Unit 4: Inference for numerical data

1. Decision errors, significance levels, sample size & power

Sta 104 - Summer 2015

Duke University, Department of Statistical Science

June 1, 2015

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Slides posted at http://bit.ly/sta104su15

1. Housekeeping

2. Main ideas

    1. Hypothesis tests and confidence intervals at equivalent significance/confidence levels should agree
    2. Results that are statistically significant are not necessarily practically significant
    3. Calculate the sample size a priori to achieve desired margin of error
    4. Hypothesis tests are prone to decision errors
    5. Power depends on the effect size, α, n, and s

3. Summary

- PS3 due tonight
- Project proposals due Thursday night
- MT corrections extra credit: Work **as a team** to write up a collective exam corrections document that discusses all questions missed by any member of the team. Your corrections should show full work and explain reasoning, even for the multiple choice questions. Due by the end of the day on Wednesday, June 3. **Extra credit:** +2 points on the exam.

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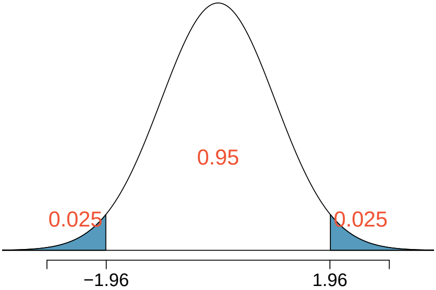
Two sided



95% confidence level
is equivalent to
two sided HT with $\alpha = 0.05$

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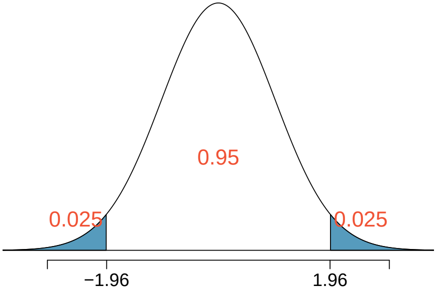

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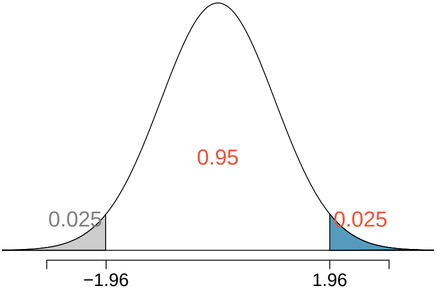

95% confidence level
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Clicker question

What is the significance level of a two-sided hypothesis test that is equivalent to a 90% confidence interval? *Hint: Draw a picture and mark the confidence level in the center.*

(a) 0.001
(b) 0.01
(c) 0.025
(d) 0.05
(e) 0.10

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A 95% confidence interval for the average normal body temperature of humans is found to be (98.1 F, 98.4 F). Which of the following is <u>true</u>?

(a) The hypothesis $H_0 : \mu = 98.2$ would be rejected at $\alpha = 0.05$ in favor of $H_A : \mu \neq 98.2$.

(b) The hypothesis $H_0 : \mu = 98.2$ would be rejected at $\alpha = 0.025$ in favor of $H_A : \mu > 98.2$.

(c) The hypothesis $H_0 : \mu = 98$ would be rejected using a 90% confidence interval.

(d) The hypothesis $H_0 : \mu = 98.2$ would be rejected using a 99% confidence interval.

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(c) *The hypothesis $H_0 : \mu = 98$ would be rejected using a 90% confidence interval.*

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    1. Hypothesis tests and confidence intervals at equivalent significance/confidence levels should agree

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2. Results that are statistically significant are not necessarily practically significant

Clicker question

All else held equal, will p-value be lower if $n = 100$ or $n = 10,000$?

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Suppose $\bar{x} = 5$, $s = 2$, $H_0 : \mu = 4.5$, *and* $H_A : \mu \geq 4.5$.

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$$Z_{n=100} \quad = \quad \frac{5 - 4.5}{\frac{2}{\sqrt{100}}}$$

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$$Z_{n=10000} = \frac{5 - 4.5}{\frac{2}{\sqrt{10000}}} = \frac{5 - 4.5}{\frac{2}{100}} = \frac{0.5}{0.02} = 25, \quad \textit{p-value} \approx 0$$

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$$
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As n *increases -* $SE \downarrow$, $Z \uparrow$, *p-value* $\downarrow$

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    5. Power depends on the effect size, α, n, and s

3. Summary

3. Calculate the sample size *a priori* to achieve desired margin of error

Application exercise: 4.1 Sample size

See course website for details.

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		fail to reject H_0	reject H_0
Truth	H_0 true		
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- A *Type 1 Error* is rejecting the null hypothesis when H_0 is true: α
  - For those cases where H_0 is actually true, we do not want to incorrectly reject it more than 5% of those times
  - Increasing α increases the Type 1 error rate, hence we prefer to small values of α

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Truth	H_0 true	✓	*Type 1 Error,* α
	H_A true	*Type 2 Error,* β	*Power,* $1-\beta$

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  - For those cases where H_0 is actually true, we do not want to incorrectly reject it more than 5% of those times
  - Increasing α increases the Type 1 error rate, hence we prefer to small values of α
- A *Type 2 Error* is failing to reject the null hypothesis when H_A is true: β
- *Power* is the probability of correctly rejecting H_0, and hence the complement of the probability of a Type 2 Error: $1-\beta$

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- increasing the *effect size*

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