

Question 1:

about $\frac{\pi}{3}$ for $g(x) = \tan x$

$$y = f'(a)(x-a) + f(a)$$

$$y = f'\left(\frac{\pi}{3}\right)\left(x - \frac{\pi}{3}\right) + f\left(\frac{\pi}{3}\right)$$

$$y = \frac{1}{\cos^2\left(\frac{\pi}{3}\right)}\left(x - \frac{\pi}{3}\right) + \tan\left(\frac{\pi}{3}\right)$$

$$y = 4\left(x - \frac{\pi}{3}\right) + \sqrt{3}$$

$$g(x) = \tan x$$

$$g'(x) = \sec^2 x$$

or

$$\frac{1}{\cos^2 x}$$

Question 2:

$$p(x) = f(\pi) + f'(\pi)(x-\pi) + \frac{f''(\pi)}{2!}(x-\pi)^2 + \frac{f'''(\pi)}{3!}(x-\pi)^3$$

$$f(x) = \sin(2x)$$

$$f'(x) = 2\cos(2x)$$

$$f''(x) = -4\sin(2x)$$

$$f'''(x) = -8\cos(2x)$$

$$\frac{f'''(\pi)}{3!} = \frac{-8\cos(2\pi)}{3!} = -\frac{4}{3}$$

Question 3:

$$p(x) = f(-3) + f'(-3)(x+3) + \frac{f''(-3)}{2!}(x+3)^2 + \frac{f'''(-3)}{3!}(x+3)^3$$

$$f(x) = (x+1)^{-2}$$

$$f'(x) = -\frac{2}{(x+1)^3}$$

$$f''(x) = \frac{6}{(x+1)^4}$$

$$f'''(x) = -\frac{24}{(x+1)^5}$$

$$\frac{f'''(-3)}{3!} = \frac{-\frac{24}{(-3+1)^5}}{3!} = \frac{1}{8}$$