Trinity College Dublin
Coláiste na Tríonóide, Baile Átha Cliath
The University of Dublin

05 – Floating Point Numbers

CS1022 – Introduction to Computing II

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D.A.Patterson, J.L.Hennessy, "Computer Organisation and Design: ARM Edition", Morgan-Kaufmann, 2016.

Section 3.5: Floating Point

(Available in the Library, doesn't have to be the ARM Edition!!)

Binary Number Representation

32-bits … 2^{32} unique values that we can use to represent different things

e.g. unsigned integers

0 … 2^{32}-1 or

0 … 4,294,967,295

e.g. signed integers using 2's complement

-2^{31} … 0 … $+2^{31}$-1

-2,147,483,648 … 0 … +2,147,483,647

How do we represent **real** numbers like 2½ or 3.14159265… ?

How do we represent values with really large or really small magnitudes?

e.g. 2.2×10^{11}

e.g. 1.3×10^{-8}

Scientific notation – decimal

The values 2.2×10^{11} and 1.3×10^{-8} are examples of (normalized) scientific notation in decimal form

$$f \times 10^{e}$$

Values expressed in normalised scientific notation satisfy the condition:

$$1 \leq |f| < 10$$

Normalized scientific notation give us one *canonical form* in which to express a value using scientific notation and allows quick, visual comparison of magnitude

Generally, as computer scientists, we avoid expressing the same thing in different ways (a==b?)

Binary Floating-Point Numbers

Convert the following binary numbers to decimal numbers with fractions

10010101

1.1

101000.01

Convert the following decimal numbers to binary floating point numbers

10½

5¼

7.75

2.1

Scientific notation –binary

Like decimal values, we can express binary values using scientific notation (again, in normalized form)

e.g.

$1010.1 = 1.0101 \times 2^3$

$0.00101 = 1.01 \times 2^{-3}$

The general form is again:

$$f \times 2^e$$

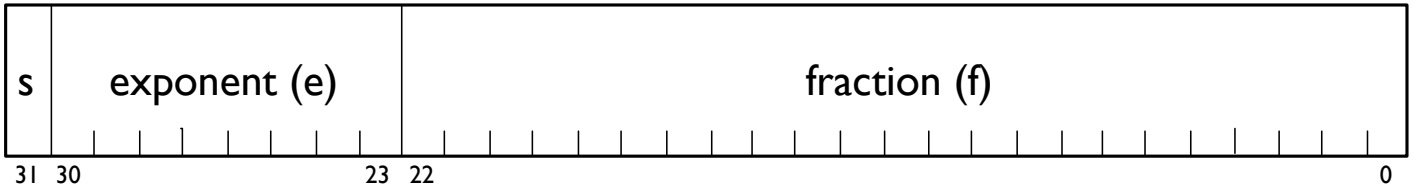
and in normalised form, f satisfies the following condition:

$$1_2 \leq |f| < 10_2$$

The normalized form of a binary number expressed using scientific notation forms the basis for its representation in a computer

IEEE 754 Floating-Point Representation

Use a different interpretation of a 32-bit value to represent floating point numbers, e.g. IEEE 754



$$(-1)^s \times f \times 2^e$$

How can we represent ...

... positive and negative values?

... values with positive and negative exponents?

Where is the binary (radix) point?

Sign of Exponent and Fraction

Sign bit?

0 ⇒ positive floating-point number

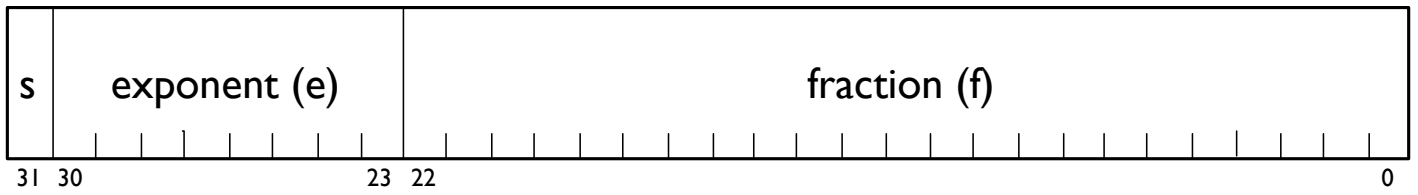
1 ⇒ negative floating-point number

Positive and negative exponents?

Option 1: 2's Complement exponents ✗

Option 2: Biased exponents ✓

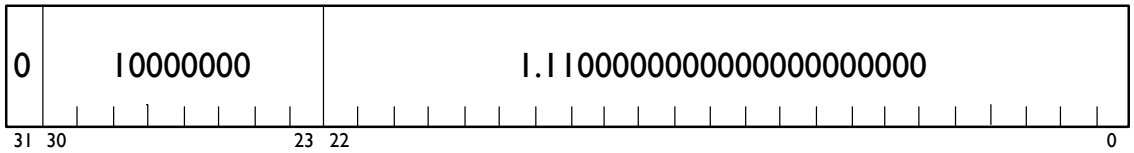
Subtract a constant bias (b = 127) from stored exponent to obtain signed exponent



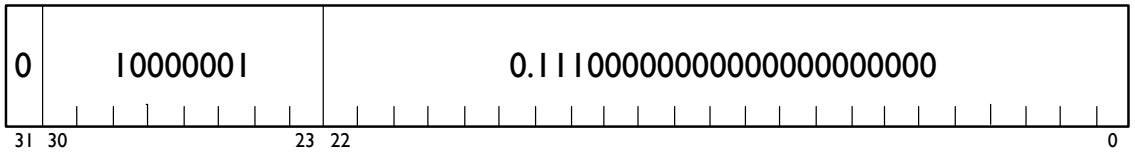
$$(-1)^s \times f \times 2^{e-b}$$

Storing the fraction

The following two representations are of the same value (3.5_{10})



$+1.11 \times 2^{(128 - 127)} = 11.1_2 = 3.5_{10}$



$+0.111 \times 2^{(129 - 127)} = 11.1_2 = 3.5_{10}$ (same value!)

We don't want multiple representations of the same value!

if (a == b) ...

Storing floating-point numbers in normalized form avoids this problem:

$$1 \leq |f| < 2, \text{ so } f \text{ is in the form } 1.ddddd\ldots$$

Normalization and the *Hidden Bit*

With normalisation

$$0.0101 \times 2^{-4}$$

... becomes ...

$$1.0100 \times 2^{-6}$$

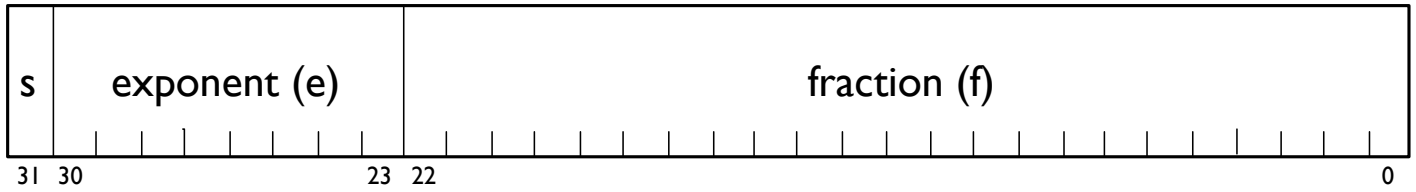
adjust fraction so there is a single 1 to left of radix point

compensate by adjusting exponent accordingly

If there is always going to be a 1 to the left of the radix point, we don't need to store it!

Increases precision (by one bit) – similar to not storing the 2 LSBs of a branch target offset!

Final IEEE 754 Floating-Point Representation



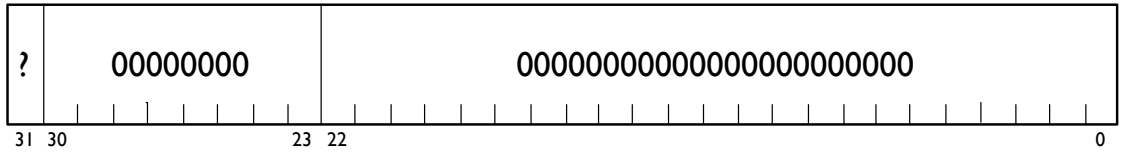
$$(-1)^s \times 1.f \times 2^{(e-b)}$$

Examples?

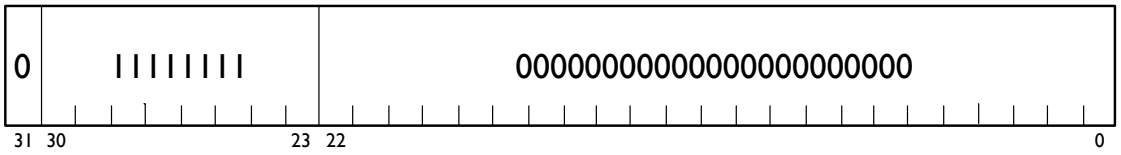
Special Values

Special bit patterns, e.g.

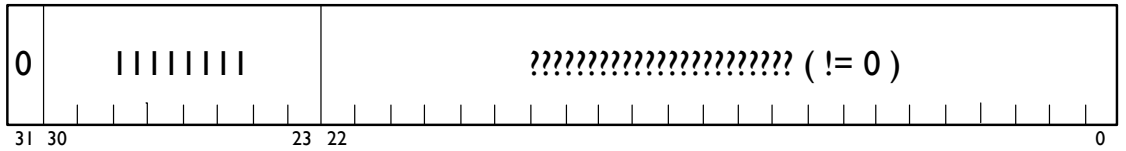
Zero (±)



Infinity (±)

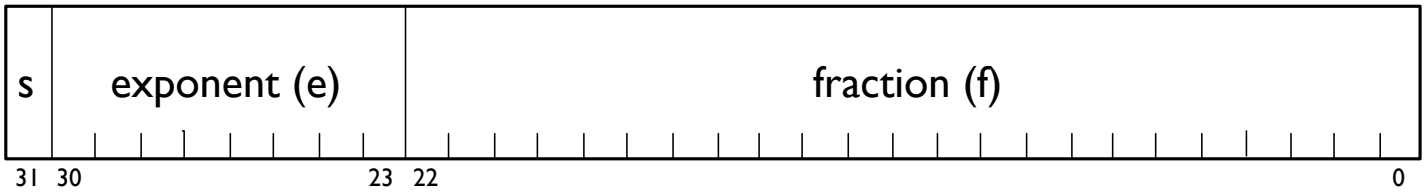


Not a Number (NaN)

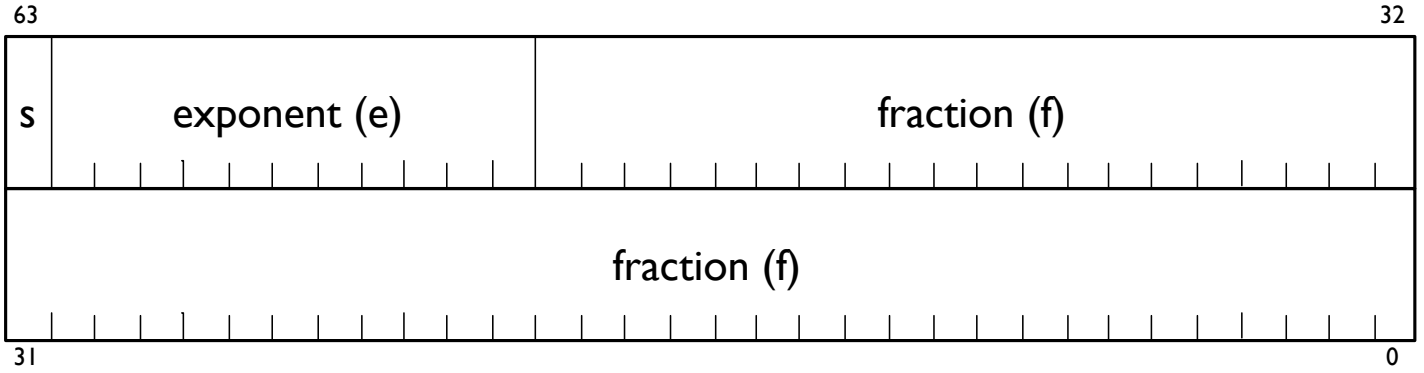


Single and Double Precision

32-Bit Single Precision (bias = 127)



64-Bit Double Precision (bias = 1023)



Floating Point Addition

We can add the fractions of two floating point values if their exponents are the same

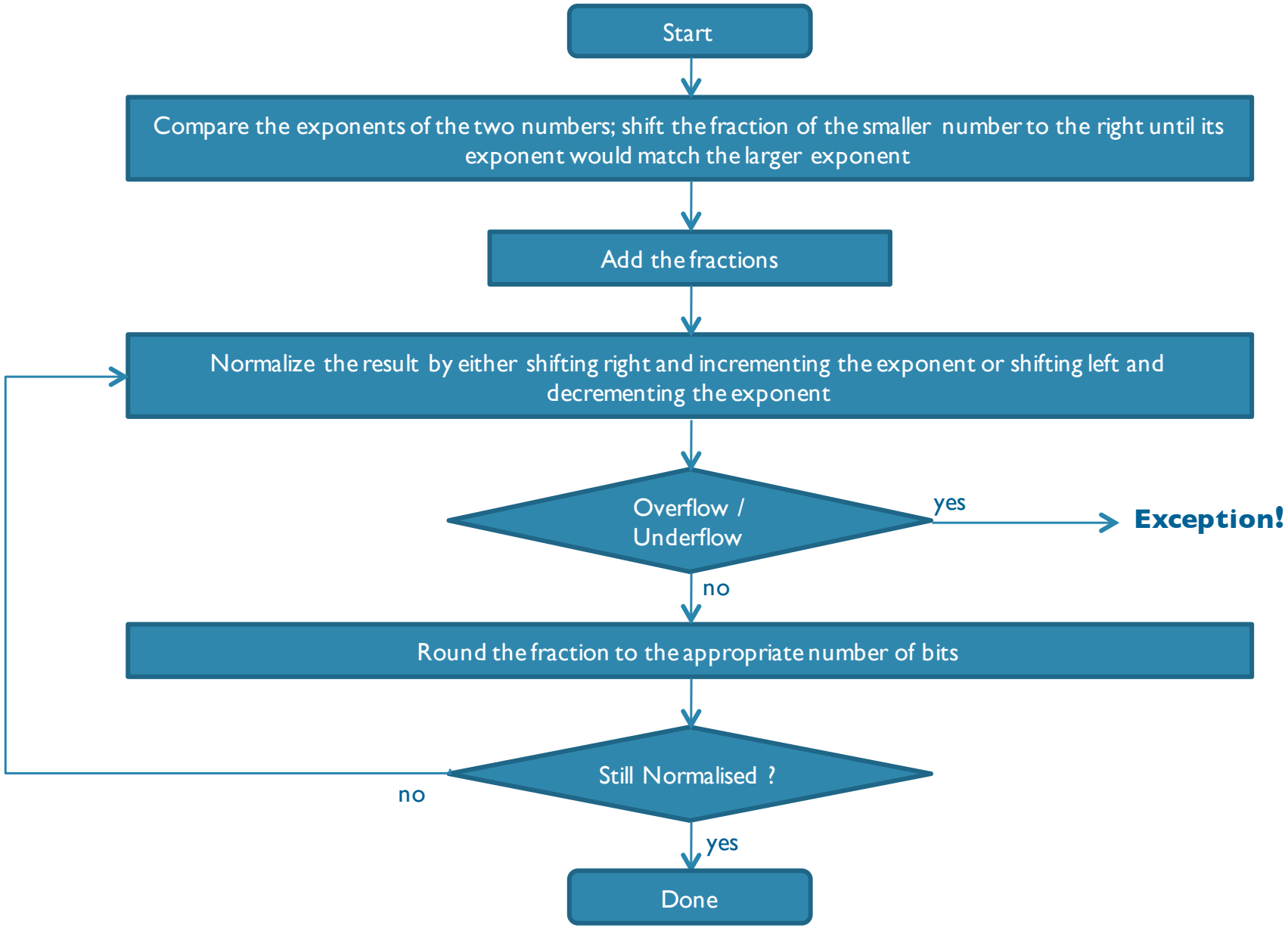
If their exponents are not the same to begin, shift the fraction of the value with the smaller exponent to compensate

e.g.

$1.01101 \times 2^3 + 1.00110 \times 2^{-2}$

$= 1.01101 \times 2^3 + 0.0000100110 \times 2^3$

$= 1.0111000110 \times 2^3$


Start
Compare the exponents of the two numbers; shift the fraction of the smaller number to the right until its exponent would match the larger exponent
Add the fractions
Normalize the result by either shifting right and incrementing the exponent or shifting left and decrementing the exponent
Overflow / Underflow
yes
Exception!
no
Round the fraction to the appropriate number of bits
Still Normalised ?
no
yes
Done

Floating Point Addition

What about adding negative values (S==1)?

Proceed as before but before adding, the fractions of values with S==1 should be converted to their 2's Compliment

A Closer Loo

By Jack Robertson

Warning: Intel Inside

WASHINGTON — Intel forgot the saw, "Bad news travels fast." The ports of the Pentium bug are far out ing Intel's attempts to contain the Intel's initial spin on the situatio the Pentium flaw is a once-in-a- time quirk affecting only a few l haired rocket scientists—was wr The bug impacts any floating-p Pentium user, including legions spreadsheet and financial users, possibly those running encryption, s ulation, and indus- trial control appli- cations. In short, the computing world is only now waking up to the Pentium booby- trap and its risk in calculations.

It is true that the Pentium er- rors—triggered by a limited combina- tion of numbers—occur very quently. But for legions of fi security, and real-time control ing applications, even the risk spurious errors can be devast

So far, Intel's addressing of t market has been bumbling at best.

F

FRIDAY
DECEMBER 9, 1994
San Jose Mercury News

BUSINESS

MARKETS ◆ HIGH TECH ◆ ECONOMY

TALKING BUSINESS

"The yield curve has had so many contractions that it's about to give birth. The birth ... might be recession."

DIRK VAN DIJK, equity strategist

Intel to work on software patch for Pentium bug

Quick fix: Progr... will work around the ...

Grove says I sorry about Pentium bu

By Dean Takahashi
Mercury News Staff Writer

Andrew S. Grove, chief executive of apologized Monday to the nation's scient nity for a flaw in the company's flagsh

Flawed Chip Bruises Intel

Investors react, stock plunges

Associated Press

New York

Intel Corp. stock dropped sharply yesterday after a minor flaw in the company's Pentium microprocessor triggered a rash of negative publicity for the world's largest chipmaker.

The problem, fixed months ago, received national attention Thursday after two weeks of discussion by researchers and mathematicians on the Internet, the global computer network, and some coverage in high-technology industry publications.

Reacting for the first time, investors sent Intel's stock down 2 percent, or 1¼, to 63⅝ on the Nasdaq stock market, which was closed for Thanksgiving on Thursday.

But analysts dismissed the idea of a long-term problem for the Santa Clara company.

"Interesting but irrelevant," said David Wu, an analyst with S.G. Warburg. "These things are quickly forgotten. People won't remember it by Monday."

The flaw occurred in early versions of the Pentium. Once every few billion calculations, the chip might produce a wrong answer to a complex division problem — one with at least nine digits to the right of the decimal point.

Analysts estimate about 2 mil...

INTEL'S MISTAKE

Daily closing price of Intel stock (INTC:NASDAQ)

Source: Datastream International

Pentium woes continue

Faulty FPU flubs math in certain equations

By Brooke Crothers

ADVANCED MATH DOESN'T SEEM TO BE Intel Corp.'s strong suit.

An estimated 2 million units of the company's Pen- ...

Multithreading gets lost on P100 systems

By Brooke Crothers

INTEL CORP. PROBABLY FOUND it difficult to find something to be thankful for last week.

On top of its floating-point-unit problems (see relat- ... recently, multithreading has emerged as another ...

DECEMBER 5, 1994 VOLUME 16, ISSUE 49 Founded in 1978

INFO WORLD

The Voice of Personal Computing in the Enterprise

Photo finish: Although Photoshop outdoes Picture P... has an in...

Flawe...

■ Pentiums wi...

By Brooke Crothers and Bob Francis

INTEL CORP. IS NOW chip-deep in the Pentium controversy with last ... floating-point unit (FPU), a special processor built into the Pentium for accelerating these calculations. (See "Pentium woes continue," Nov. 28, ...

Bug Dodge Booed

Intel Knocked For Response On Flaw

By Jonathan Cassell

SAN MATEO, CALIF. — Amid criticism of how it has handled a flaw in an early version of its Pentium microprocessor, Intel Corp.'s problems mounted last week as reports surfaced that the bug could be worse than the company has claimed.

The division flaw occurs in normal spreadsheet calculations up to the third significant digit, according to a report issued by Mark Edelstone, an analyst with Prudential Securities Research, San Francisco. Intel has claimed the problem could appear in equations that have been calculated out to nine digits beyond the decimal point. Such instances arise only in obscure mathematical operations for scientific equations and should affect the average user once every 27,000 years, according to Intel.

But Edelstone said the error is far more common than Intel claims. "We believe the

[to page 60]

Previous problems with 386es and 486es were "so esoteric that it didn't matter," said Thomas Nicely, a mathematics professor at Lynch- ...

San Francisco Chronicle

BUSINESS

CITYLINE: 24-Hour
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■ Stock Quotes, enter 9

Intel's Pentium Problem Persi...

By David Einstein
Chronicle Staff Writer

Business Machines began ... up phone lines for users ...

IBM to stop shipping Pentium PCs

ERSONAL TECHNOLOGY

By Walter S. Mossberg

't Serving Who Bought ium Campaign

...esn't seem to want us ...nical computer users, ...puter-buying rabble, to ...dea that we might want ...d replacements for the ...pa in PCs millions of us ...t recently, even though ...any admits they're ...But you might want to ...ew chip anyway.

...spent the past few years ...s massive consumer campaign designed to ...ime and the name of its ...d Pentium computer chip ...ords. But over the past ...n it was forced to disclose ... the Pentium chip that ...do some math calcula- ..., Intel has done virtually ...each out directly to that ...s audience of garden- ...puter owners in homes ...usinesses. It has directly ...technical and scientific ... users, big companies, computer ... It has issued press releases ...

...t the Uni- ...s Informa- ...he patches ...L, Page 6F

Chip Shot

Computer Giants' War Over Flaw in Pentium Jolts the PC Industry

Who Is Twisting the Truth? Intel Stands by Product As IBM Halts Shipments

'I'd Be Totally Confused'

By Hart Zeller and Don Clark
Staff Reporters of The Wall Street Journal

IBM fired a broadside at Intel Corp. yesterday in the controversy over the ...

Some Scientists Are Angry Over Flaw In Pentium Chip, and Intel's Response

By Don Clark
Staff Reporter of The Wall Street Journal

A flaw in Intel Corp.'s Pentium chip has angered some scientists who feel the company's response to the bug has been cava- ...

Dr. Nicely said the flaw occurs once in every 11 billion calculations when a computer is set to pick random numbers for division operations. But any chance of inaccuracy is a serious matter for mathematicians in cutting-edge fields such as chaos theory, and some sensitive areas of ...

TECHNOLOGY

Intel's Grove Airs Apology for Pentium Over the Internet

• • •

But CEO, Defending Policy Not to Replace All Chips, Doesn't Silence Criticism

By Don Clark
Staff Reporter of The Wall Street Journal

Intel Corp.'s chief executive officer, Andrew Grove, took to cyberspace to apolo- ... of relief" at such a minor issue, he said.

An Intel spokesman said publicly about the bug last week had caused a sharp increase in calls to the company, with Intel technicians discussing the type of work users are doing and deciding if a new chip is appropriate. Mr. Grove asked computer users for patience, because it will take time to answer all of the calls.

"Please don't be concerned that the passing of time will deprive you of the opportunity to get your problem resolved – we will stand behind these chips for the life of the computer," Mr. Grove said.

But electronic responses to Mr. Grove's message suggest that some computer users are still upset that Intel didn't immediately publicize the problem and halt all ...

calculations that results in an inaccurate ...

...Moler, chairman of Mathworks in Natick, ... having a chip replaced."

calling Intel's toll-free number (800-628-8686), even if you're not an MIT professor. Intel doesn't make it easy. But in a test I conducted over the past week, I managed to get Intel to agree to send me a new Pentium even though I refused to tell Intel what I did for a living or what software I used. The company said I'd have it within 30 days. "If you insist on a new chip, we'll supply a new chip," Intel Senior Vice President Carl Everett ...

... reluctantly conceded in an interview before I called the 800 number.

But it isn't a consumer-friendly process. I had to go through multiple conversations (it took me three phone calls over six days). When I was asked if I wanted to talk to an expert or receive a "white paper" about the nature of the defect in the chip, I said no thank you, that I just wanted a new chip.

THE INTEL person then wanted to know what software I used. I replied that it was none of his business, and I wanted a new chip, period. I had to give Intel a credit-card number and agree to a potential charge of $1,000 if I failed to return my old Pentium within 29 days after receiving my new one – an ironic requirement. From a company that delayed for months even disclosing the defect. Intel pays for shipping both the new chip to you and the old one back, but not for any labor costs you may incur in hiring somebody to swap the chips.

Patterson & Hennessy, "Computer Organisation and Design", 4th edition

“To owners of Pentium™ processor-based computers and the PC community: We at Intel wish to sincerely apologize for our handling of the recently publicized Pentium processor flaw. The Intel Inside ® symbol means that your computer has a microprocessor second to none in quality and performance. Thousands of Intel employees work very hard to ensure that this is true. But no microprocessor is ever perfect. What Intel continues to believe is technically an extremely minor problem has taken on a life of its own. Although Intel firmly stands behind the quality of the current version of the Pentium processor, we recognize that many users have concerns. We want to resolve these concerns. Intel will exchange the current version of the Pentium processor for an updated version, in which this floating-point divide flaw is corrected, for any owner who requests it, free of charge anytime during the life of their computer.**”**

Andrew S. Grove, Craig R. Barrett, and Gordon E. Moore
President, Vice President, and Chairman of the Board, respectively, of Intel

Patterson & Hennessy,
“Computer Organisation
and Design”, 4th edition