. anti symmetric

$$[\underline{0} = \underline{0}]$$

MCQ: A diagonal matrix having equal elements is called a

- A. square matrix
- B. identical matrix
- ✓ C. scalar matrix
- D. rectangular matrix

MCQ: A Matrix with only 1 column is called

- A. unit or identical matrix
- ✓ A. column matrix
- B. row matrix
- C. identical matrix

✓ **MCQ: If all elements in a matrix are zeros, then it is called a**

- A. column matrix
- B. diagonal matrix
- C. identical matrix
- D. null/zero matrix

✓ **MCQ: A matrix with only 1 row is called**

- A. column matrix
- B. row matrix
- C. identical matrix
- D. square matrix

MCQ: If two matrices A and B have same order and their corresponding elements are equal then it is called

- ✓ A. matrix equality
- B. rectangular matrix
- C. square matrix
- D. identical matrix

ملاحظات : * في حالة انطباق خطين , السيستم سيكون ليه عدد لا نهائي من الحلول
* في حالة التقاطع , السيستم ليه حل واحد
* في حالة التوازي , السيستم ليس له حل

* system that has infinity or unique solution ,, is called consistent.

* system that has no solution ,, is called inconsistent.

CH 2: MATRICES ARITHMETIC

MCQ: If A and B matrices are of same order and $A + B = B + A$, this law is known as

- A. distributive law
- ✓ B. commutative law
- C. associative law
- D. cramer's law

MCQ: If sum of two matrices A and B is zero matrix, then A and B are said to be

- A. multiplicative inverse of each other
- ✓ B. additive inverse of each other
- C. transpose of each other
- D. determinant of each other

MCQ: Law which does not hold in multiplication of matrices is known as

- A. distributive law
- B. Inverse law
- C. associative law
- ✓ D. commutative law

MCQ: If A, B and C matrices are of same order and $(A + B) + C = A + (B + C)$, this law is known as

- A. cramer's law
- B. distributive law
- C. commutative law
- ✓ D. associative law


MCQ: We can add or subtract two matrices having real numbers A and B if their

- ✓ A. order is same
- B. rows are same
- C. columns are same
- D. elements are same

$$\begin{bmatrix} 10 & -6 \\ 12 & 0 \end{bmatrix} + \begin{bmatrix} -6 & 2 \\ -5 & 3 \end{bmatrix}$$

is equal to which matrix?

a) $\begin{bmatrix} -4 & 21 \\ 9 & 4 \end{bmatrix}$

 b) $\begin{bmatrix} 4 & -4 \\ 7 & 3 \end{bmatrix}$

c) $\begin{bmatrix} 4 & -12 \\ 12 & -5 \end{bmatrix}$


d) $\begin{bmatrix} 16 & -8 \\ 17 & -3 \end{bmatrix}$

$$Q = \begin{bmatrix} 2 & 1 \\ -1 & 1 \\ 3 & 4 \end{bmatrix} \quad R = \begin{bmatrix} -7 & 3 \\ -4 & 1 \\ 3 & -2 \end{bmatrix}$$

$$Q - R =$$

A. $\begin{bmatrix} 2 & 9 \\ 0 & 3 \\ 6 & 9 \end{bmatrix}$

C. $\begin{bmatrix} -5 & 2 \\ -5 & 0 \\ -9 & -6 \end{bmatrix}$

 B. $\begin{bmatrix} 9 & -2 \\ 3 & 0 \\ 0 & 6 \end{bmatrix}$

D. $\begin{bmatrix} 7 \\ 3 \\ -11 \end{bmatrix}$

$$I_3$$

$$(I_3)^{-1}$$

9. Let I_3 be the Identity matrix of order 3 then $(I_3)^{-1}$ is equal to:

a) 0

b) $3I_3$

✓ c) I_3

d) None of the mentioned.

The multiplication of the matrices $\begin{bmatrix} 3 & 2 & 1 \\ 1 & 5 & 0 \\ 2 & 2 & 1 \end{bmatrix}$ and $\begin{bmatrix} 1 \\ 3 \\ 5 \end{bmatrix}$ gives a

✓ ☒ 3 x 1 matrix.

☐ 1 x 3 matrix.

☐ 1 x 1 matrix.

☐ 3 x 3 matrix.

When you multiply a matrix by the identity matrix, you obtain the

☐ inverse matrix.

☐ the transpose matrix.

☐ adjoint matrix.

☐ cofactor matrix.

✓ ☒ original matrix.

Multiply matrix $\begin{bmatrix} 5 & 3 \\ -3 & -2 \end{bmatrix}$ by $\begin{bmatrix} -4 \\ 3 \end{bmatrix}$

☐ $\begin{bmatrix} -11 & 6 \end{bmatrix}$

☐ $\begin{bmatrix} 11 \\ -6 \end{bmatrix}$

☐ $\begin{bmatrix} -6 & 11 \end{bmatrix}$

✓ ☒ $\begin{bmatrix} -11 \\ 6 \end{bmatrix}$

☐ none of the above

$$\begin{bmatrix} -20 + 9 \\ 12 - 6 \end{bmatrix}$$

MCQ: If A is a matrix of order $(m \times n)$ then a matrix $(n \times m)$ obtained by interchanging rows and columns of A is called the

A ~~/~~ additive inverse of A

✓ B. transpose of A

C. determinant of A

D. order of A

$$B = A^T$$

The matrix $B = A^T$, where A is any matrix is

- a) skew symmetric
- b) symmetric about the secondary diagonal
- ✓ c) always symmetric
- d) not symmetric

MCQ: In matrices $(A + B)^t$ equals to

- A. A^t
- B. B^t
- ✓ C. $A^t + B^t$
- D. $A^t B^t$

MCQ: In matrices $(AB)^t$ equals to

- a) A^t
- b) B^t
- ✓ c) $B^t A^t$
- d) $A^t B^t$

If A and B be real symmetric matrices of size $n \times n$, then

- a) $AA^T = I$
- b) $A = A^{-1}$
- ✓ c) $AB = BA$
- ✓ d) $A = A^t$

$$✓ e) (AB)^T = BA$$

For a matrix A , B and identity matrix I , if a matrix $AB=I=BA$ then:

- a) B is inverse of A
- b) A is inverse of B
- c) $A^{-1} = B$, $B^{-1} = A$
- ✓ d) All of the mentioned

✓ MCQ: In matrices $(AB)^{-1}$ equals to

- a) A^{-1}
- b) $B^{-1} A^{-1}$
- c) $A^{-1} B^{-1}$
- d) B^{-1}

Cramer's rule leads easily to a general formula for:

- a) The adjugate of a matrix A.
- b) The determinant of a matrix A.
- ✓ c) The inverse of $n \times n$ matrix A.
- d) None of the above

✓ **If A is an invertible square matrix then:**

- a) $(A^T)^{-1} = (A^{-1})^T$
- b) $(A^T)^T = (A^{-1})^T$
- c) $(A^T)^{-1} = (A^{-1})^{-1}$
- d) None of the mentioned

The inverse of the matrix $\begin{bmatrix} 5 & 3 \\ 10 & 6 \end{bmatrix}$ is

☐ $\begin{bmatrix} .2 & .33 \\ .1 & .167 \end{bmatrix}$

☐ $\begin{bmatrix} .5 & .3 \\ 1.0 & .6 \end{bmatrix}$

☐ $\begin{bmatrix} .167 & .1 \\ .33 & .2 \end{bmatrix}$

✓ ☒ **undefined.**

☐ none of the above

} 0 - 30

If A, B, C are square matrices of the same order, then $(ABC)^{-1}$ is equal to

a) $C^1 A^{-1} B^{-1}$

✓ ☒ **$C^{-1} B^{-1} A^{-1}$**

c) $A^{-1} B^{-1} C^{-1}$

d) $A^{-1} C^{-1} B^1$

NOTES :

- $(A^T)^T = A$
- $(kA)^T = kA^T$
- $(AB)^T = B^T A^T$
- $(A+B)^T = A^T + B^T$
- If A is nonsingular, then so is A^{-1} and $(A^{-1})^{-1} = A$
- If A and B are nonsingular **matrices**, then AB is nonsingular and.
 $(AB)^{-1} = B^{-1}A^{-1}$
- if A is nonsingular then $(A^T)^{-1} = (A^{-1})^T$
- if A and B are matrices with $AB = I_n$ then A and B are inverses of each other.
- A is nonsingular
- $Ax = 0$ has only the trivial solution
- A is row equivalent to I
- The linear system $Ax = b$ has a unique solution for every $n \times 1$ matrix b .
- When the matrix is invertible , it's inverse is given by the formula:

$$A^{-1} = \frac{1}{ad-bc(\det)} \begin{vmatrix} d & -b \\ -c & a \end{vmatrix}$$

CH 3: DETERMINANTS

MCQ: Value of determinant is computed by adding multiples of one row to

- A. another dimension
- ✓ B. another row
- C. another column
- D. another matrix



In computation of determinant of a matrix, significant efficiencies are introduced by combining row to another which

- A. contains subtraction
- ✓ B. contains zero
- C. contains ones
- D. contains addition

According to determinant properties, when two rows are interchanged then signs of determinant

- ✓ A. must changes
- B. remains same
- C. multiplied
- D. divided

According to determinant properties, determinant equals to zero if row is

- ✓ A. multiplied to row
- B. multiplied to column
- ✓ C. divided to row
- D. divided to column

AccorDing to determinant properties, determinant of resulting matrix equals to $k \Delta$ if elements of rows are

- A. multiplied to constant k
- B. added to constant k
- ✓ C. multiplied to constant k
- D. divided to constant

MCQ: If determinant of a matrix is not equal to zero, then it is said to be

- ✓ **A. non-singular matrix**
- B. square matrix
- C. singular matrix
- D. identical matrix

The value of the determinant $\begin{vmatrix} a & b \\ c & d \end{vmatrix}$ is

- ☐ db - ca.
- ✓ ☒ **da - cb.**
- ☐ dc + ba.
- ☐ db + ca.
- ☐ dc - ba.

✓ **The correct determinant value for the determinant $\begin{vmatrix} 7 & 8 \\ 5 & 2 \end{vmatrix}$ would be**

- ☐ (7)(5) - (8)(2).
- ☐ (7)(2) + (5)(8).
- ☐ (7)(8) - (5)(2).
- ☐ (7)(2)(5)(8).
- ✓ ☒ **(7)(2) - (5)(8).**

Let A be a square matrix of order 3 x 3 with det (A)= 21, then det (2A) is:

- ✓ ☒ **a) 168**
- b) 186
- c) 126
- d) None of the above.

$$2^3 \times 21 = 168$$

✓ **If A and B are square matrices of size n x n, then which of the following statement is not true?**

- a) $\det. (AB) = \det (A) \det (B)$
- b) $\det (kA) = k^n \det (A)$
- ✓ ☒ **c) $\det (A + B) = \det (A) + \det (B)$**
- d) $\det (A^T) = 1/\det (A^{-1})$

$(\text{cofactors})^T$

State True or False:

For a matrix A of order n, the $\det(\text{adj}(A)) = (\det(A))^n$, where $\text{adj}()$ is adjoint of matrix

a) True

✓ b) False

Rule which provides method of solving determinants is classified as:

✓ a) Cramer's rule.

b) Determinant rule.

c) Solving rule.

d) None of the above

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix}$$

$$M_{11} = \begin{vmatrix} 5 & 6 \\ 8 & 9 \end{vmatrix} = (5)(9) - (8)(6) = -3$$

$$C_{ij} = (-1)^{i+j} + M_{ij}$$

$$C_{11} = (-1)^{1+1} + (-3) = -2$$

NOTES :

➤ The adjoint of a matrix A is the transpose of the cofactor matrix. It is denoted by $\text{adj } A$. An adjoint matrix is also called an adjugate matrix.

➤ The **determinant** of a **lower triangular matrix** (or an **upper triangular matrix**) and diagonal matrix is the product of the diagonal entries.

➤ the **determinant** of A is equal to zero then A is not invertible.

↳ singular

CH 4: VECTORS

The two vectors are said to be equivalent if:

- a) Same length.
- b) Same direction.
- ✓ c) Both (a) and (b).
- d) None of the above.

If one of the vectors is a scalar multiple of the other, then the vectors lie on a common line, so such vectors are:

- a) Parallel.
- ✓ b) Collinear.
- c) Perpendicular.
- d) None of the above

Unit vectors are normally used to represent other vector's

- ✓ a) direction
- b) place
- c) velocity
- d) magnitude

The Norm of the vector $v = -3, 2, 1$ is:

- a) 14.
- b) 7.
- ✓ c) $\sqrt{14}$
- d) None of the above.

A vector of norm 1 is called a:

- ✓ e) Unit vector.
- f) Parallel vector.
- g) Zero vectors.
- h) None of the above.

The two vectors ^a $(-2, 1)$ and ^b $(1, 2)$ are...

- a) linearly dependent of each other
- b) forming an orthonormal basis
- ✓ c) perpendicular to each other
- d) pointing in the opposite direction of each other

$$\cos \theta = \frac{|a \cdot b|}{\|a\| \|b\|}$$

Commutative law is valid for the cross product of two vectors.
(Commutative law: $P \times Q = Q \times P$; for two vectors P and Q)

a) True

✓ b) False

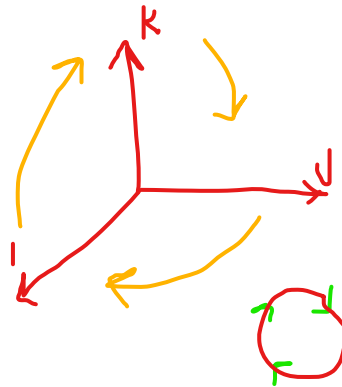
Which of the following is true?

a) $i \times i = 1$

b) $j \times i = -j$

✓ c) $k \times j = -i$

d) $k \times k = 1$



Which of them is not correct?

a) $j \times j = 0$

b) $j \times k = i$

✓ c) $j \times i = k \rightarrow -k$

d) $j \times i = -k$

x zero

✓ What is $\{(i \cdot i) + (-i \cdot j) + (-k \cdot k) + (k \cdot i)\} \cdot (Ai + Bj + Cz)$?

a) 1

b) 0

c) $A + B + C$

d) -1

$$\begin{aligned} x \rightarrow i &\rightarrow (1, 0, 0) \\ y \rightarrow j &\rightarrow (0, 1, 0) \\ z \rightarrow k &\rightarrow (0, 0, 1) \end{aligned}$$

The scalar product (aka dot product) of two perpendicular vectors is

✓ a) 0

b) 1

c) 2π

d) -2π

e) none of the above.

If the angle between two vectors (both having a non-zero magnitude) is greater than 90° and smaller than 270° , then the scalar product (dot product) of these vectors is...

a) positive

✓ b) negative

c) undefined

d) positive when the angle is smaller than 180° , negative when the angle is greater than 180°



If the scalar product (dot product) of two unit vectors is zero, they are...

- a) linearly dependent
- ✓ b) forming an orthonormal basis
- c) pointing in the same direction
- d) at an angle of 180 degrees to each other.

The process of multiplying a nonzero vector by the reciprocal of its length to obtain a unit vector is called:

- a) Triangular vector.
- ✓ b) Normalizing.
- c) Cramer's rule.
- d) None of the above.

If $u = 1, 3, -2, 7$ and $v = 0, 7, 2, 2$ are two vectors then the distance between u and v is:

- a) 28.
- b) 12.
- ✓ c) $\sqrt{58}$
- d) None of the above.

For two vectors A and B , what is $\underline{A \cdot B}$ (if they have angle α between them)?

- ✓ a) $|A||B| \cos \alpha$
- b) $|A||B|$
- c) $\sqrt{(|A||B|)} \cos \alpha$
- d) $|A||B| \sin \alpha$

Mathematically, for two vectors A and B of any magnitude, the cross product of both, i.e. $A \times B$ = given by:

- ✓ a) $|A||B| \sin \theta$
- b) $|A||B|$
- c) $|A||B| \cos \theta$
- d) $|A||B| \sin(180^\circ + \theta)$

✓ What is the Euclidean length $\|v\|$ of vector $v = (0, 3, 4, 0)$?

Answer: $\|v\| = \sqrt{0^2 + 3^2 + 4^2 + 0^2} = \sqrt{25} = 5$

✓ What is the scalar product (or dot product) $v \cdot w$ of the two vectors $v = (0, 5, -2)$ and $w = (3, 1, -2)$?

Answer: $v \cdot w = (0 \cdot 3) + (5 \cdot 1) + (-2 \cdot -2) = 9$

✓ What is the cross product $v \times w$ of the two vectors $v = (0, 5, -2)$ and $w = (3, 1, -2)$?

$$\begin{vmatrix} 0 & 5 & -2 \\ 3 & 1 & -2 \end{vmatrix}$$

Answer: $v \times w = (-8, -6, -15)$

$(-10+2, -(0+6), (0-15))$

Projection of vector A in direction of x-axis is represented by angle of

$$\text{Proj}_x A = \frac{A \cdot (1, 0, 0)}{\|x\|^2} \rightarrow \|A\| \|x\| \cos(\theta)$$

a) Cos

b) Sin

c) Tan

d) both a and b

Orthographic projection represents three dimensional objects in

a. One dimension

✓ b. Two dimension

c. Three dimension

d. All of the above

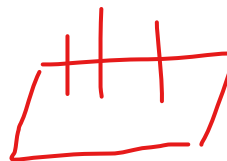
In orthographic projection, the projection lines are _____ to the projection plane.

a. Parallel

✓ b. Orthogonal

c. Inclined

d. Any of the above



The point, from which the observer is assumed to view the object, is called

- ✓ a. Center of projection
- b. Point of projection
- c. Point of observer
- d. View point

NOTES :

- A scalar is a physical quantity with magnitude only.
- A vector is a physical quantity with magnitude and direction.
- Two vectors are equal if they have the **same** magnitude and the **same** direction
- A negative vector is a vector that has the *opposite* direction to the reference positive direction.
- The resultant vector is the single vector whose effect is the same as the individual vectors acting together.
- The equilibrant is the vector which has the *same magnitude* but *opposite direction* to the resultant vector.

COLLECTION OF LAWS

$$U=(u_1, u_2, u_3) \quad V=(v_1, v_2, v_3)$$

$$\triangleright V \cdot U = \|u\| \|v\| \cos \theta.$$

$$\triangleright \cos \theta = \frac{u \cdot v}{\|u\| \|v\|}$$

$$\begin{aligned} \circ U \cdot V > 0 &\rightarrow \cos \theta > 0 \rightarrow 0 \leq \theta < \frac{\pi}{2} \text{ acute} \\ \circ U \cdot V < 0 &\rightarrow \cos \theta < 0 \rightarrow \frac{\pi}{2} < \theta \leq \pi \text{ obtuse} \\ \circ U \cdot V = 0 &\rightarrow \cos \theta = 0 \rightarrow \theta = \frac{\pi}{2} \rightarrow u \perp v \end{aligned}$$

$$\triangleright \|U\| = \sqrt{u_1^2 + u_2^2 + u_3^2}$$

$$\triangleright U \cdot U = (\|u\|)^2 = u^2$$

$$\triangleright \text{proj}_a^u = \frac{u \cdot a}{\|a\|^2} \cdot a$$

$$\circ \|\text{proj}_a^u\| = \left\| \frac{u \cdot a}{\|a\|^2} \cdot a \right\| = \frac{|u \cdot a|}{\|a\|}$$

$$\triangleright \|KU\| = |k| \|u\|$$

$$\triangleright D(u, v) = \|u - v\|$$

$$\triangleright D(u, v) = D(v, u)$$

$$\triangleright D(u, v) \leq D(u, w) + D(w, v)$$

$$\begin{aligned} \triangleright U \times V &= \left(\begin{vmatrix} u_2 & u_3 \\ v_2 & v_3 \end{vmatrix}, \begin{vmatrix} u_1 & u_3 \\ v_1 & v_3 \end{vmatrix}, \begin{vmatrix} u_1 & u_2 \\ v_1 & v_2 \end{vmatrix} \right) \\ &= (u_2 v_3 - v_2 u_3, u_1 v_3 - v_1 u_3, u_1 v_2 - v_1 u_2). \end{aligned}$$

$$U = (u_1 i, u_2 j, u_3 k)$$

$$\circ U \times V = \begin{vmatrix} i & j & k \\ u_1 & u_2 & u_3 \\ v_1 & v_2 & v_3 \end{vmatrix} \rightarrow i, j, k \text{ (متجهات الوحدة القياسية)}$$

$$\triangleright \|U \times V\|^2 = \|u\|^2 \|v\|^2 - (u \cdot v)^2$$

$$\triangleright U \times V = -(V \times U)$$

$$\triangleright U \cdot (U \times V) \rightarrow u \perp (u \times v)$$

- $V \cdot (U \times V) \rightarrow v \perp (u \times v)$
- $U \times U = 0$ (المتجه الصفري)
- $U \times (V \times W) = (U \cdot W)V - (U \cdot V)W$
- $(U \times V) \times W = (U \cdot W)V - (V \cdot W)U$
- $||U + V||^2 = ||u||^2 + ||v||^2 + 2 ||u|| ||v|| \cos\theta$
- $||U - V||^2 = ||u||^2 + ||v||^2 - 2 ||u|| ||v|| \cos\theta$
- $||U + V||^2 = ||U - V||^2 \rightarrow u \perp v$
- $||U \times V|| = \text{area of parallelogram which determined by } u, v$

$$\text{➤ } U \cdot (V \times W) = \begin{vmatrix} u_1 & u_2 & u_3 \\ v_1 & v_2 & v_3 \\ w_1 & w_2 & w_3 \end{vmatrix} = 0 \rightarrow u, w, v \text{ يقعوا في نفس المستوي}$$

$$\text{➤ } Ax + By + Cz = 0 \text{ (معادلة المستوي)}$$

$$\circ a(x-x_0) + b(y-y_0) + c(z-z_0) = 0$$

$$\text{➤ } P_0 = (x_0, y_0, z_0), \quad Ax + By + C = 0$$

$$\circ D = \frac{|ax_0 + by_0 + c|}{\sqrt{a^2 + b^2}}$$

$$\text{➤ } P_0 = (x_0, y_0, z_0), \quad Ax + By + Cz + d = 0$$

$$\circ D = \frac{|ax_0 + by_0 + cz_0 + d|}{\sqrt{a^2 + b^2 + c^2}}$$

$$\text{➤ } U \parallel V \rightarrow u = ku$$

$$\text{➤ } a_1 x + b_1 y + c_1 z + d_1 = 0 \quad n_1 = (a_1, b_1, c_1)$$

$$\text{➤ } a_2 x + b_2 y + c_2 z + d_2 = 0 \quad n_2 = (a_2, b_2, c_2)$$

$$\circ n_1 \parallel n_2 \rightarrow n_1 = kn_2 \rightarrow \frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2} = k$$

$$\circ n_1 \perp n_2 \rightarrow n_1 \cdot n_2 = 0 \rightarrow a_1 a_2 + b_1 b_2 + c_1 c_2 = 0$$