Unit 6: Simple linear regression

2. Outliers & Inference for SLR

Sta 104 - Summer 2015

Duke University, Department of Statistical Science

June 15, 2015

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Slides posted at http://bit.ly/sta104su15

1. Housekeeping

2. Main ideas
    1. R^2 assesses model fit -- higher the better
    2. Inference for regression uses the T distribution
    3. Conditions for regression
    4. Type of outlier determines how it should be handled

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Response: annual_murders_per_mil
          Df  Sum Sq Mean Sq F value    Pr(>F)
perc_pov   1 1308.34 1308.34  43.064 3.638e-06 ***
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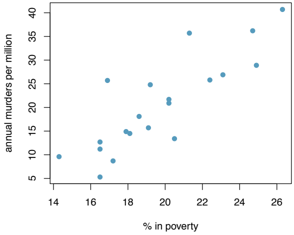
Clicker question

R^2 for the regression model for predicting annual murders per million based on percentage living in poverty is roughly 71%. Which of the following is the correct interpretation of this value?



(a) 71% of the variability in percentage living in poverty is explained by the model.

(b) 84% of the variability in the murder rates is explained by the model, i.e. percentage living in poverty.

(c) 71% of the variability in the murder rates is explained by the model, i.e. percentage living in poverty.

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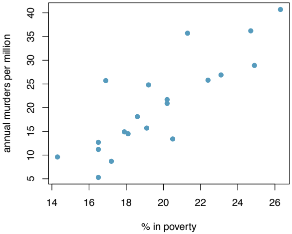

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- Confidence intervals for a slope:
  - $b_1 \pm T^{\star}_{n-2} SE_{b_1}$

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- Independence of residuals (and hence observations) $\rightarrow$ depends on data collection method, often violated for time-series data – important for inference

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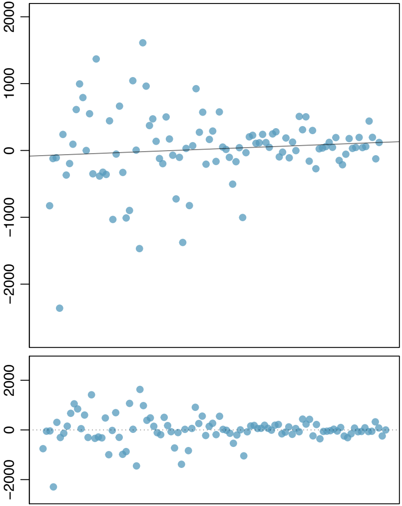
What condition is this linear model obviously and definitely violating?

(a) Linear relationship
(b) Non-normal residuals
(c) Constant variability
(d) Independence of observations



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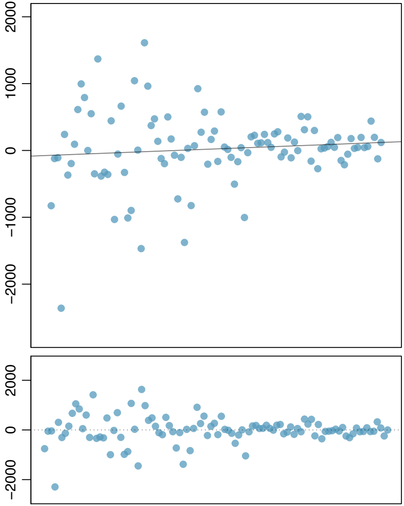

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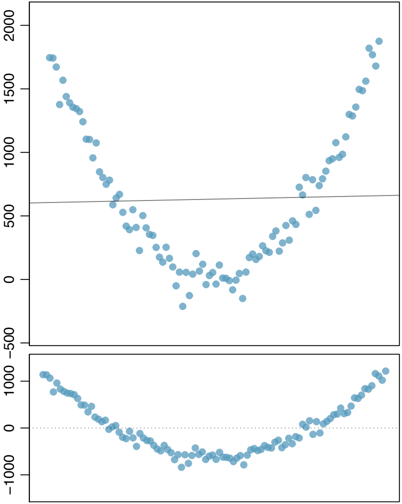

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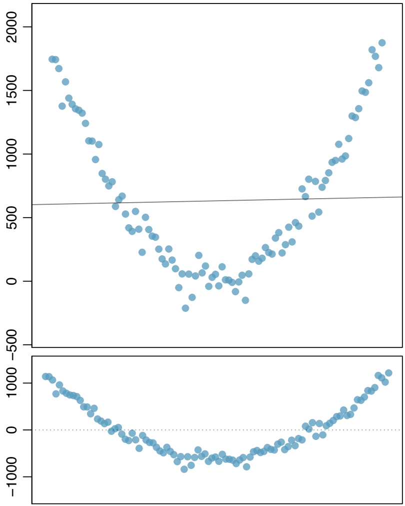

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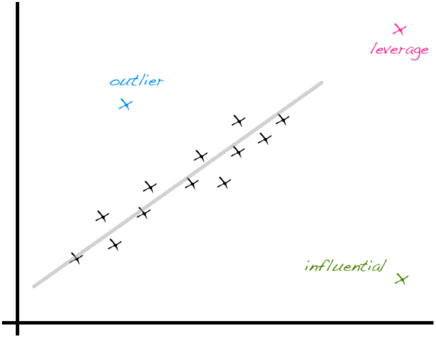
- *Influential* point changes the slope (most likely also has high leverage) – run the regression with and without that point to determine



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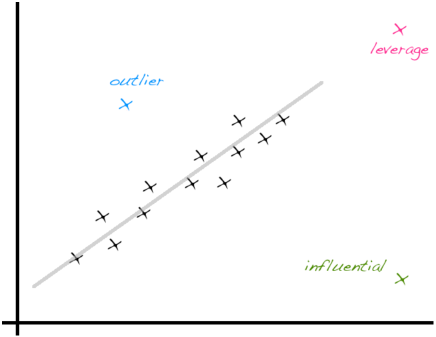

- *Outlier* is an unusual point without these special characteristics (this one likely affects the intercept only)
- If clusters (groups of points) are apparent in the data, it might be worthwhile to model the groups separately.

Application exercise: 6.2 Linear regression

See course website for details

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