Course Work Assignment #1

(Do all 4 questions)

For a set, X, $\overline{X}$ is the complement of X.

1. [10 marks]
   Which of the following is equal to $\overline{A \cap \overline{B}}$. Justify the answer.

   (a) $A \cup B$
   (b) $A \cup \overline{B}$
   (c) $\overline{A} \cup B$
   (d) $\overline{A} \cup \overline{B}$

2. [30 marks]
   Let the set operator, $\gg$, be defined so that

   $$X \gg Y = \overline{X} \cup Y$$

   Determine by Karnaugh Map whether:

   (a) $A \gg B = \overline{A \cap \overline{B}}$
   (b) $(A \cap B) \gg C = A \gg (B \gg C)$
   (c) $A \gg (B \gg C) = (A \gg B) \gg C$

3. [24 Marks]
   Each student in a group of 139 speaks either French, German or Spanish.

   63 speak French, 91 speak German, 44 speak Spanish.

   25 speak French and German.

   23 speak French and Spanish.

   21 speak German and Spanish.

   How many students speak all 3 languages?

4. [36 marks]
   Let the Universal set, $U = \{0, 1, 2, 3, 4, 5, 6, 7\}$.
   The Universal set is represented as the 8-bit array, 11111111
   and the empty set is represented as 00000000
   For example, the set $S = \{1, 3, 5, 7\}$ is represented by the 8-bit array $s = 01010101$

For a set, $S \subseteq U$, if $(k \in S)$ then $s[k] = 1$ else $s[k] = 0$.
where the bit array, s, is the representation of the set, S.
Example: Let $S = \{1, 2, 3\}$ and let the bit array, s, represent the set S, then
$s[0] = 0$, $s[1] = 1$, $s[2] = 1$, $s[3] = 1$, $s[4] = 0$ etc.
Writing this in 'table form'

k	0	1	2	3	4	5	6	7
$s[k]$	0	1	1	1	0	0	0	0

i.e. $s = 01110000$.

Let $A = \{3, 4, 5\}$, $B = \{1, 3, 6\}$.

(a) Express the sets A and B as bit arrays, a and b respectively.

(b) Express the set $A \cap B$ and $A \cup B$ as bit arrays.

(c) Express the set $A \oplus B$ as a bit array
where $A \oplus B = (A \cap \overline{B}) \cup (\overline{A} \cap B)$