Week 3 Exercises

1. Let the set operator, $\hat{}$, be defined so that:

$$X \hat{} Y = X \cup \overline{Y}$$

where $\overline{X}$ is the complement of X.
Determine by Karnaugh Maps whether:

   (a) $A \hat{} B = \overline{\overline{A} \cap B}$

   (b) $A \cap B = \overline{\overline{A} \hat{} B}$

   (c) $A \hat{} (B \cap C) = (A \hat{} B) \hat{} C$

   (d) $A \hat{} (B \cup C) = (A \hat{} B) \cap (A \hat{} C)$

   (e) $(A \cap B) \hat{} C = (A \hat{} C) \cap (B \hat{} C)$

2. Show $Y \subseteq X \equiv X \cap Y = Y$

   i.e. Show

   (a) $Y \subseteq X \rightarrow X \cap Y = Y$
   i.e. Assume $Y \subseteq X$ show
      i. $X \cap Y \subseteq Y$
      ii. $Y \subseteq X \cap Y$

   (b) $X \cap Y = Y \rightarrow Y \subseteq X$
   i.e. Assume $X \cap Y = Y$
   show $Y \subseteq X$.

3. Among 73 students, 20 play the guitar, 52 play the piano and 25 play the violin..
Also,
17 play both the piano and the violin,
12 play the piano and guitar,
7 play the violin and guitar,
but only one student can play all three instruments.

   (a) How many students do not play any of these instruments.

   (b) How many students play the violin but not the piano.