Unit 2: Probability and distributions

1. Probability and conditional probability

Sta 104 - Summer 2015

Duke University, Department of Statistical Science

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Slides posted at http://bit.ly/sta104su15

1. Housekeeping

2. Main ideas
    1. Disjoint and independent do not mean the same thing
    2. Application of the addition rule depends on disjointness of events
    3. Bayes' theorem works for all types of events

3. Summary

- Lab: Put your code in R chunks so that the markdown can process it as code and produce the desired output and plots.
- PS1:
  - 1.6 (c): How is income recorded? (Under 2,600; 10,400 to 15,600; above 36,400; ...)
  - 1.14 (b): What type of a sample is it if you only ask your friends to respond?
  - 1.46 (c): Is the histogram or the intensity map more informative?

- 15 min individual
- 10 min teams

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- *Disjoint (mutually exclusive) events* cannot happen at the same time
  - A voter cannot register as a Democrat and a Republican at the same time
  - But s/he might be a Republican and a Moderate at the same time – *non-disjoint events*
  - For disjoint A and B: $P(A \ and \ B) = 0$

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  - For disjoint A and B: $P(A\ and\ B) = 0$
- If A and B are *independent events*, having information on A does not tell us anything about B (and vice versa)
  - If A and B are independent:
    - $P(A \mid B) = P(A)$
    - $P(A\ and\ B) = P(A) \times P(B)$

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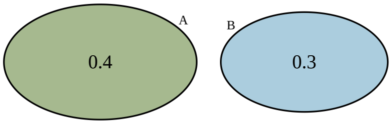
= 0.4 + 0.3 - 0 = 0.7



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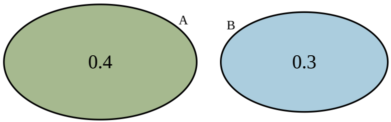
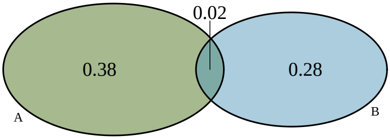
= 0.4 + 0.3 - 0.02 = 0.68





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  = P(A | B) × P(B)
  = *0* × P(B) = 0

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  = *P(A)* × P(B)

Application exercise: 2.1 Probability and conditional probability

See the course website for instructions.

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