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Question 1:

$$A = \begin{pmatrix} 1 & 2 & 1 \\ 3 & 4 & 2 \\ 2 & 4 & 3 \end{pmatrix}$$ find inverse of A

$$A^{-1} = \frac{1}{\det A}(\tilde{A})^T$$

$$\det A = +1\begin{vmatrix} 4 & 2 \\ 4 & 3 \end{vmatrix} - 2\begin{vmatrix} 3 & 2 \\ 2 & 3 \end{vmatrix} + 1\begin{vmatrix} 3 & 4 \\ 2 & 4 \end{vmatrix}$$

$$= 4(3) - 2(4) - 2[3(3) - 2(2)] + 3(4) - 4(2)$$

$$= -2$$

$$\frac{1}{\det A} = \frac{1}{-2}$$

$$\hat{A} = \begin{pmatrix} \begin{vmatrix} 4 & 2 \\ 4 & 3 \end{vmatrix} & \begin{vmatrix} 3 & 2 \\ 2 & 3 \end{vmatrix} & \begin{vmatrix} 3 & 4 \\ 2 & 4 \end{vmatrix} \\ \begin{vmatrix} 2 & 1 \\ 4 & 3 \end{vmatrix} & \begin{vmatrix} 1 & 1 \\ 2 & 3 \end{vmatrix} & \begin{vmatrix} 1 & 2 \\ 2 & 4 \end{vmatrix} \\ \begin{vmatrix} 2 & 1 \\ 4 & 3 \end{vmatrix} & \begin{vmatrix} 1 & 1 \\ 3 & 2 \end{vmatrix} & \begin{vmatrix} 1 & 2 \\ 3 & 4 \end{vmatrix} \end{pmatrix} = \begin{pmatrix} 4 & -5 & 4 \\ -2 & 1 & 0 \\ 0 & 1 & -2 \end{pmatrix}$$

$$\tilde{A}^T = \begin{pmatrix} 4 & -2 & 0 \\ -5 & 1 & 1 \\ 4 & 0 & -2 \end{pmatrix}$$

$$A^{-1} = \frac{1}{-2}\begin{pmatrix} 4 & -2 & 0 \\ -5 & 1 & 1 \\ 4 & 0 & -2 \end{pmatrix} = \begin{pmatrix} -2 & 1 & 0 \\ 5/2 & -1/2 & -1/2 \\ -2 & 0 & 1 \end{pmatrix}$$

Question 2:

Let
$$A = \begin{pmatrix} 6 & 2 & 0 \\ 2 & 3 & 0 \\ 0 & 0 & -1 \end{pmatrix}$$

Find $\det(A - \lambda I)$
$$\begin{vmatrix} 6-\lambda & 2 & 0 \\ 2 & 3-\lambda & 0 \\ 0 & 0 & -1-\lambda \end{vmatrix}$$

$$= (6-\lambda)\begin{vmatrix} 3-\lambda & 0 \\ 0 & -1-\lambda \end{vmatrix} - 2\begin{vmatrix} 2 & 0 \\ 0 & -1-\lambda \end{vmatrix} + 0\begin{vmatrix} 2 & 3-\lambda \\ 0 & 0 \end{vmatrix}$$
$$= 6\lambda^2 - 12\lambda - 18 - \lambda^3 + 2\lambda^2 + 3\lambda + 4 + 4\lambda$$
$$= -\lambda^3 + 8\lambda^2 - 5\lambda - 14$$

The characteristic equation is
$$\lambda^3 - 8\lambda^2 + 5\lambda + 14$$

Eigenvalues : $\lambda = -1$ or $\lambda = 2$ or $\lambda = 7$

For $\lambda = -1$
$$\left(\begin{array}{ccc|c} 7 & 2 & 0 & 0 \\ 2 & 4 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right)$$

$R1 \div 7$
$R2 - 2R1$
$$\left(\begin{array}{ccc|c} 1 & 2/7 & 0 & 0 \\ 0 & 24/7 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right)$$

$R1 - \frac{2}{7}R2$
$R2 \div 24/7$
$$\left(\begin{array}{ccc|c} 1 & 2/7 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right)$$

infinite solutions for $\lambda = -1$

parameters

$x_1 = 0$

$x_2 = 0$ $\quad x = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} t$ for $t \in \mathbb{R} \setminus \{0\}$ Eigenvectors for $\lambda = -1$

$x_3 = t$

For $\lambda = 2$

$$\left(\begin{array}{ccc|c} 4 & 2 & 0 & 0 \\ 2 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{array}\right)$$

$R1 \div 4$

$R2 - 2R1$

$$\left(\begin{array}{ccc|c} 1 & \frac{1}{2} & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{array}\right)$$

swap R3 and R2

$$\left(\begin{array}{ccc|c} 1 & \frac{1}{2} & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right)$$

infinite solutions for $\lambda = 2$

parameters

$x_1 + \frac{1}{2}x_2 = 0$

$x_3 = 0$

$t = x_2$

$x_1 + \frac{1}{2}t = 0$

$x_1 = -\frac{1}{2}t$

$x = \begin{pmatrix} -\frac{1}{2} \\ 1 \\ 0 \end{pmatrix} t$ for $t \in \mathbb{R} \setminus \{0\}$ Eigenvectors for $\lambda = 2$

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For $\lambda = 7$

$$\left(\begin{array}{ccc|c} -1 & 2 & 0 & 0 \\ 2 & -4 & 0 & 0 \\ 0 & 0 & -8 & 0 \end{array}\right)$$

$R1 \div -1$
$R2 - 2R1$
$$\left(\begin{array}{ccc|c} 1 & -2 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & -8 & 0 \end{array}\right)$$

$R3 \div -8$
$$\left(\begin{array}{ccc|c} 1 & -2 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{array}\right)$$

swap R3 and R2
$$\left(\begin{array}{ccc|c} 1 & -2 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right)$$

infinite solutions for $\lambda = 7$

parameters

$$x_1 - 2x_2 = 0$$
$$x_3 = 0$$

$x_2 = t$

$x_1 = 2t$

$$x = \begin{pmatrix} 2 \\ 1 \\ 0 \end{pmatrix} t \quad \text{for } t \in \mathbb{R} \setminus \{0\}$$

eigenvectors for $\lambda = 7$