

CS1003 Mathematics, Taylor Polynomials Tutorial Sheet

- Q1. Find the linear Taylor polynomial about 1 for the function $f(x) = \frac{1}{x^2}$.
- Q2. Find the linear Taylor polynomial about 0 for the function $f(x) = (8+x)^{\frac{1}{3}}$. Hence find an approximate value for $\sqrt[3]{8.01}$ and use your calculator to find the associated remainder to six decimal places.
- Q3. Find the linear Taylor polynomial about 0 for each of the functions below. Use the polynomial to find an approximation for $f(0.02)$, and use your calculator to find the value of the associated remainder to eight decimal places.
- (a) $f(x) = e^{-x}$
 - (b) $f(x) = (4-x)^{\frac{1}{2}}$ ($x < 4$)
- Q4. Find the linear Taylor polynomial about 1 for each of the functions below:
- (a) $f(x) = e^{-x}$
 - (b) $f(x) = \frac{1}{1+x}$
- Q5. (a) Use the linear Taylor polynomial about 1 from Q4 above to find an approximate value for the reciprocal of 2.01. Use your calculator to find the value of the associated remainder to eight decimal places.
- (b) Find the linear Taylor polynomial about 0 for $f(x) = \frac{1}{(1+x)^3}$. Use your answer to find an approximate value for the reciprocal of 1.01. Use your calculator to find the value of the associated remainder to eight decimal places.
- Q6. Find the quadratic Taylor polynomial about 0 for each of the functions below. Use the polynomial to find an approximation for $f(0.01)$, and use your calculator to find the value of the associated remainder to eight decimal places.
- (a) $f(x) = e^{\frac{x}{2}}$
 - (b) $f(x) = x \cos x$
- Q7. (a) Show that the linear Taylor polynomial about 0 for the function

$$f(x) = (1+x)^k, \quad \text{where } k > 0,$$

is

$$p(x) = 1 + kx.$$

- (b) Use the polynomial p from part (a) with $k = \frac{1}{2}$ and $x = 0.01$ to find an approximate value for $\sqrt{1.01}$. Use your calculator to find, to six decimal places, the value of the associated remainder.

Q8. Find the quadratic Taylor polynomial about 0 for the function $f(x) = \tan x$. Hence find an approximate value for $\tan(-0.1)$, and use your calculator to find the associated remainder to six decimal places.

Q9. Find the Taylor polynomial of degree n about 0 for the function

$$f(x) = \frac{1}{1-x}.$$

Q10. Find the cubic Taylor polynomial about $\frac{1}{6}\pi$ for the function $f(x) = \sin x$.

Q11. Find the quintic Taylor polynomial for each of the following functions about the given point

- (a) $f(x) = x^6$ about 1
- (b) $f(x) = \ln(1-x)$ about 0
- (c) $f(x) = \sin x$ about $\frac{1}{4}\pi$
- (d) $f(x) = (1+x)^{\frac{1}{2}}$ about 0

Q12. By expressing 10 as $9(1 + \frac{1}{9})$ and using your result from the previous question, evaluate $\sqrt{10}$ to three decimal places.

Q13. Find the quartic Taylor polynomial about 0 for each of the functions below.

- (a) $f(x) = \cos(2x)$
- (b) $f(x) = \ln\left(\frac{1}{1+x}\right)$
- (c) $f(x) = \sqrt{1-x}$

- Q14. (a) Find the quintic Taylor polynomial about π for the function $f(x) = \cos x$.
- (b) Find the quintic Taylor polynomial about e for the function $f(x) = \ln x$.

Q15. (a) Given that the Taylor polynomial of degree n about 0 for the function $f(x) = \ln(1+x)$ is

$$p_n(x) = x - \frac{1}{2}x^2 + \frac{1}{3}x^3 - \frac{1}{4}x^4 + \dots + \frac{(-1)^{n+1}}{n}x^n,$$

for $n = 1, 2, 3, \dots$ calculate the value of $\ln(1.05)$ to four decimal places.

- (b) Given that the Taylor polynomial of degree $2n$ about 0 for the function $f(x) = \cos(2x)$ can be expressed as

$$p_{2n}(x) = 1 - \frac{(2x)^2}{2!} + \frac{(2x)^4}{4!} - \dots + (-1)^n \frac{(2x)^{2n}}{(2n)!},$$

for $n = 0, 1, 2, \dots$, calculate the value of $\cos(0.2)$ to four decimal places.