MAT120: Integral Calculus and Differential Equations
BRAC University
Assignment 4

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1 Evaluate the double integral $\int\int_R (3x - 2y)\ dA$; R is the region enclosed by the circle $x^2 + y^2 = 1$

Solution

$-1 \leq x \leq 1 \quad ; \quad -\sqrt{1-x^2} \leq y \leq \sqrt{1-x^2}$

$$Now,$$

$$\begin{aligned}
\int\int_R (3x-2)\ dA &= \int_{-1}^{1}\int_{-\sqrt{1-x^2}}^{\sqrt{1-x^2}} (3x-2y)\ dy\ dx \\
&= \int_{-1}^{1} \left[3xy - y^2\right]_{-\sqrt{1-x^2}}^{\sqrt{1-x^2}}\ dx \qquad Let, u = 1-x^2 \\
&= \int_{-1}^{1} 6x\sqrt{1-x^2}\ dx \qquad \Rightarrow \frac{du}{dx} = -2x \\
&= -3\int_{-1}^{1} 2x\ dx\sqrt{u} \qquad \therefore du = -2xdx \\
&= -3\int_{-1}^{1} u^{\frac{1}{2}}\ du \\
&= -3\ \frac{2}{3}\left[u^{\frac{3}{2}}\right]_{-1}^{1} \\
&= 2\left[(1-x^2)^{\frac{3}{2}}\right]_{-1}^{1} \qquad [\because u = 1-x^2] \\
&= 2\ (0-0) \\
&= 0
\end{aligned}$$

[ANSWER]

2 Evaluate the double integral $\int\int_R\ y\ dA$; R is the region in the first quadrant enclosed between the circle $x^2 + y^2 = 25$ and the line $x + y = 5$

Solution

$0 \le x \le 5 \quad ; \quad 5 - x \le y \le \sqrt{25 - x^2}$

$$Now,$$

$$\begin{aligned}
\int\int_R y\ dA &= \int_0^5 \int_{5-x}^{\sqrt{25-x^2}} y\ dy\ dx \\
&= \int_0^5 \left[\frac{y^2}{2}\right]_{5-x}^{\sqrt{25-x^2}} dx \\
&= \int_5^0 \left(\frac{25-x^2}{2} - \frac{(5-x)^2}{2}\right) dx \\
&= \int_0^5 \left(\frac{10x - 2x^2}{2}\right) dx \\
&= \int_0^5 (5x - x^2)\ dx \\
&= \int_0^5 5x\ dx\ - \int_0^5 x^2\ dx \\
&= 5\left[\frac{x^2}{2}\right]_0^5 - \left[\frac{x^3}{3}\right]_0^5 \\
&= \frac{125}{2} - \frac{125}{3} \\
&= \frac{125}{6}
\end{aligned}$$

[ANSWER]

3 Evaluate the double integral $\int\int_R x(1+y^2)^{-\frac{1}{2}}\,dA$; R is the region in the first quadrant enclosed between $y = x^2, y = 4,$ and $x = 0$

Solution

$$0 \le x \le \sqrt{y} \quad ; \quad 0 \le y \le 4$$

$$Now,$$

$$\begin{aligned}
\int\int_R x(1+y^2)^{-\frac{1}{2}}\,dA &= \int_0^4 \int_0^{\sqrt{y}} x(1+y^2)^{-\frac{1}{2}}\,dx\,dy \\
&= \int_0^4 (1+y^2)^{-\frac{1}{2}}\,dy \int_0^{\sqrt{y}} x\,dx && Let, u = 1+y^2 \\
&= \int_0^4 (1+y^2)^{-\frac{1}{2}}\,\frac{1}{2}\left[x^2\right]_0^{\sqrt{y}} && \Rightarrow \frac{du}{dy} = 2y \\
&= \int_0^4 \frac{1}{2}\,(1+y)^{-\frac{1}{2}}\,y\,dy && \therefore ydy = \frac{du}{2} \\
&= \frac{1}{2}\int_0^4 u^{-\frac{1}{2}}\,\frac{du}{2} \\
&= \frac{1}{4}\int_0^4 u^{-\frac{1}{2}}\,du \\
&= \frac{1}{4}\,2\,\left[u^{\frac{1}{2}}\right]_0^4 \\
&= \frac{1}{2}\left[\left(1+y^2\right)^{\frac{1}{2}}\right]_0^4 && [\because u = (1+y^2)] \\
&= \frac{1}{2}\,\sqrt{17}-1 \\
&= \frac{\sqrt{17}-1}{2}
\end{aligned}$$

[ANSWER]

4 Evaluate the double integral $\int\int_R x\ cos\ y\ dA$; R is the triangular region bounded by the lines $y = x, x = 0$ and $x = \pi$

Solution

$$0 \le x \le \pi \quad ; \quad 0 \le y \le x$$

$$Now,$$

$$\begin{aligned}
\int\int_R x\ cos\ y\ dA &= \int_0^\pi \int_0^x x\ cos\ y\ dy\ dx \\
&= \int_\pi^0 x\ [sin\ y]_0^x\ dx && Let, u = x\ \therefore \frac{du}{dx} = 1 \\
&= \int_0^\pi x\ sin\ x\ dx && \frac{dv}{dx} = sin\ x\ \therefore v = -cos\ x \\
&= [sin\ x - x\ cos\ x\ ]_0^\pi && \left[\because \int u\ \frac{dv}{dx} = uv - \int v\ \frac{du}{dx}\right] \\
&= \pi
\end{aligned}$$

[ANSWER]

5 Evaluate the double integral $\int\int_R\ x\ dA$; R is the triangular region bounded by $y = sin^{-1}x, y = 0$ and $x = \frac{1}{\sqrt{2}}$

Solution

$$sin\ y \leq x \leq \frac{1}{\sqrt{2}} \quad ; \quad 0 \leq y \leq \frac{\pi}{4}$$

$$Now,$$

$$\begin{aligned}
\int\int_R x\ dA &= \int_0^{\frac{\pi}{4}} \int_{sin\ y}^{\frac{1}{\sqrt{2}}} x\ dx\ dy \\
&= \int_0^{\frac{\pi}{4}} \left[\frac{x^2}{2}\right]_{sin\ y}^{\frac{1}{\sqrt{2}}} dy \\
&= \int_0^{\frac{\pi}{4}} \left(\frac{1}{4} - \frac{sin^2\ y}{2}\right) dy \\
&= \int_0^{\frac{\pi}{4}} \left(\frac{2 - 4\ sin^2\ y}{8}\right) dy \qquad\qquad Let, u = 2y \Rightarrow \frac{du}{dy} = 2 \\
&= \int_0^{\frac{\pi}{4}} \frac{1}{4}\ cos\ 2y\ dy\ \ [\because 1 - 2sin^2\ A = cos\ 2A] \qquad \therefore dy = \frac{du}{2} \\
&= \frac{1}{4}\ \frac{1}{2} \int_0^{\frac{\pi}{4}} cos\ u\ du \\
&= \frac{1}{8}\ [sin\ u]_0^{\frac{\pi}{4}} \\
&= \frac{1}{8}\ [sin\ 2y]_0^{\frac{\pi}{4}} \quad [\because u = 2y] \\
&= \frac{1}{8}
\end{aligned}$$

[ANSWER]