Maths

1 (a)

p	q	r	b(p,q,r)
F	F	F	F
F	F	T	(T)
F	T	F	F
F	T	T	F
T	F	F	(T)
T	F	T	(T)
T	T	F	F
T	T	T	(T)

Rows 1, 4, 5, 7 returns true

Row 1: $\neg p \wedge \neg q \wedge r = T$

Row 4: $p \wedge \neg q \wedge \neg r = T$

Row 5: $p \wedge \neg q \wedge r = T$

Row 7: $p \wedge q \wedge r = T$

$(\neg p \wedge \neg q \wedge r) \vee (p \wedge \neg q \wedge \neg r) \vee (p \wedge \neg q \wedge r) \vee (p \wedge q \wedge r)$

(ii)

$(\neg p \wedge \neg q \wedge r) \vee p \wedge [(\neg q \wedge \neg r) \vee (\neg q \wedge r) \vee (q \wedge r)]$

$(\neg p \wedge \neg q \wedge r) \vee p \wedge [\neg q \wedge (\neg r \vee r) \vee (q \wedge r)]$

$(\neg p \wedge \neg q \wedge r) \vee p \wedge [\neg q \vee (q \wedge r)]$

$\therefore p \wedge [(\neg q \wedge q) \vee (\neg q \wedge r)]$

$(\neg p \wedge \neg q \wedge r) \vee p \wedge (\neg q \wedge r)$

1(b)

$$(\neg p \wedge \neg q \wedge r) \vee (p \wedge \neg q \wedge r) \wedge (r \vee \neg b \vee d) \wedge (r \vee b \vee \neg d) \vee (r \vee b \vee d)$$

$$\neg q \wedge [(p \wedge r) \vee (p \vee r)] \vee b$$

$$\neg q \wedge (p \wedge q \wedge r) \vee (\neg r \vee b \vee \neg d) \wedge (d \vee q \wedge r) \vee b$$

$$\neg q \wedge (p \wedge r) \vee (r \vee d) \vee b$$

$$\boxed{\neg q \wedge (p \wedge r) \vee (r \vee d) \vee b}$$

2(a)(i) $\models ((p \to q) \to p) \to p$

p	q	$((p \to q) \to p) \to p$
F	F	T F T
F	T	T F T
T	F	F T T
T	T	T T T

(ii) $p \vee q \models (p \to q) \to q$

p	q	$p \vee q \models (p \to q) \to q$
F	F	F T T F
F	T	T T T T
T	F	T T F T
T	T	T T T T

(iii) $\neg(p \vee q) = \neg p \wedge \neg q$

DeMorgans Law: $\models \neg(p \vee q) \equiv \neg p \wedge \neg q$

p	q	$\neg (p \vee q) \equiv \neg p \wedge \neg q$
F	F	T F T T T T
F	T	F T T T F F
T	F	F T T F F T
T	T	F T T F T F

(iv) $p \to q = (q \equiv p \vee q)$

$\models p \to q \equiv (q \equiv p \vee q)$

p	q	$p \to q \equiv (q \equiv p \vee q)$
F	F	T T F T F
F	T	T T T T T
T	F	F T F F T
T	T	T T T T T

(b)(i) $p \wedge (q \equiv r) \equiv (p \wedge q) \equiv (p \wedge r)$

p	q	r	$p \wedge q \equiv r$	$\equiv$	$(p \wedge q)$	$\equiv$	$(p \wedge r)$
F	F	F	F	T	F	T	F
F	F	T	F	F	F	T	F
F	T	F	F	T	F	T	F
F	T	T	F	F	F	T	F
T	F	F	T	T	F	T	F
T	F	T	F	F	F	F	T
T	T	T	T	T	T	T	T
T	T	F	F	F	T	F	F

FALSE

(b) $p \rightarrow q \rightarrow r \equiv (p \rightarrow q) \rightarrow r$

p	q	r	$p \rightarrow q \rightarrow r$	$\equiv$	$(p \rightarrow q) \rightarrow r$
F	F	F	T	T	F
F	F	T	T	T	T
F	T	F	T	T	F
F	T	T	T	T	T
T	F	F	F	T	F
T	F	T	F	T	F
T	T	F	T	T	F
T	T	T	T	T	T

~~TRUE~~

FALSE