

Question 1:

$$\begin{aligned} x + 3y + z &= 0 \\ -x + y + z &= 0 \\ x + 5y + 2z &= 0 \end{aligned} \quad = \quad \begin{pmatrix} 1 & 3 & 1 & 0 \\ -1 & 1 & 1 & 0 \\ 1 & 5 & 2 & 0 \end{pmatrix}$$

$$R_2 = R_2 + R_1$$

$$\begin{pmatrix} 1 & 3 & 1 & 0 \\ 0 & 4 & 2 & 0 \\ 1 & 5 & 2 & 0 \end{pmatrix}$$

$$R_3 = R_3 - R_1$$

$$\begin{pmatrix} 1 & 3 & 1 & 0 \\ 0 & 4 & 2 & 0 \\ 0 & 2 & 1 & 0 \end{pmatrix}$$

$$R_2 = R_2 \div 4$$

$$\begin{pmatrix} 1 & 3 & 1 & 0 \\ 0 & 1 & \frac{1}{2} & 0 \\ 0 & 2 & 1 & 0 \end{pmatrix}$$

$$R_3 = R_3 - 2R_2$$

$$\begin{pmatrix} 1 & 3 & 1 & 0 \\ 0 & 1 & \frac{1}{2} & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

$$R_1 = R_1 - 3R_2$$

$$\begin{pmatrix} 1 & 0 & \frac{1}{2} & 0 \\ 0 & 1 & \frac{1}{2} & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

The equation has infinite solutions

→ Introducing Parameters

$$\text{Equation 1} = x_1 + \frac{1}{2}x_3 = 0$$

$$\text{Equation 2} = x_2 + \frac{1}{2}x_3 = 0$$

$$x_3 = t$$

$$\text{Equation 1} = x_1 = -\frac{1}{2}t$$

$$\text{Equation 2} = x_2 = -\frac{1}{2}t$$

Question 2:

$$\begin{pmatrix} 1 & -1 & 1 \\ 1 & 1 & 2 \\ 1 & 2 & 2 \end{pmatrix}$$

$$\begin{array}{l} R_2 = R_2 - R_1 \\ R_3 = R_3 - R_1 \end{array} \begin{pmatrix} 1 & -1 & 1 & | & 1 & 0 & 0 \\ 0 & 2 & 1 & | & -1 & 1 & 0 \\ 0 & 3 & 1 & | & -1 & 0 & 1 \end{pmatrix}$$

$$R_2 = R_2 \div 2$$

$$\begin{pmatrix} 1 & -1 & 1 & | & 1 & 0 & 0 \\ 0 & 1 & 0.5 & | & -0.5 & 0.5 & 0 \\ 0 & 3 & 1 & | & -1 & 0 & 1 \end{pmatrix}$$

$$\begin{array}{l} R_1 = R_1 + R_2 \\ R_3 = R_3 - 3R_2 \end{array} \begin{pmatrix} 1 & 0 & 1.5 & | & 0.5 & 0.5 & 0 \\ 0 & 1 & 0.5 & | & -0.5 & 0.5 & 0 \\ 0 & 0 & -0.5 & | & 0.5 & -1.5 & 1 \end{pmatrix}$$

$$R_3 = R_3 \div 0.5$$

$$\begin{pmatrix} 1 & 0 & 1.5 & | & 0.5 & 0.5 & 0 \\ 0 & 1 & 0.5 & | & -0.5 & 0.5 & 0 \\ 0 & 0 & 1 & | & -1 & 3 & -2 \end{pmatrix}$$

$$R_1 = R_1 - 1.5R_3$$

$$\begin{pmatrix} 1 & 0 & 0 & | & 2 & -4 & 3 \\ 0 & 1 & 0.5 & | & -0.5 & 0.5 & 0 \\ 0 & 0 & 1 & | & -1 & 3 & -2 \end{pmatrix}$$

$$R_2 = R_2 - 0.5R_3$$

$$\begin{pmatrix} 1 & 0 & 0 & | & 2 & -4 & 3 \\ 0 & 1 & 0 & | & 0 & -1 & 1 \\ 0 & 0 & 1 & | & -1 & 3 & -2 \end{pmatrix}$$

← inverse of the equation

Question 3:

$$2x + 3y = 11$$

$$3x + 2y = 11$$

$$D = \begin{vmatrix} 2 & 3 \\ 3 & 2 \end{vmatrix}$$

$$= 2(2) - 3(3)$$

$$= -5$$

$$D_x = \begin{vmatrix} 11 & 3 \\ 11 & 2 \end{vmatrix}$$

$$= 11(2) - 11(3)$$

$$= -11$$

$$D_y = \begin{vmatrix} 2 & 11 \\ 3 & 11 \end{vmatrix}$$

$$= 2(11) - 3(11)$$

$$= -11$$

$$x = \frac{D_x}{D} = \frac{-11}{-5} = \frac{11}{5}$$

$$y = \frac{D_y}{D} = \frac{-11}{-5} = \frac{11}{5}$$