AHMED HAMED ALY CS1003

Question 1

$$A = \begin{pmatrix} 1 & 5 \\ 2 & 4 \end{pmatrix} \qquad B = \begin{pmatrix} 1 & 4 \\ 2 & 3 \end{pmatrix}$$

(i)

$\det A = ad - bc$ $\qquad$ $\det B = ad - bc$

$\det A = 4 - 10$ $\qquad$ $\det B = 3 - 8$

$\det A = -6$ $\qquad$ $\det B = -5$

(ii)

$$(AB) = \begin{pmatrix} \boxed{1 \quad 5} \\ \boxed{2 \quad 4} \end{pmatrix} \begin{pmatrix} \boxed{\begin{matrix} 1 \\ 2 \end{matrix}} & \boxed{\begin{matrix} 4 \\ 3 \end{matrix}} \end{pmatrix} = \begin{pmatrix} 11 & 19 \\ 10 & 20 \end{pmatrix}$$

$$AB^T = \begin{pmatrix} 11 & 10 \\ 19 & 20 \end{pmatrix}$$

$$B^T = \begin{pmatrix} 1 & 2 \\ 4 & 3 \end{pmatrix} \qquad A^T = \begin{pmatrix} 1 & 2 \\ 5 & 4 \end{pmatrix}$$

show that $(AB)^T = B^T A^T$

$$\begin{pmatrix} 11 & 10 \\ 19 & 20 \end{pmatrix} = \begin{pmatrix} \boxed{1 \quad 2} \\ \boxed{4 \quad 3} \end{pmatrix} \begin{pmatrix} \boxed{\begin{matrix} 1 \\ 5 \end{matrix}} & \boxed{\begin{matrix} 2 \\ 4 \end{matrix}} \end{pmatrix}$$

$$\begin{pmatrix} 11 & 10 \\ 19 & 20 \end{pmatrix} = \begin{pmatrix} 11 & 10 \\ 19 & 20 \end{pmatrix}$$

Therefor $(AB)^T = B^T A^T$

Question 2:

$$A = \begin{pmatrix} 1 & 1 & 2 \\ 1 & 2 & 3 \\ 2 & 1 & 3 \end{pmatrix} \qquad B = \begin{pmatrix} 1 & 2 & 2 \\ 2 & 5 & 3 \\ 2 & 1 & 1 \end{pmatrix}$$

(i) Product of AB is:

$$\begin{pmatrix} \boxed{1 \quad 1 \quad 2} \\ \boxed{1 \quad 2 \quad 3} \\ \boxed{2 \quad 1 \quad 3} \end{pmatrix} \begin{pmatrix} 1 & 2 & 2 \\ 2 & 5 & 3 \\ 2 & 1 & 1 \end{pmatrix}$$

$$AB = \begin{pmatrix} 7 & 9 & 7 \\ 11 & 15 & 11 \\ 10 & 12 & 10 \end{pmatrix}$$

Product of BA is:

$$\begin{pmatrix} \boxed{1 \quad 2 \quad 2} \\ \boxed{2 \quad 5 \quad 3} \\ \boxed{2 \quad 1 \quad 1} \end{pmatrix} \begin{pmatrix} 1 & 1 & 2 \\ 1 & 2 & 3 \\ 2 & 1 & 3 \end{pmatrix}$$

$$BA = \begin{pmatrix} 7 & 7 & 14 \\ 13 & 15 & 28 \\ 5 & 5 & 10 \end{pmatrix}.$$

(ii) $AB \neq BA$ because the multiplication of matrices isn't commutative which means AB is not equal to BA

Question 3:

$$x + y + z = 2$$
$$2x + 3y + z = 3$$
$$x - y - 2z = -6$$

$$\begin{pmatrix} 1 & 1 & 1 & 2 \\ 2 & 3 & 1 & 3 \\ 1 & -1 & -2 & -6 \end{pmatrix}$$

step 1: $R_2 = R_2 - 2R_1$

$$\begin{pmatrix} 1 & 1 & 1 & 2 \\ 0 & 1 & -1 & -1 \\ 1 & -1 & -2 & -6 \end{pmatrix}$$

step 2: $R_3 = R_3 - R_1$

$$\begin{pmatrix} 1 & 1 & 1 & 2 \\ 0 & 1 & -1 & -1 \\ 0 & -2 & -3 & -8 \end{pmatrix}$$

step 3: $R_3 = R_3 + 2R_2$

$$\begin{pmatrix} 1 & 1 & 1 & 2 \\ 0 & 1 & -1 & -1 \\ 0 & 0 & -5 & -10 \end{pmatrix}$$

step 4: $R_3 = R_3 \div -5$

$$\begin{pmatrix} 1 & 1 & 1 & 2 \\ 0 & 1 & -1 & -1 \\ 0 & 0 & 1 & 2 \end{pmatrix}$$

step 5: $R_2 = R_2 + R_3$

$$\begin{pmatrix} 1 & 1 & 1 & 2 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 1 & 2 \end{pmatrix}$$

step 6: $R_1 = R_1 - R_3$

$$\begin{pmatrix} 1 & 1 & 0 & 2 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 2 \end{pmatrix}$$

step 7: $R_1 = R_1 - R_2$

$$\begin{pmatrix} 1 & 0 & 0 & -1 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 2 \end{pmatrix}$$

Therefor $\boxed{x = -1}$, $\boxed{y = 1}$, $\boxed{z = 2}$