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Assignment # 4   Artificial Intelligence

Q1:-

a)   True   False   False |= True

T   F   True

b)   True   False   (True |= False

T   F   False

c)   (A∧B) |= (A⟷B)

| A | B | A∧B | A⟷B | (A∧B)|= (A⟷B) |
|---|---|-----|-----|---------------|
| T | T | T   | T   | T |
| T | F | F   | F   | T |
| F | T | F   | F   | T |
| F | F | F   | T   | T |

d)   A⟷B |= A∨B

| A | B | A⟷B | A∨B | A⟷B|= A∨B |
|---|---|-----|-----|-----------|
| T | T | T   | T   | T |
| T | F | F   | T   | T |
| F | T | F   | T   | T |
| F | F | T   | F   | T |

e, $\quad A \longleftrightarrow B \quad \models \quad \sim A \lor B$

| A | B | $A \longleftrightarrow B$ | $\sim A$ | $\sim A \lor B$ | $A \leftrightarrow B \models \sim A \lor B$ |
|---|---|---|---|---|---|
| T | T | T | F | T | T |
| T | F | F | F | F | T |
| F | T | F | T | T | T |
| F | F | T | T | T | T |

f, $\quad (A \land B) \rightarrow C \models (A \rightarrow C) \lor (B \rightarrow C) =$ Final (F)

| A | B | C | $(A \land B)$ | $(A \land B) \rightarrow C$ | $(A \rightarrow C)$ | $(B \rightarrow C)$ | $(A \rightarrow C) \lor (B \rightarrow C)$ | F |
|---|---|---|---|---|---|---|---|---|
| T | T | T | T | T | T | T | T | T |
| T | T | F | T | F | F | F | F | T |
| T | F | T | F | T | T | T | T | T |
| T | F | F | F | T | F | T | T | T |
| F | T | T | F | T | T | T | T | T |
| F | T | F | F | T | T | F | T | T |
| F | F | T | F | T | T | T | T | T |
| F | F | F | F | T | T | T | T | T |

g) $(C \lor (\sim A \land \sim B)) \equiv ((A \to C) \land (B \to C)) =$ Final (F)

| A | B | C | ~A | ~B | (~A ∧ ~B) | (i) | A→C | B→C | (ii) |
|---|---|---|----|----|-----------|-----|-----|-----|------|
| T | T | T | F | F | F | T | T | T | T |
| T | T | F | F | F | F | F | F | F | F |
| T | F | T | F | F | F | T | T | T | T |
| T | F | F | F | T | F | F | F | T | F |
| F | T | T | T | F | F | T | T | T | T |
| F | T | F | T | F | F | F | T | F | F |
| F | F | T | T | T | T | T | T | T | T |
| F | F | F | T | T | T | T | T | T | T |

⇒ where (i) is equivalent to (ii)

(h part on next page) ⇒

h) $(A \lor B) \land (\sim C \lor \sim D \lor E) \models (A \lor B) =$ Final (F)

| A | B | C | D | E | (A∨B) | (~C∨~D∨E) | (i) | F | (ii) |
|---|---|---|---|---|-------|-----------|-----|---|------|
| T | T | T | T | T | T | F | T | T | F |
| T | T | T | T | F | T | F | T | T | T |
| T | T | T | F | T | T | T | T | T | T |
| T | T | T | F | F | T | T | T | T | T |
| T | T | F | T | T | T | T | T | T | F |
| T | T | F | T | F | T | T | T | T | T |
| T | T | F | F | T | T | F | F | T | T |
| T | F | T | T | T | T | F | T | T | F |
| T | F | T | T | F | T | F | T | T | T |
| T | F | T | F | T | T | T | T | T | T |
| T | F | T | F | F | T | T | T | T | T |
| T | F | F | T | T | T | T | T | T | F |
| T | F | F | T | F | T | T | T | T | T |
| T | F | F | F | T | T | T | T | T | T |
| F | T | T | T | T | T | F | T | T | T |
| F | T | T | T | F | T | F | T | T | F |
| F | T | T | F | T | T | T | T | T | T |
| F | T | T | F | F | T | T | T | T | T |
| F | T | F | T | T | T | T | T | T | F |
| F | T | F | T | F | T | T | T | T | T |
| F | T | F | F | T | T | T | T | T | T |
| F | T | F | F | F | T | T | T | T | T |
| F | F | T | T | T | F | F | F | F | F |
| F | F | T | T | F | F | F | F | F | F |
| F | F | T | F | T | F | T | F | F | F |
| F | F | T | F | F | F | T | F | F | F |
| F | F | F | T | T | F | T | F | F | F |
| F | F | F | T | F | F | T | F | F | F |
| F | F | F | F | T | F | T | F | F | F |
| F | F | F | F | F | F | T | F | F | F |

i) $(A\lor B)\land(\sim(\lor\sim D\lor E))\models (A\cup B)\land(\sim D\lor E)$

→ using the truth table made in (h) to solve this

j) 
$(A\lor B)\land\sim(A\to B)=F$

$(A\lor B)\land\sim(A\to B)=F$

| A | B | A∨B | A→B | ∼(A→B) | Final |
|---|---|-----|-----|--------|-------|
| T | T | T | T | F | F |
| T | F | T | F | T | T |
| F | T | T | T | F | F |
| F | F | F | T | F | F |

From the truth table, we know the statement is satisfiable

k) 
$(A\leftrightarrow B)\land(\sim A\lor B)=$ Final

| A | B | A↔B | ∼A | (∼A∨B) | Final |
|---|---|-----|----|--------|-------|
| T | T | T | F | T | T |
| T | F | F | F | F | F |
| F | T | F | T | T | F |
| F | F | T | T | T | T |

l) 
$(A\leftrightarrow B)\leftrightarrow L$

| A | B | L | (A↔B) | (A↔B)↔L |
|---|---|---|-------|---------|
| T | T | T | T | T |
| T | F | F | T | F |
| T | F | T | F | F |
| T | F | F | F | T |
| F | T | T | F | F |
| F | F | F | F | T |
| F | F | T | | |
| F | F | F | | |

Problem # 2

Smoke = S
Fire = F
Heat = H

a) S → S
   ~S ∨ S      (True)

b) S → F
   ~S ∨ F    (Satisfiable)

c) (S → F) → (~S → ~F)
   (~S ∨ F) → (S ∨ ~F)
   ~(~S ∨ F) ∨ (S ∨ ~F)
   (S ∧ ~F) ∨ (S ∨ ~F)
   (S ∨ S ∨ ~F) ∧ (F ∨ S ∨ ~F)
   (S ∨ ~F) ∧ (T)
   S ∨ ~F   (Satisfiable)

d) S ∨ F ∨ ~F

   S ∨ (True)
   S  (valid)

e) ((S ∧ H) → F) ⟷ ((S → F) ∨ (H → F))
   (~(S ∧ H) ∨ F)) ⟷ ((~S ∨ F) ∨ (~H ∨ F))
   ((~S ∨ H) ∨ F) ⟷ ((~S ∨ F) ∨ (~H ∨ F))
   [((~S ∨ ~H) ∨ F) → ((~S ∨ F) ∨ (~H ∨ F))] ∧ [((~S ∨ F) ∨ (~H ∨ F)]
   → ((~S ∨ ~H) ∨ F)]

i) (~((~S ∨ ~H) ∨ F) ∨ ((~S ∨ F) ∨ (~H ∨ F))
   (S ∧ H ∧ ~F) ∨ ((~S ∨ F) ∨ (~H ∨ F))

$(S \land H \land \sim F) \lor (\sim S \lor H \lor F)$

$\Rightarrow$ True

ii) $\quad \sim ((\sim S \lor F) \lor (\sim H \lor F)) \lor (\sim S \lor \sim H \lor F)$

$\qquad (S \land H \land F) \lor (\sim S \lor \sim H \lor F)$

$\Rightarrow$ True

Comparing (i) and (ii)

$T \land T$

$\Rightarrow$ Valid

f) 

$\quad (S \rightarrow F) \rightarrow ((S \land H) \rightarrow F)$

$\quad (\sim S \lor F) \rightarrow (\sim (S \land H) \lor F)$

$\quad \sim (\sim S \lor F) \rightarrow \lor ((\sim S \lor \sim H) \lor F)$

$\quad (S \land \sim F) \lor ((\sim S \lor \sim H \lor F))$

$\quad (S \lor \sim S \lor \sim H \lor F) \land (\sim F \lor \sim S \lor \sim H \lor F)$

$\quad (T \lor (S \lor \sim H) \lor (S \lor F)) \land ((\sim F \lor \sim S) \lor (\sim F \lor \sim H) \lor T)$

$\qquad T \land T$

$\Rightarrow$ Valid

g) $\quad$ Big $\lor$ Dumb $\lor$ (Big $\Rightarrow$ Dumb)

$\quad (B \lor D) \lor (\sim B \lor D)$

$\quad (B \lor \sim B) \land (B \lor D) \land (D \lor \sim B) \land (D \lor D)$

$\qquad$ True

$\Rightarrow$ Valid

# Q3-

Food=F

Party= P

Drinks = D

we are given , $[(F \to P) \vee (D \to P)] \to [(F \wedge D) \to P]$

a)

| F | P | D | F→P | D→P | F∧D | X | Y |
|---|---|---|-----|-----|-----|---|---|
| T | T | T | T | T | T | T | T |
| T | T | F | T | T | F | F | F |
| T | F | T | F | F | F | T | F |
| T | F | F | F | T | F | T | T |
| F | T | T | T | T | F | T | T |
| F | F | F | T | T | F | T | T |
| F | T | T | T | T | F | T | T |
| F | F | F | T | T | F | T | T |

=> From x and y, we can say that this statement is valid

b) For left hand side we have;

$(F \to P) \vee (D \to P)$

$(\sim F \vee P) \vee (\sim D \vee P)$

$\sim F \vee P \vee \sim D \vee P$

=> $\sim F \vee \sim D \vee P$

For right hand side we have

$(F \wedge D) \to P$

$\sim (F \wedge D) \vee P$

=> $\sim F \vee \sim D \vee P$

By comparing both LHS and RHS above, we can see that the results are identical hence the original statement is valid.

c) For resolution, we negate as

$\sim [[(F \to P) \lor (D \to P)] \to [(F \land D) \to P)]$

$\sim ((F \lor \sim D \lor P) \to (\sim F \lor \sim D \lor P))$ from (b)

$\sim ((F \land D \land \sim P) \lor (\sim F \lor \sim D \lor P))$

$\sim (True)$

$\Rightarrow$ False

we can see that the negation is unsatisfiable; hence proving (a)

Q4:-

a) Creating KB knowledge base:

$R_1: \sim W_n$

$R_2: \sim P_{11}$

$R_3: B_{21}$

$R_4: S_{12}$

$R_5: \sim B_{12}$

$R_6: S_{12} \to W_{13}$ or $\sim S_{12} \to W_{13}$

$R_7: S_{12} \to W_{22}$ or $\sim S_{12} \to W_{22}$

$R_8: B_{21} \to P_{31}$ or $\sim B_{21} \lor P_{31}$

$R_9: B_{21} \to P_{22}$ or $\sim B_{21} \lor P_{22}$

$R_{10}: \sim B_{12} \to \sim P_{22}$ or $B_{12} \lor \sim P_{22}$

$R_{11}: \sim B_{12} \to \sim P_{13}$ or $B_{12} \lor \sim P_{13}$

$R_{12}: \sim B_{12} \to \sim P_{13}$ or $B_{12} \lor \sim P_{13}$

$R_{13}: \sim S_{21}$

$R_{14}: \sim S_{21} \to \sim W_{22}$ or

$S_{21} \lor \sim V_{22}$

b) $KB \models \alpha_1$ where $\alpha_1 = \sim P_{22}$

$R_{15}: P_{22}$ [Negated conclusion]

$R_{16}: \sim P_{22}$ [5, 10 Propositional Resolution]

$R_{17}:$ [15, 16 Propositional Resolution]

c) $KB \models \alpha_2$ where $\alpha_2 = W_{13}$

R6: $S_{12} \longrightarrow W_{13}$

R4: $S_{12}$

R18: [Modus Ponens]

R6: $S_{12} \longrightarrow W_{13}$

R4: $S_{12}$

R18: [Modus Ponens]