DIGITAL SIGNAL PROCESSING (DSP) COURSE

Lecture One

Introduction

Amr Wageeh

Objective

Our main goal is to be able to design digital LTI filters. Such filters are using widely in applications such as audio entertainment systems, telecommunication and other kinds of communication systems, radar, video enhancement, and biomedical engineering.

Grading

- Work year:
  midterm (15).
  quizes (15).
  project (10).
  attendance (10).

Reference Books

- "Discrete Time Signal Processing", Alan V . Oppenheim and R. W . Schafer, 3rd ed., Prentice Hall, 2003.
- "Digital Signal Processing, Signals, Systems and Filters", Andreas Antoniou, The McGraw-Hill, 2006.

COURSE OBJECTIVE

1. Determine if a DSP system is linear, time-invariant, causal, and memory less, determine stability of systems given in frequency domain.
2. Perform Z and inverse Z transforms using the definitions, Tables of Standard Transforms and Properties, and Partial Fraction Expansion.
3. Design FIR and IIR filters by hand to meet specific magnitude and phase requirements.
4. Analyze a DSP system in time and frequency domains.
5. Use computers and MATLAB to create, analyze and process signals, and to simulate and analyze systems sound and image synthesis and analysis, to plot and interpret magnitude and phase of LTI system frequency responses.

COURSE OUTLINES

- Introduction to Digital Signal Processing
- Review of Signals, Systems, and Fourier Transform.
- Sampling of Continuous-Time Signals.
- Discrete-Time Signals and System.
- The Z-Transform.
- Transform Analysis of Linear Time-Invariant Systems.
- Filter Design Techniques.
- Computation of the Discrete-Fourier Transform.
- Applications of Digital Signal Processing.

CHAPTER ONE
INTRODUCTION TO DIGITAL SIGNAL PROCESSING

CHAPTER OUTLINES

- what is a signal.
- What is Signal Processing.
  - Analog Signal Processing System
  - Digital Signal Processing System
- Time and Frequency Domain Representations of Signals.

SIGNAL

- Signal:

Signal is defined as any physical quantity that varies with Time, Space, or any other independent variables. For Example, the functions.

- Example:

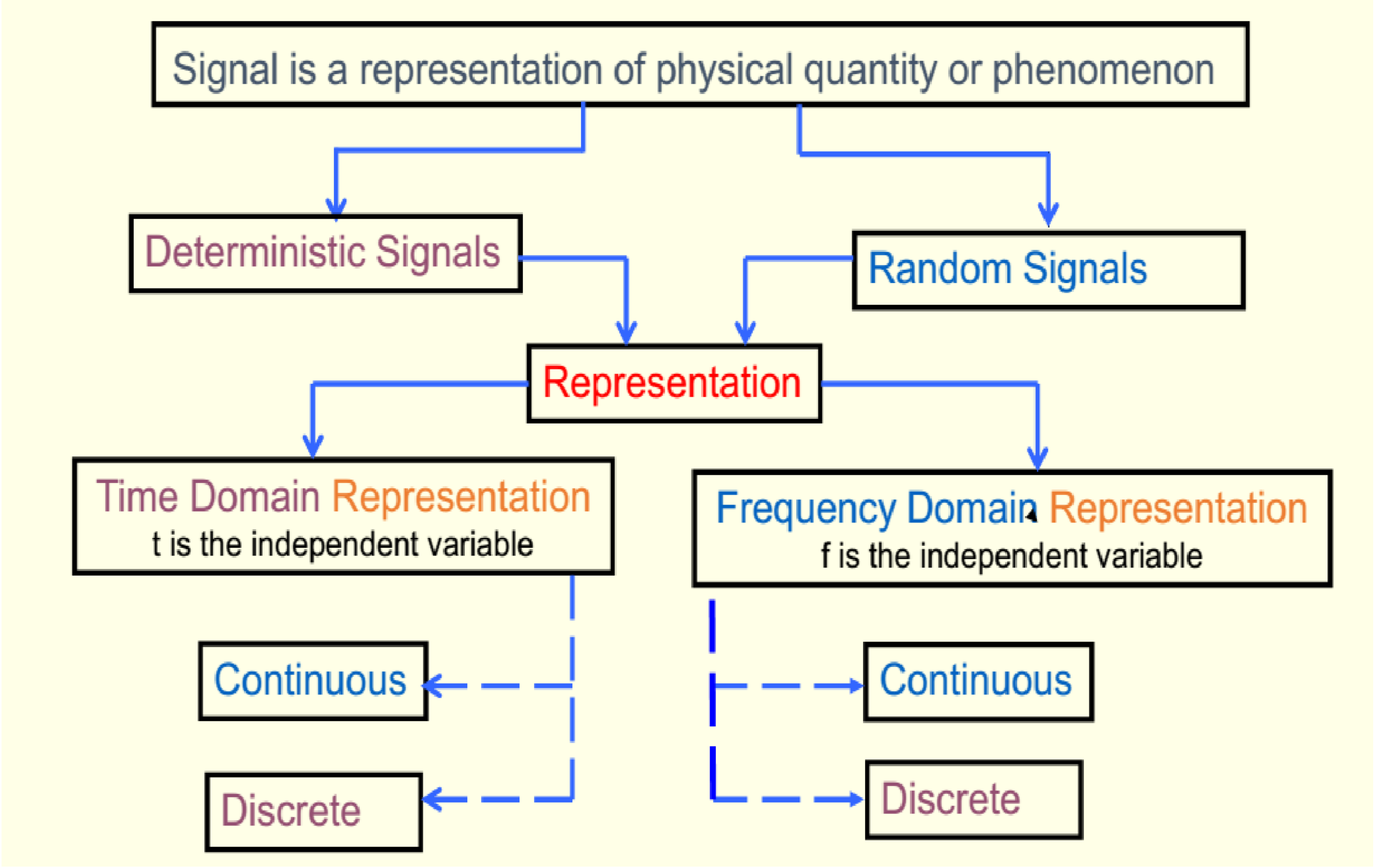
$s_1(t) = 5t$ or $s_1(t) = 5t^2$

➔ one variable

$S(x, y) = 3x + 4xy + 6x^2$

➔ two variables x and y

SIGNAL CLASSIFICATION



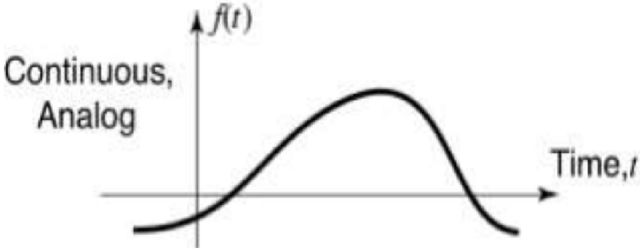
ANALOG SIGNAL

- Defined for every value of time and they take on values in the continuous interval (a,b).
- Analog Signal
  - Continuous in time.
  - Amplitude may take on any value in the continuous range of (-∞, ∞).
- Analog Processing

Differentiation, Integration, Filtering, Amplification.

Differential Equations

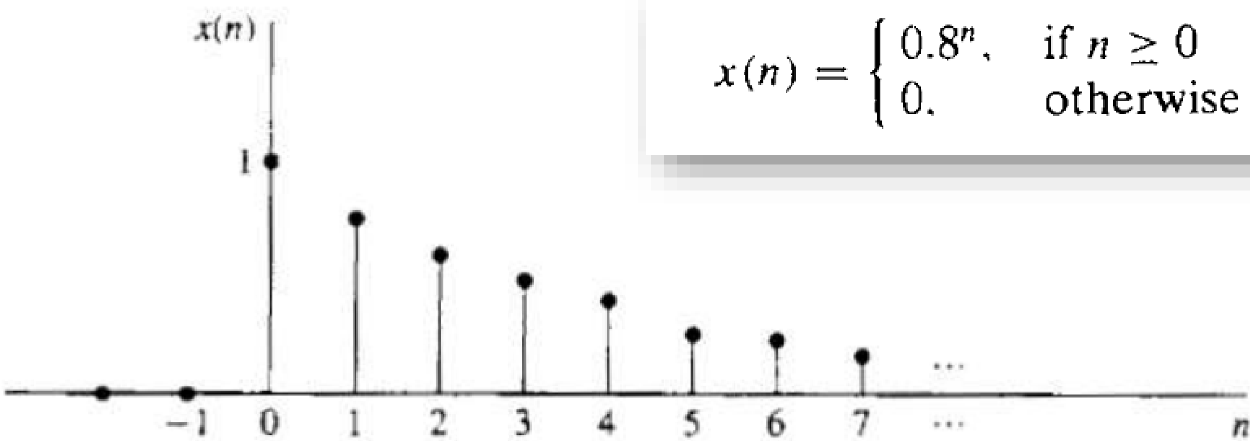
Implemented via passive or active electronic circuitry .



DISCRETE SIGNAL

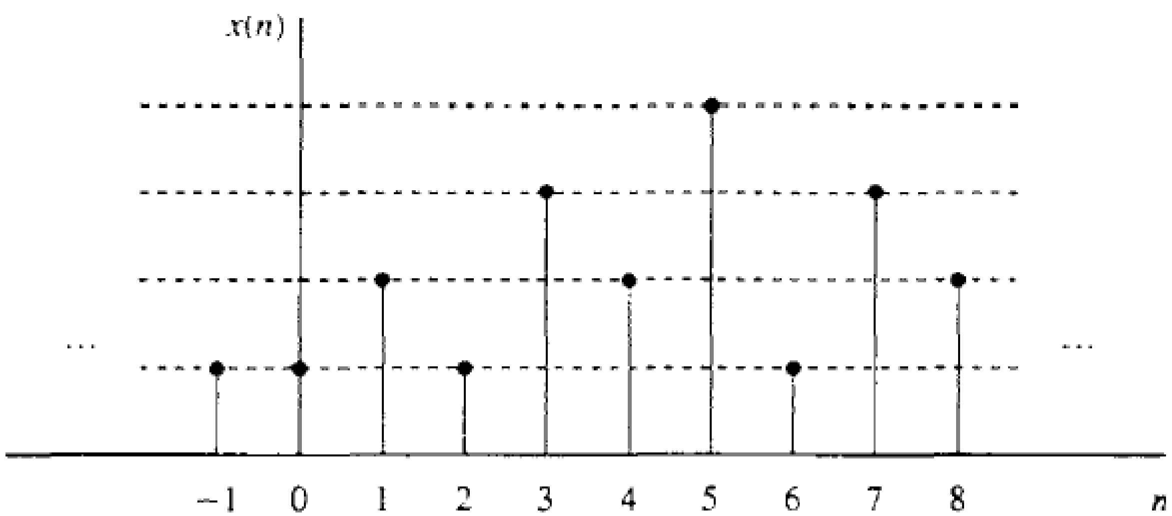
- Discrete-Time signals: are defined only at certain specific value of time.
- Continuous in Amplitude but Discrete in Time
- Only defined for certain time instances.
- Can be obtained from analog signals via sampling.

$$x(n) = \begin{cases} 0.8^n, & \text{if } n \geq 0 \\ 0, & \text{otherwise} \end{cases}$$



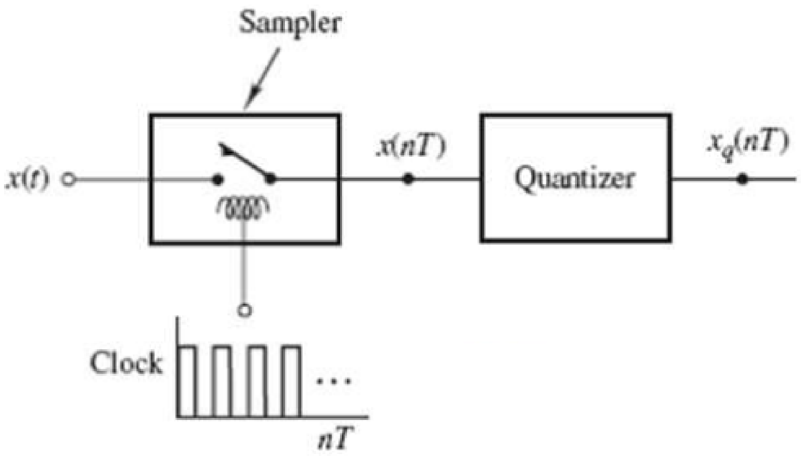
DIGITAL SIGNAL

- Digital Signal: is the signal that takes on values from a finite set of possible values.
- Discrete in Amplitude & Discrete in Time.
- Can be obtained from Discrete signals via quantization.



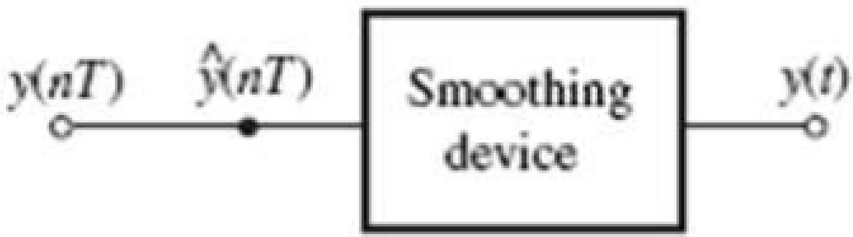
DIGITAL SIGNAL

- Discrete-time signals are often generated from corresponding continuous-time signals through the use of an analog-to-digital (A/D) interface.
- An A/D interface typically comprises three components, namely, a sampler, a quantizer , and an encoder.

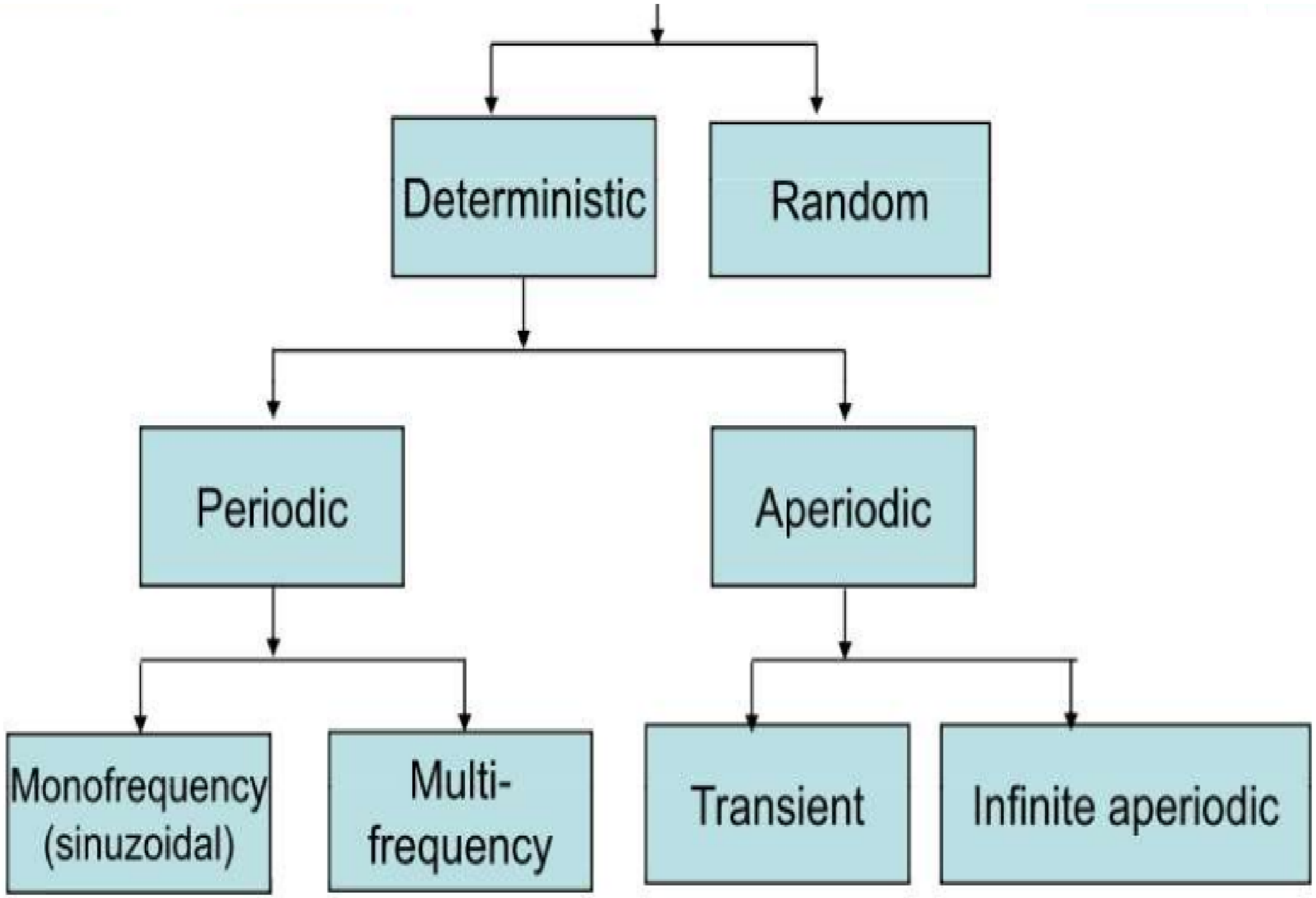


DIGITAL SIGNAL

- Similarly, continuous-time signals can be obtained by using a digital-to-analog (D/A) interface. The D/A interface comprises two modules, a decoder and a smoothing device.

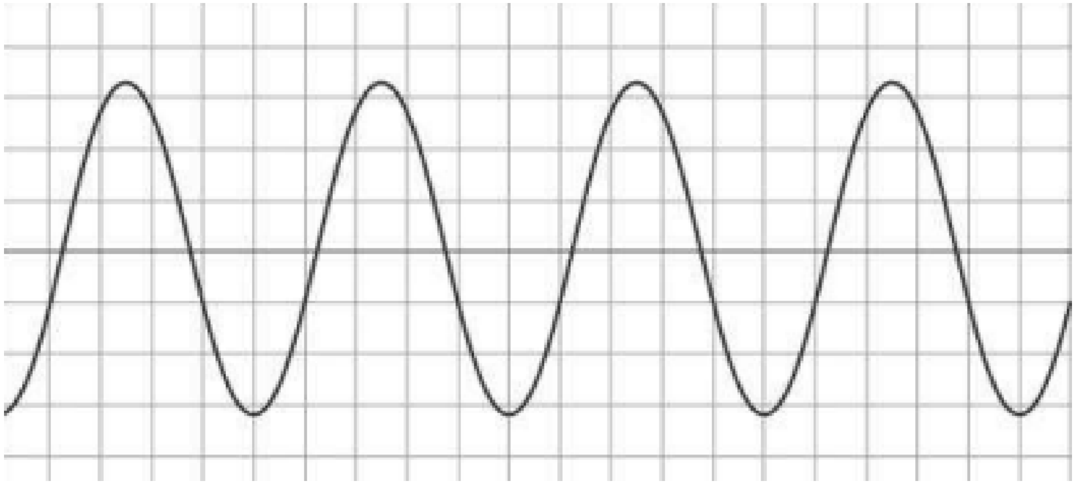


CLASSIFICATION OF SIGNALS



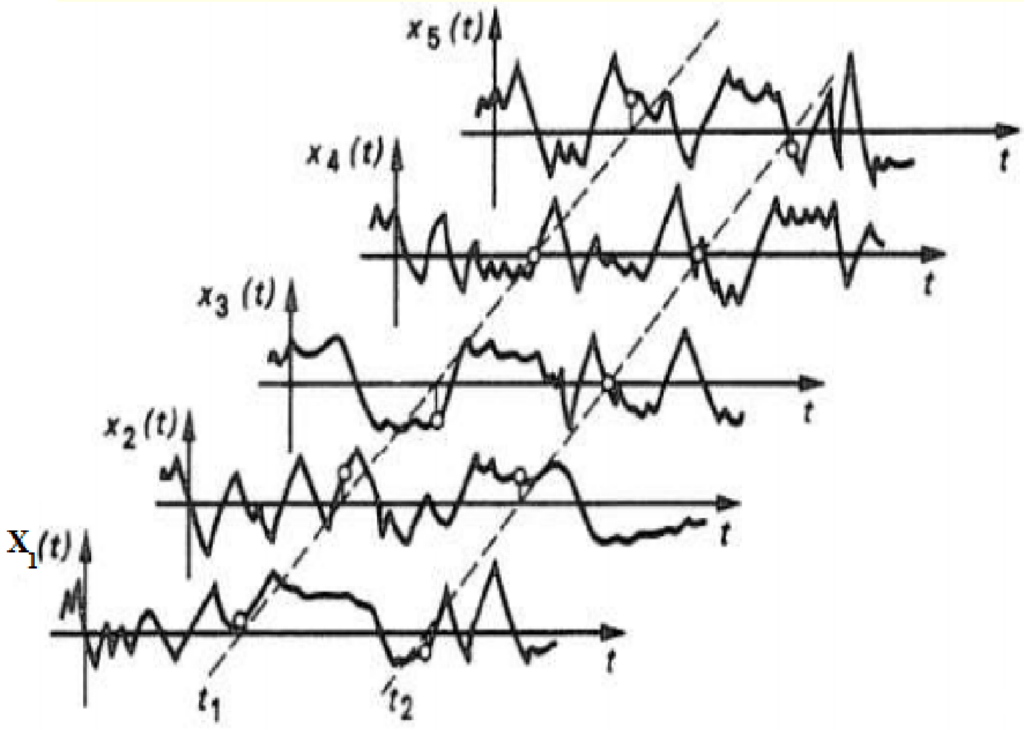
DETERMINISTIC SIGNAL

- Deterministic Signal: Any signal whose past, present and future values are precisely known without any uncertainty .



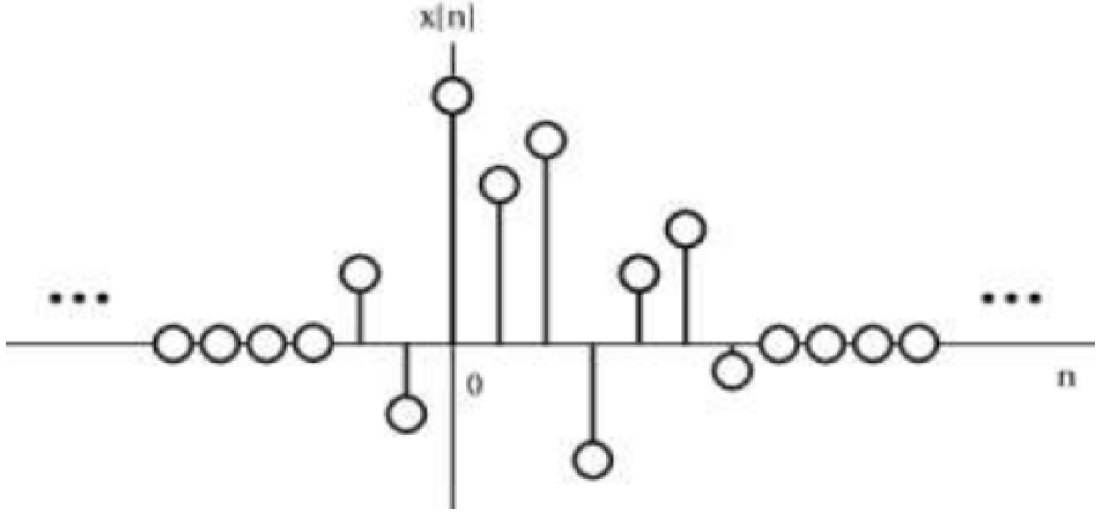
RANDOM SIGNAL

- Random Signal: A signal in which cannot be approximated by a formula to a reasonable degree of accuracy (i.e. noise).



FINITE AND INFINITE SIGNAL

- Finite-length signal: non zero over a finite interval $t_{min} < t < t_{max}$



- Infinite-length signal: nonzero over all real numbers.

MULTI-CHANNEL SIGNALS

- Signals are generated by multiple source or multiple sensor .

- This signals, can represented in vector form Example: ECG (Electrocardiogram) are often used 3-channel and 12-channel.

SINGLE AND MULTIDIMENSIONAL SIGNAL

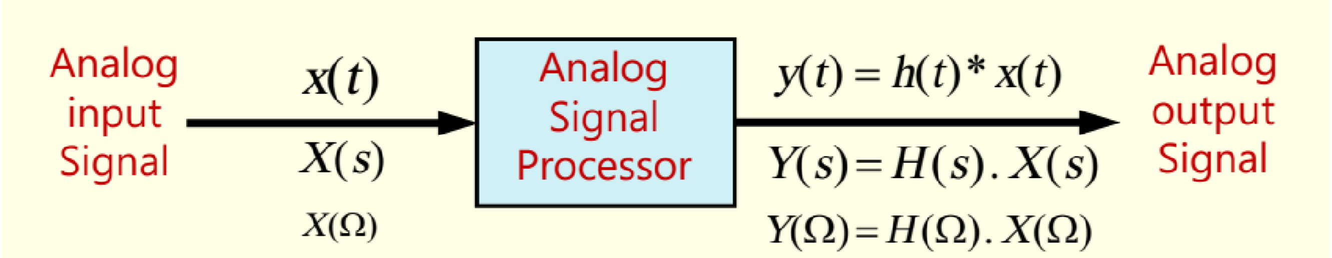
- If the signal is a function of a single independent variable, the signal called a one-dimensional signal.

- A signal called M-dimensional if its value is a function of M independent variables.

SIGNAL PROCESSING

- Signals may have to be transformed in order to Amplify or filter out embedded information
- Detect patterns
- Prepare the signal to survive a transmission channel
- Undo distortions contributed by a transmission channel
- To do so, we also need:
- Methods to measure, characterize, model, and simulate signals.
- Mathematical tools that split common channels and transformations into easily.
- manipulated building blocks.

ANALOG SIGNAL PROCESSING (ASP)



$h(t)$: The System Impulse Response.

$H(s)$: The System Transfer Function.

$H(\Omega)$: The System Frequency Response.

LIMITATION OF ANALOG SIGNAL PROCESSING

- Accuracy limitations due to

Component tolerances   Undesired nonlinearities

- Limited repeatability due to

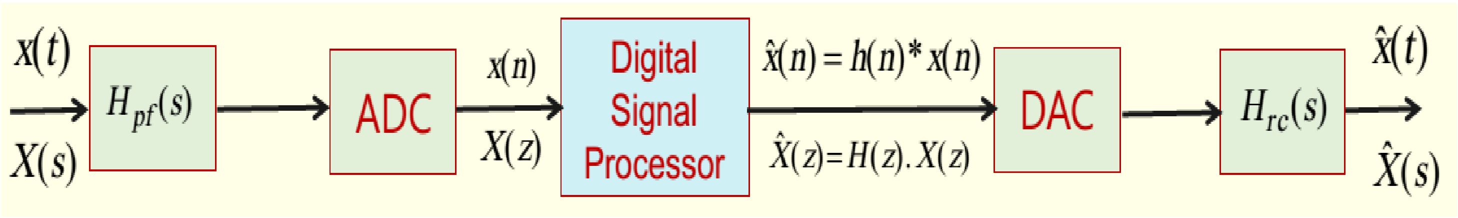
Tolerances

Changes in environmental conditions

Temperature   Vibration

- Sensitivity to electrical noise
- Limited dynamic range for voltage and currents
- Difficulty of implementing certain operations
- Nonlinear operations
- Time-varying operations
- Difficulty of storing information

DIGITAL SIGNAL PROCESSING (DSP)



- $h(n)$: The System Impulse Response (Weighted Sequence)
- $H(z)$: The System Transfer Function
- $H_{pf}(s)$: Prefilter (Band-limited – Reduce noise)
- $H_{rc}(s)$: reconstruction filter (smoothing)
- Digital signal processing techniques are now so powerful that sometimes it is extremely difficult, if not impossible, for analogue signal processing to achieve similar performance.

DSP APPLICATIONS

- Sound applications

Compression, recognition, echo cancellation, Cell Phones, MP3 Players,…

- Communication

Modulation, coding, detection, equalization, Cell Phones, dial-up modem, DSL modem, Satellite Receiver,…

- Medical

Tomography, Electrocardiogram,…

- Military

Radar, Sonar, Space photographs, remote sensing,…

- Image and Video Applications

DVD, JPEG, Movie special effects, video conferencing,…

- Mechanical

Motor control, process control, oil and mineral prospecting,…

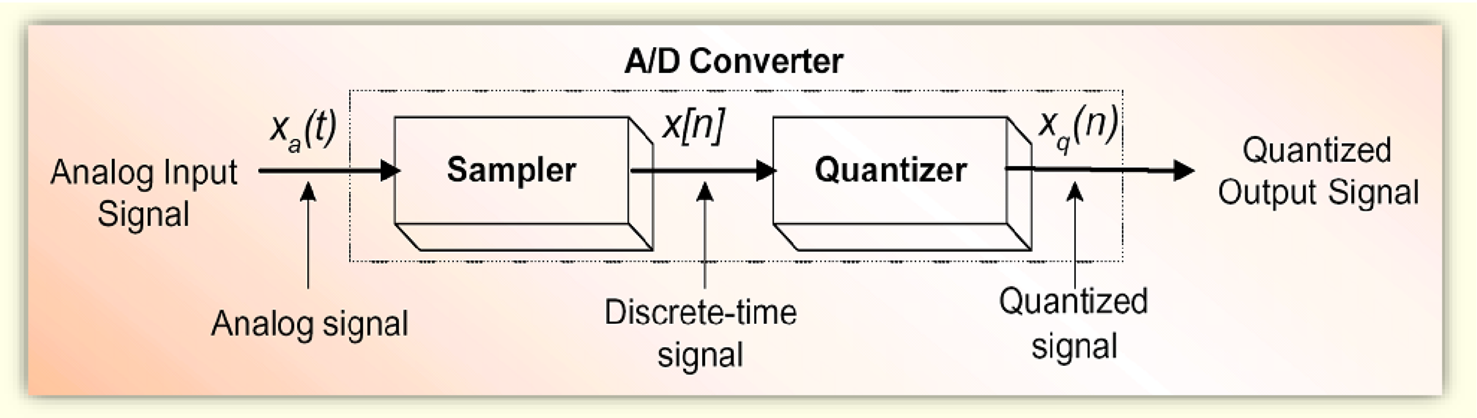
SIGNAL TYPES

- Analog signals: continuous in time and amplitude
  - Example: voltage, current, temperature,…
- Digital signals: discrete both in time and amplitude
  - Example: attendance of this class, digitizes analog signals,…
- Discrete-time signal: discrete in time, continuous in amplitude
  - Example: hourly change of temperature in Austin

WHY DIGITAL

- Digital techniques need to distinguish between discrete symbols allowing regeneration versus amplification.
- Good processing techniques are available for digital signals, such as medium.
  - Data compression (or source coding)
  - Error Correction (or channel coding)
  - Equalization
  - Security
- Easy to mix signals and data using digital techniques (Time Division Multiplexing)

ANALOG TO DIGITAL CONVERTER



Sampling

SAMPLING

- Sampling is the processes of converting continuous-time analog signal, $x_a(t)$, into a discrete-time signal by taking the "samples" at discrete-time intervals.
- Sampling analog signals makes them **discrete** in time but still **continuous** valued.

- **Sampled values:**
  - The value of the function at the sampling points.

- **Sampling interval:**
- The time that separates sampling points (interval b/w samples), T_s
- If the signal is slowly varying, then fewer samples per second will be required than if the waveform is rapidly varying.
- So, the optimum sampling rate depends on the maximum frequency component present in the signal.

IDEAL SAMPLING

- Sampling Rate (or sampling frequency f_s):

The rate at which the signal is sampled, expressed as the number of samples per second (reciprocal of the sampling interval), $1/T_s = f_s$.

- Nyquist Sampling Theorem (or Nyquist Criterion):
  - If the sampling is performed at a proper rate, no info is lost about the original signal, and it can be properly reconstructed later .

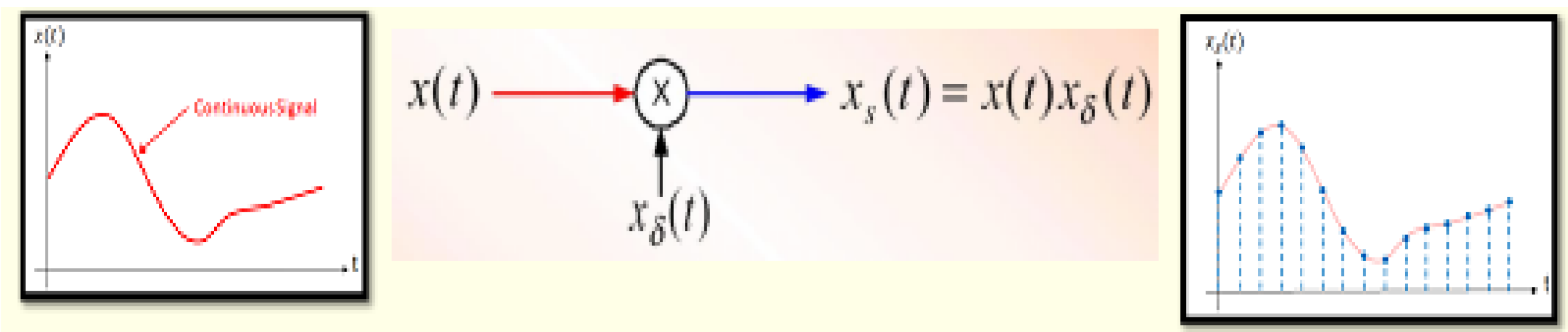
- Statement:

"If a signal is sampled at a rate at least, but not exactly equal to twice the max frequency component of the waveform, then the waveform can be exactly reconstructed from the samples without any distortion"

$$f_s \geq 2\, f_{max}$$

IDEAL SAMPLING (IMPULSE SAMPLING

- Is accomplished by the multiplication of the signal x(t) by the uniform train of impulses.
- Consider the instantaneous sampling of the analog signal x(t).



- Train of impulse functions select sample values at regular intervals

$$x_s(t) = x(t)\sum_{n=-\infty}^{\infty}\delta(t-nT_s) = \sum_{n=-\infty}^{\infty}x(nT_s)\,\delta(t-nT_s)$$

- Fourier Series representation:

$$\sum_{n=-\infty}^{\infty}\delta(t-nT_s) = \frac{1}{T_s}\sum_{n=-\infty}^{\infty}e^{jn\omega_s t}, \quad \omega_s = \frac{2\pi}{T_s}$$

IDEAL SAMPLING

- Therefore, we have:

$$x_s(t) = x(t).\delta_T(t) = x(t).\frac{1}{T_s}\sum_{n=-\infty}^{\infty} e^{jn\omega_s t}$$

- Take Fourier Transform (frequency convolution)

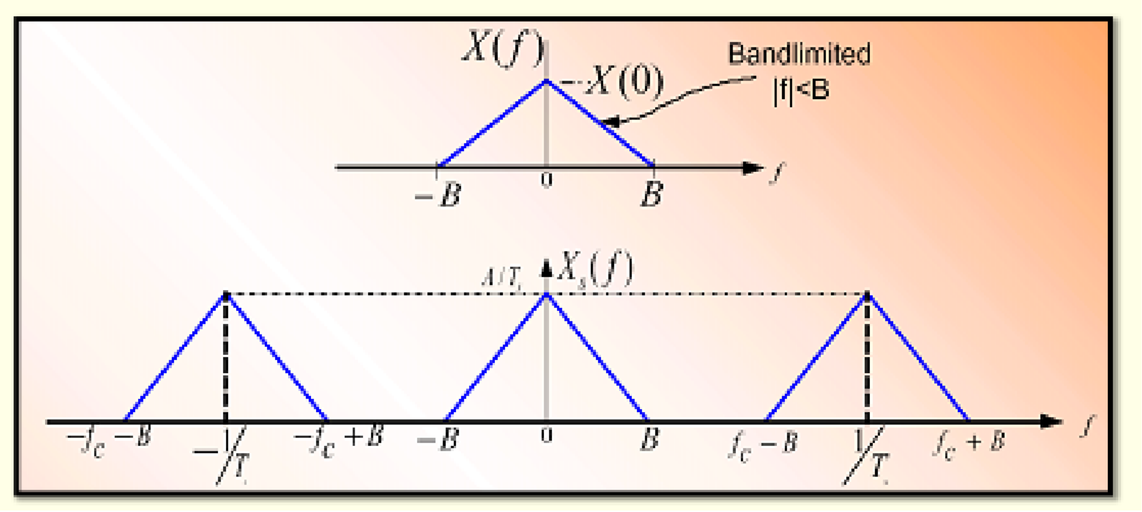
$$X_s(\omega) = \frac{1}{2\pi}.X(\omega) * \mathcal{F}\left\{\frac{1}{T_s}\sum_{n=-\infty}^{\infty} e^{jn\omega_s t}\right\} = \frac{1}{2\pi}X(\omega) * \frac{1}{T_s}\sum_{n=-\infty}^{\infty}\mathcal{F}\{e^{jn\omega_s t}\}$$

$$X_s(\omega) = \frac{1}{2\pi}X(\omega) * \frac{2\pi}{T_s}\sum_{n=-\infty}^{\infty}\delta(\omega - n\omega_s)$$

$$\boxed{X_s(\omega) = \frac{1}{T_s}\sum_{n=-\infty}^{\infty} X(\omega - n\omega_s)}$$

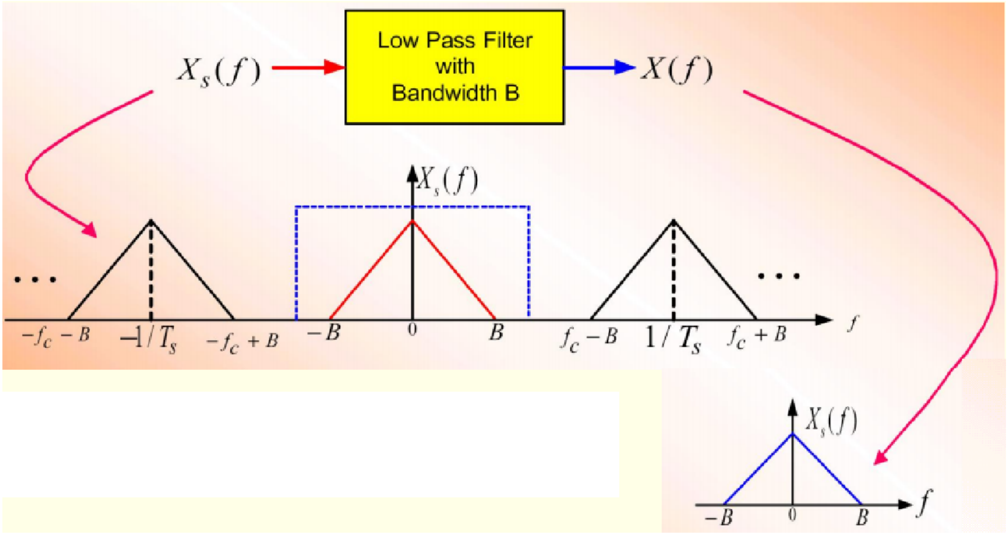
IDEAL SAMPLING

- This shows that the Fourier Transform of the sampled signal is the Fourier Transform of the original signal at rate of $1/T_s$.



IDEAL SAMPLING

- As long as $f_s > 2f_m$, no overlap of repeated replicas $X(f - n/T_s)$ will occur in $X_s(f)$.



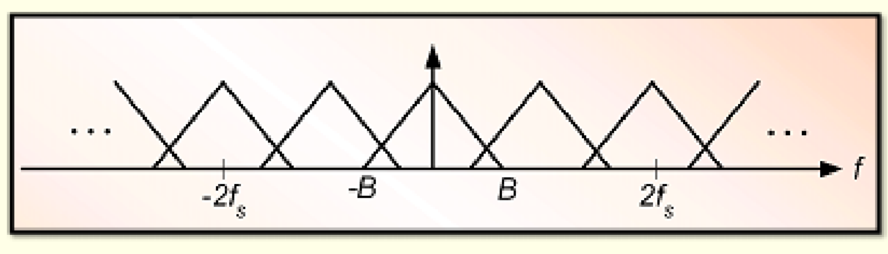
- Minimum Sampling Condition:

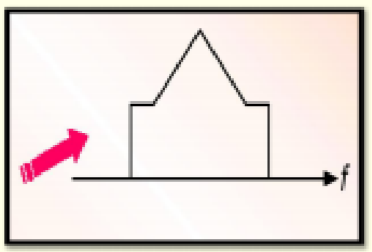
$$\boxed{f_s - f_m > f_m \;\Rightarrow\; f_s > 2f_m}$$

IDEAL SAMPLING

- Under-Sampling and Aliasing

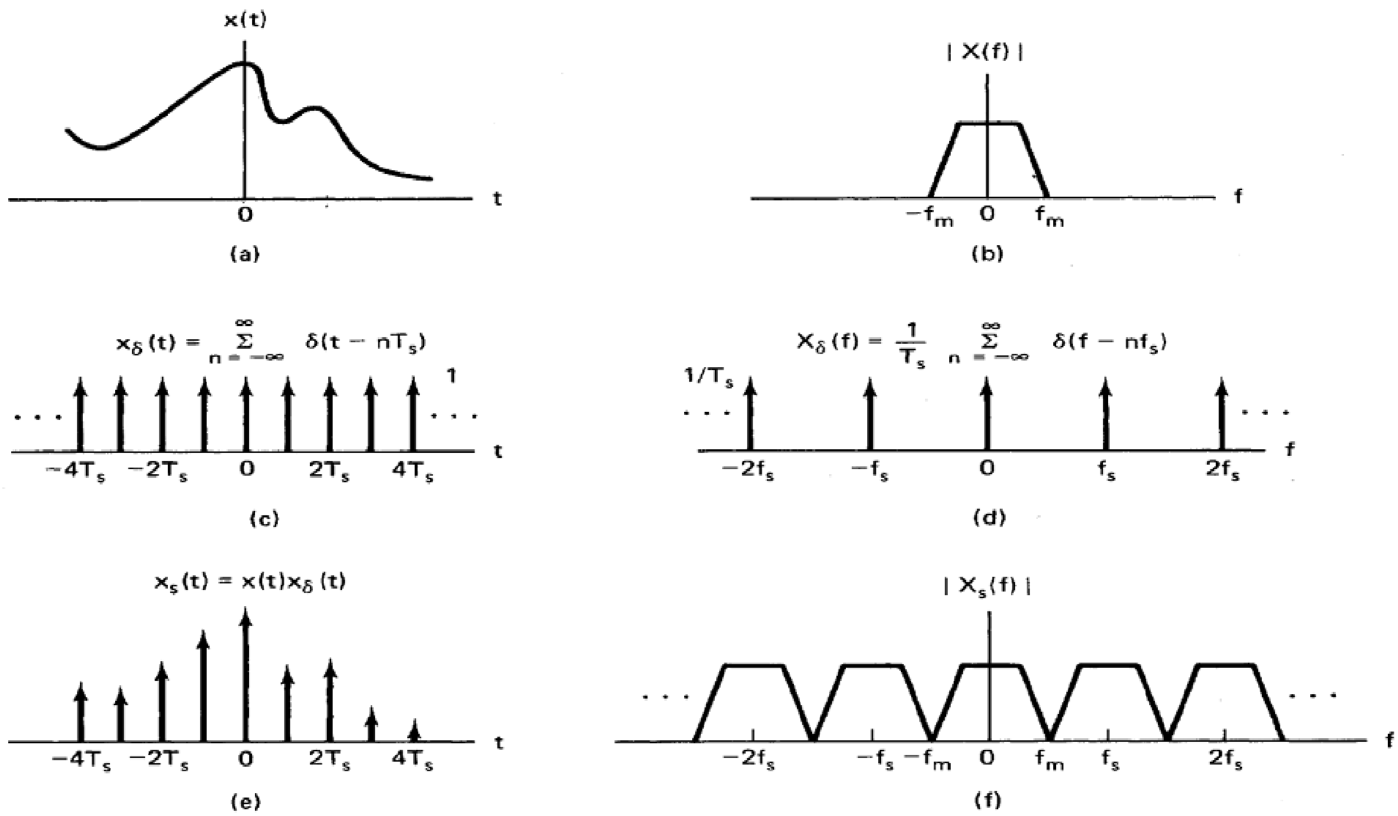
- If the waveform is under sampled (i.e. $f_s < 2B$) then there will be spectral overlap in the sampled signal.





- The signal at the output of the filter will be different from the original signal spectrum. [This is the outcome of aliasing!]
- This implies that whenever the sampling condition is not met, an irreversible overlap of the spectral replicas is produced.

* T_S is called the Nyquist interval: It is the longest time interval that can be used for sampling a band limited signal and still allow reconstruction of the signal at the receiver without distortion.



IDEAL SAMPLING

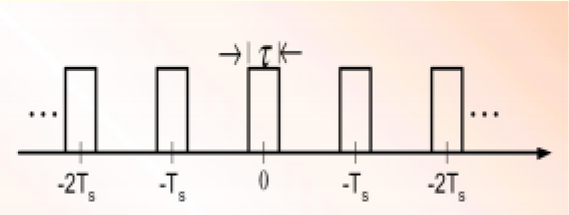
- Sampling Theorem: A finite energy function x(t) can be completely reconstructed from its sampled value x(nTs) with

$$x(t)=\sum_{n=-\infty}^{\infty} T_s x(nT_s)\left\{\frac{\sin\left[\dfrac{2\pi f(t-nT_s)}{2T_s}\right]}{\pi(t-nT_s)}\right\}=\sum_{n=-\infty}^{\infty} T_s\, x(nT_s)\,\sin c(2\,f_s(t-nT_s))$$

PRACTICAL SAMPLING (EXACT OR NATURAL SAMPLING)

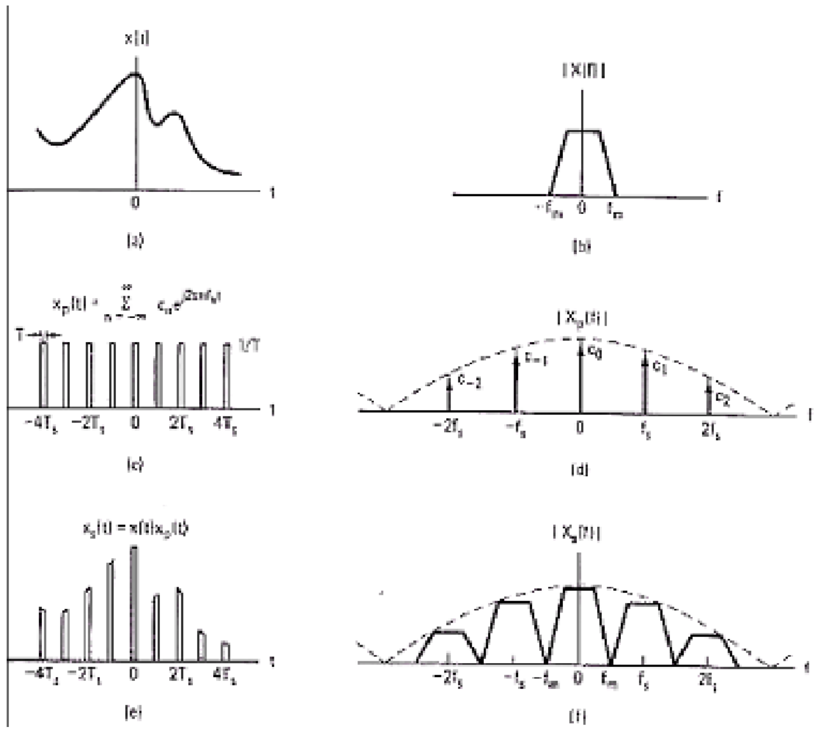
- In practice we cannot perform ideal sampling
  - It is practically difficult to create a train of impulses
- Thus a non-ideal approach to sampling must be used
- We can approximate a train of impulses using a train of very thin rectangular pulses:

$$x_p(t) = \sum_{n=-\infty}^{\infty} \Pi\left(\frac{t - nT_s}{\tau}\right)$$



NATURAL SAMPLING

- If we multiply x(t) by a train of rectangular pulses xp(t), we obtain a gated waveform that approximates the ideal sampled waveform, known as natural sampling or gating.



$$x_s(t) = x(t)\,x_p(t)$$
$$= x(t)\sum_{n=-\infty}^{\infty} c_n e^{j2\pi n f_s t}$$
$$X_s(f) = \Im[x(t)\,x_p(t)]$$
$$= \sum_{n=-\infty}^{\infty} c_n \Im[x(t)e^{j2\pi n f_s t}]$$
$$= \sum_{n=-\infty}^{\infty} c_n X[f - nf_s]$$