MAT120: Integral Calculus and Differential Equations
BRAC University
Assignment 3

Name - Ishraq Ahmed Esha
ID - 19301261
Section - 06
Semester- Fall 2020
Set- 20

November 15, 2020

1 Find the arc length of the curve $x = ln(sec\, y)$ on the interval $y = 0$ and $y = \frac{\pi}{4}$

Solution

$$
\begin{aligned}
x &= ln(sec\, y) \\
\therefore \frac{dx}{dy} &= \frac{d}{dy}(ln(sec\, y))
\end{aligned}
$$

$using\ chain\ rule,\ Let,\ u = sec\, y\ \ x = ln\, u$

$$
\frac{du}{dy} = sec\, y\, tan\, y\ \ \frac{dx}{du} = \frac{1}{u}
$$

$$
\begin{aligned}
\therefore \frac{dx}{dy} &= \frac{dx}{du}\frac{du}{dy} \\
&= \frac{1}{u}\ sec\, y\, tan\, y \\
&= tan\, y \\
\therefore\ \left(\frac{dx}{dy}\right)^2 &= tan^2\, y \\
Now, & \\
s &= \int_0^{\frac{\pi}{4}} \sqrt{1 + tan^2\, y}\, dy \\
&= \int_0^{\frac{\pi}{4}} \sqrt{sec^2\, y}\, dy && [\because\ 1 + tan^2\, x = sex^2\, x] \\
&= \int_0^{\frac{\pi}{4}} sec\, y\, dy \\
Let, & \\
u &= sec\, y\, tan\, y \\
\Rightarrow \frac{du}{dy} &= sec\, y\, tan\, y\ + sec^2\, y \\
\therefore du &= sec\, y\, tan\, y\ + sec^2\, y\, dy \\
Now, & \\
\int sec\, y\, \frac{sec\, y\ +\ tan\, y}{sec\, y\ +\ tan\, y}\, dy &= \int \frac{sec^2\, y\ +\ sec\, y\, tan\, y}{sec\, y\ +\ tan\, y}\, dy \\
&= \int \frac{du}{u} \\
&= ln|u| + c \\
&= ln|sec\, y\ +\ tan\, y| + c \\
So, & \\
\int_0^{\frac{\pi}{4}} sec\, y\, dy &= ln\, [sec\, y\ +\ tan\, y]_0^{\frac{\pi}{4}} \\
&= ln|sec\, \frac{\pi}{4} +\ tan\, \frac{\pi}{4}| - ln|sec(0) + tan(0)| \\
&= ln(\sqrt{2} + 1)
\end{aligned}
$$

[ANSWER]

2 Evaluate the following indefinite integral by using a trigonometric substitution or otherwise

$$\int \frac{1}{\sqrt{4-9x^2}}dx$$

Solution

$$Let,$$

$$x = \frac{3u}{2} \qquad\qquad \therefore\ u = \frac{3w}{2}$$

$$\therefore dx = \frac{2}{3}du$$

$$Now,$$

$$\begin{aligned}
\int \frac{1}{\sqrt{2-9x^2}}dx &= \int \frac{1}{\sqrt{4-9\left(\frac{2}{3}u\right)^2}}\ \frac{2}{3}du \\
&= \frac{2}{3}\int \frac{1}{\sqrt{4-9\ \frac{4}{9}u^2}}du \\
&= \frac{2}{3}\int \frac{1}{\sqrt{4-4u^2}}du \\
&= \frac{2}{3}\int \frac{1}{2\sqrt{1-u^2}}du \\
&= \frac{2}{6}\int \frac{1}{\sqrt{1-u^2}}du \\
&= \frac{1}{3}\ sin^{-1}(u) + c \qquad \left[\because\ \int \frac{1}{\sqrt{1-x^2}}dx\ = sin^{-1}(x) + c\right] \\
&= \frac{1}{3}\ sin^{-1}\left(\frac{3x}{2}\right) + c
\end{aligned}$$

[ANSWER]

3 Find the arc length of the curve $x(t) = t^3 - 4t$, $y(t) = t^2 - 3t$ for $-2 \leq t \leq 2\pi$

Solution

$$\therefore \frac{dx}{dt} = 3t^2 - 4 \quad \therefore \frac{dy}{dt} = 2t - 3$$

$$Now,$$

$$\begin{aligned} s = \int_{-2}^{2\pi} \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}\, dt &= \int_{-2}^{2\pi} \sqrt{(3t^2 - 4)^2 + (2t - 3)^2}\, dt \\ &= \int_{-2}^{2\pi} \sqrt{9t^4 - 20t^2 - 12t + 25}\, dt \end{aligned}$$

[ANSWER]

4 Find the area of the surface generated by revolving the curve $x = \sqrt[3]{y}$ over the interval $1 \leq y \leq 8$ about the x-axis

Solution

$$x = \sqrt[3]{y}$$
$$\therefore y = x^3$$
$$\Rightarrow \frac{dy}{dx} = \frac{d}{dx}(x^3) \ = 3x^2$$
$$\therefore \left(\frac{dy}{dx}\right)^2 = 9x^4$$

$$Now,$$

$$s = \int_1^8 2\pi y\sqrt{1+9x^4}\ dx = \int_1^8 2\pi(x^3)\sqrt{1+9x^4}\ dx \qquad Let,$$
$$= \int_{10}^{36865} \sqrt{1+9x^4}\ \frac{1}{36}\ du \qquad u = 1+9x^4 \quad \therefore \frac{du}{dx} = 36x^3$$
$$= 2\pi\ \frac{1}{36}\int_{10}^{36865} u^{\frac{1}{2}}\ du \qquad du = 36x^3\ dx \quad \therefore \frac{1}{36} = x^3\ dx$$
$$= \frac{2\pi}{36}\ \frac{2}{3}\left[u^{\frac{3}{2}}]\right]_{10}^{36865} \qquad x \to 1\ u \to 10\ and\ x \to 8\ u \to 36865$$
$$\frac{\pi}{27}\left(36865^{\frac{3}{2}} - 10^{\frac{3}{2}}\right)$$

[ANSWER]

5 Find the area of the surface generated by revolving the curve $x = 2\sqrt{1-y}$ over the interval $-1 \le y \le 0$ about the y axis

Solution

$$\begin{aligned}
x &= 2\sqrt{1-y} \\
\frac{dx}{dy} &= 2\,\frac{(-1)}{2\sqrt{1-y}} \qquad = \frac{-1}{\sqrt{1-y}} \\
\therefore \left(\frac{dx}{dy}\right)^2 &= \frac{1}{1-y}
\end{aligned}$$

$$Now,$$

$$\begin{aligned}
s = \int_{-1}^{0} 2\pi x\sqrt{1+\frac{1}{1-y}}\,dy &= \int_{-1}^{0} 2\pi x\sqrt{\frac{1-y+1}{1-y}}\,dy && Let, \\
&= \int_{-1}^{0} 2\pi x\sqrt{\frac{2-y}{1-y}}\,dy && u = 2-y \\
&= 2\pi\int_{-1}^{0} 2\sqrt{1-y}\frac{\sqrt{2-y}}{\sqrt{1-y}}\,dy && du = -dy \\
&= -4\pi\int_{3}^{2} u^{\frac{1}{2}}\,du && y \to -1\ u \to 3\ and\ y \to 0\ u \to 2 \\
&= -4\pi\,\frac{2}{3}\left[u^{\frac{3}{2}}\right]_3^2 \\
&= -\frac{8\pi}{3}\left(2^{\frac{3}{2}} - 3^{\frac{3}{2}}\right) \\
&= \frac{8\pi}{3} \times (2.36)
\end{aligned}$$

[ANSWER]