Tractable Closure-Based Possibilistic Repair for Partially Ordered DL-Lite Ontologies (supplementary material)

Ahmed Laouar[1], Sihem Belabbes[2], and Salem Benferhat[1]

[1] CRIL, Univ. Artois & CNRS, Lens, France
{laouar, benferhat}@cril.fr
[2] LIASD, IUT de Montreuil, Univ. Paris 8, Montreuil, France
belabbes@iut.univ-paris8.fr

Table of Contents

A Proofs for Section 4: Characterization of Cπ-repair

Lemma 1. *Let $\mathcal{K}_{\rhd} = \langle \mathcal{T}, \mathcal{A}_{\unrhd} \rangle$ be a consistent, partially preordered KB, with $\mathsf{Cf}(\mathcal{A}_{\unrhd}) = \emptyset$. Let $cl(\cdot)$ be as in Definition 3. Let φ be an assertion. Then: $c\pi(\mathcal{A}_{\unrhd}) = cl(\mathcal{A}_{\unrhd})$. Equivalently: $\varphi \in c\pi(\mathcal{A}_{\unrhd})$ iff $\exists \mathcal{B} \subseteq \mathcal{A}_{\unrhd}$ s.t. $\langle \mathcal{T}, \mathcal{B} \rangle \models \varphi$.*

Proof. The proof here is straightforward. Let $\mathcal{A}_{\unrhd}$ be a consistent ABox, let $c\pi(\mathcal{A}_{\unrhd})$ be as in definition 4 and $\mathcal{A}_{\geq}$ a compatible base of $\mathcal{A}_{\unrhd}$ preserving its strict relations.
Since the ABox is consistent, $\forall \mathcal{A}_{\geq}$ compatible base, the possibilistic-based repair can be computed as: $\mathcal{R}(\mathcal{A}_{\geq}) = \mathcal{A}_{\geq}$, hence $c\pi(\mathcal{A}_{\unrhd}) = \bigcap\{cl(\mathcal{A}_{\geq})\}$ which is equivalent to $cl(\mathcal{A}_{\unrhd})$ (extending the partially preordered ABox to a total order affects only the order between the assertions).

Proposition 1. *Let $\mathcal{K}_{\rhd} = \langle \mathcal{T}, \mathcal{A}_{\unrhd} \rangle$ be an inconsistent, partially preordered KB, $\mathsf{Cf}(\mathcal{A}_{\unrhd})$ be its conflict set and φ be an assertion. If $\forall \mathcal{C} \in \mathsf{Cf}(\mathcal{A}_{\unrhd})$, $\exists \mathcal{B} \subseteq \mathcal{A}_{\unrhd}$ s.t.:*
1. *$\mathcal{B}$ supports φ (as per Definition 5), and*
2. *$\mathcal{B} \rhd^{dom} \mathcal{C}$ (as per Definition 6),*

then $\varphi \in c\pi(\mathcal{A}_{\unrhd})$.

Proof. Let $\mathcal{K}_{\rhd} = \langle \mathcal{T}, \mathcal{A}_{\unrhd} \rangle$ be a partially preordered KB, $\mathsf{Cf}(\mathcal{A}_{\unrhd})$ be its conflict set, and φ be an assertion. Assume that the following condition of our proposition: $\forall \mathcal{C} \in \mathsf{Cf}(\mathcal{A}_{\unrhd})$, $\exists \mathcal{B} \subseteq \mathcal{A}_{\unrhd}$ supporting φ s.t. $\mathcal{B} \rhd^{dom} \mathcal{C}$ ...(*) is true. Let us show that $\varphi \in c\pi(\mathcal{A}_{\unrhd})$.
From Definition 4, $\varphi \in c\pi(\mathcal{A}_{\unrhd})$ means that for each total preorder $\geq$ compatible with $\unrhd$, we have $\mathcal{R}(\mathcal{A}_{\geq}) \models \varphi$. Let $\geq_1$ be a given compatible total preorder and $(\mathcal{S}_1, \ldots, \mathcal{S}_n)$ be its associated well-ordered partition. Let $\mathcal{R}(\mathcal{A}_{\geq_1}) = (\mathcal{S}_1 \cup \cdots \cup \mathcal{S}_i)$ be the possibilistic repair (see Section 2 for the definition of possibilistic repair). Now let us show that indeed $\mathcal{R}(\mathcal{A}_{\geq_1}) \models \varphi$. Recall from Section 2 that $\langle \mathcal{T}, \mathcal{R}(\mathcal{A}_{\geq_1}) \rangle$ is consistent, but $\langle \mathcal{T}, \mathcal{R}(\mathcal{A}_{\geq_1}) \cup \mathcal{S}_{i+1} \rangle$ is inconsistent. Hence, there is at least one conflict $\mathcal{C} \subseteq (\mathcal{R}(\mathcal{A}_{\geq_1}) \cup \mathcal{S}_{i+1})$. By (*), φ has a support $\mathcal{B}$ s.t. $\mathcal{B} \rhd^{dom} \mathcal{C}$. By Definition 6, $\mathcal{B} \rhd^{dom} \mathcal{C}$ means $\forall \phi \in \mathcal{B}$, $\exists c_j \in \mathcal{C}$ s.t. $\phi \rhd c_j$. Since $\geq_1$ extends $\unrhd$, this also means that $\forall \phi \in \mathcal{B}$, $\exists c_j \in \mathcal{C}$ s.t. $\phi >_1 c_j$. Therefore, $\forall \phi \in \mathcal{B}$, $\phi \in \mathcal{S}_1 \cup \cdots \cup \mathcal{S}_i$. Hence, $\mathcal{B} \subseteq \mathcal{R}(\mathcal{A}_{\geq_1})$. By Definition 5, $\mathcal{B} \models \varphi$, so $\mathcal{R}(\mathcal{A}_{\geq_1}) \models \varphi$. Since this result applies to all the compatible orders, we conclude that $\varphi \in c\pi(\mathcal{A}_{\unrhd})$.

Proposition 2. *Let $\mathcal{K}_{\rhd} = \langle \mathcal{T}, \mathcal{A}_{\unrhd} \rangle$ be an inconsistent, partially preordered KB, $\mathsf{Cf}(\mathcal{A}_{\unrhd})$ its conflict set and φ an assertion.*
If $\varphi \in c\pi(\mathcal{A}_{\unrhd})$, then $\forall \mathcal{C} \in \mathsf{Cf}(\mathcal{A}_{\unrhd}), \exists \mathcal{B} \subseteq \mathcal{A}_{\unrhd}$ s.t.:

1. *$\mathcal{B}$ supports φ (as per Definition 5), and*
2. *$\mathcal{B} \rhd^{dom} \mathcal{C}$ (as per Definition 6).*

Proof. Let $\varphi \in c\pi(\mathcal{A}_{\unrhd})$ and assume that the following condition is false:
$\forall \mathcal{C} \in \mathsf{Cf}(\mathcal{A}_{\unrhd})$, $\exists \mathcal{B} \subseteq \mathcal{A}_{\unrhd}$ support of φ s.t. $\mathcal{B} \rhd^{dom} \mathcal{C}$ ...(*). We distinguish two cases:

1. There is no support for φ. This means that for any total preorder $\geq$ compatible with $\trianglerighteq$, there will be no support in $\mathcal{A}_{\geq}$ (the notion of support is independent from the preference relation). This also means that there will be no support in $\mathcal{R}(\mathcal{A}_{\geq})$ and $c\pi(\mathcal{A}_{\trianglerighteq})$. Hence, this means that φ does not belong neither to $\mathcal{R}(\mathcal{A}_{\geq})$ nor to its closure $cl(\mathcal{R}(\mathcal{A}_{\geq}))$. But this contradicts our assumption that $\varphi \in c\pi(\mathcal{A}_{\trianglerighteq})$.
2. There is at least one support for φ. Then, (*) is false means that there is a conflict that is not dominated by any support. Let $\mathcal{C}$ be such a conflict. We then have:

   $\forall \mathcal{B} \subseteq \mathcal{A}_{\trianglerighteq}$ a support of φ, $\mathcal{B} \triangleright^{dom} \mathcal{C}$ does not hold . . . (**).

   If $\mathcal{B} \triangleright^{dom} \mathcal{C}$ does not hold, then there is $\phi \in \mathcal{B}$ s.t. $\forall c_j \in \mathcal{C}$, either $c_j \triangleright \phi$ or $c_j \equiv_{\triangleright} \phi$ or $c_j \bowtie \phi$ (namely, ϕ is not strictly preferred to c_j). $c_j \equiv_{\triangleright} \phi$ means that both $c_j \trianglerighteq \phi$ and $\phi \trianglerighteq c_j$ hold. The idea is to construct a sequence of partial preorders ($\trianglerighteq_0 = \trianglerighteq, \trianglerighteq_1, \ldots, \trianglerighteq_n$) where $\trianglerighteq_i$ ($i \geq 1$) is obtained from $\trianglerighteq_{i-1}$ by transforming one case of incomparability between an element c_j of $\mathcal{C}$ and an element ϕ of a support $\mathcal{B}$ of φ into a preference of c_j over ϕ. The last partial preorder $\trianglerighteq_n$ is s.t. there is no incomparability between elements of $\mathcal{C}$ and of $\mathcal{B}$. One can easily check that each compatible order of $\trianglerighteq_i$ is also a compatible order of $\trianglerighteq_{j<i}$ and so of $\trianglerighteq_0 = \trianglerighteq$. One can also check that the condition (**) holds for each $\trianglerighteq_i$. At step n, $\mathcal{B} \triangleright_n^{dom} \mathcal{C}$ does not hold means that $\exists \phi \in \mathcal{B}$ s.t. $\forall c_j \in \mathcal{C}$ we have either $c_j \trianglerighteq_n \phi$ or $c_j \trianglerighteq \phi$ since there is no incomparability relation in $\trianglerighteq_n$. Lastly, let $\geq$ be a total preorder compatible with $\trianglerighteq_n$ and let $(\mathcal{S}_1, \ldots, \mathcal{S}_n)$ be its associated well-ordered partition and $\mathcal{R}(\mathcal{A}_{\geq}) = \mathcal{S}_1 \cup \cdots \cup \mathcal{S}_i$. It is easy to see that there is no support $\mathcal{B}$ of φ s.t. $\mathcal{B} \subseteq \mathcal{R}(\mathcal{A}_{\geq})$. If this was the case, then $\mathcal{C} \subseteq \mathcal{R}(\mathcal{A}_{\geq})$, which is impossible since $\mathcal{R}(\mathcal{A}_{\geq})$ is consistent. The fact that φ has no support in $\mathcal{R}(\mathcal{A}_{\geq})$ also means that $\varphi \notin cl(\mathcal{R}(\mathcal{A}_{\trianglerighteq}))$. And this contradicts our assumption that $\varphi \in c\pi(\mathcal{A}_{\triangleright})$.

Proposition 3.
1. $\langle \mathcal{T}, c\pi(\mathcal{A}_{\trianglerighteq}) \rangle$ *is consistent.*
2. $\pi(\mathcal{A}_{\trianglerighteq}) \subseteq c\pi(\mathcal{A}_{\trianglerighteq})$. *The converse is false.*

Proof. 1. The consistency of Cπ-repair follows first from the fact that if $\mathcal{A}_{\geq}$ is a totally preordered ABox, then by construction of the possibilistic repair (recalled in Section 2), the sub-base $\mathcal{R}(\mathcal{A}_{\geq})$ is consistent. Therefore, its positive deductive closure $cl(\mathcal{R}(\mathcal{A}_{\geq}))$ is also consistent since only positive axioms are considered for computing the closure. Therefore, $c\pi(\mathcal{A}_{\trianglerighteq})$ is also consistent since, by definition, $c\pi(\mathcal{A}_{\trianglerighteq})$ is obtained by intersecting all positive closures of repairs of each compatible totally preordered ABox.

2. The proof is immediate since for each compatible ABox $\mathcal{A}_{\geq}$, we have $\mathcal{R}(\mathcal{A}_{\geq}) \subseteq cl(\mathcal{R}(\mathcal{A}_{\geq}))$. Thus, $\bigcap\{\mathcal{R}(\mathcal{A}_{\geq}) | \mathcal{A}_{\geq}$ compatible with $\mathcal{A}_{\trianglerighteq}\} \subseteq \bigcap\{cl(\mathcal{R}(\mathcal{A}_{\geq})) | \mathcal{A}_{\geq}$ compatible with $\mathcal{A}_{\trianglerighteq}\}$. Example 2 is a counterexample of the converse.

Proposition 4. *Let* $\mathcal{K}_{\triangleright} = \langle \mathcal{T}, \mathcal{A}_{\trianglerighteq} \rangle$ *be a partially preordered KB and* φ *be an assertion. Checking if* $\varphi \in c\pi(\mathcal{A}_{\trianglerighteq})$ *is done in PTIME in DL-Lite*$_{\mathcal{R}}$*.*

Proof. The proof of tractability of computing $c\pi(\mathcal{A}_{\trianglerighteq})$ comes first from our characterization of Cπ-repair given in Propositions 1 and 2, that does not need to enumerate all compatible totally preordered ABoxes. We recall that this characterization consists in saying that $\varphi \in c\pi(\mathcal{A}_{\trianglerighteq})$ iff $\forall \mathcal{C}$ a conflict, $\exists \mathcal{B}$ a support of φ s.t. $\mathcal{B} \triangleright^{dom} \mathcal{C}$. Therefore, one can easily provide a polynomial algorithm to check if a given assertion φ belongs or not to Cπ-repair. Note that, In DL-Lite$_{\mathcal{R}}$, the size of a given conflict and support is two, so checking if $\mathcal{B} \triangleright^{dom} \mathcal{C}$ is in $\mathcal{O}(1)$ (in DL-Lite$_{\mathcal{R}}$, computing the conflict set is done in polynomial time [15].). Therefore the number of conflicts and supports is bounded by $|\mathcal{A}_{\trianglerighteq}|^2$ (the extreme case where all assertions are pairwise conflicting). Now, once the sets of all conflicts and all supports are computed, a simple algorithm consists of iterating over the conflict set, then for each conflict $\mathcal{C}$, parse the set of supports to check if some support $\mathcal{B}$ exists and dominate $\mathcal{C}$. Then the whole steps of checking if φ belongs to Cπ-repair is achieved in polynomial time.

B Proofs for Section 5: Rationality properties of π-accepted and Cπ-repair

In the following, we provide the study of the satisfaction of the properties of unconditional and conditional inferences with both π-accepted assertions and Cπ-repair semantics. Consider respectively, $\pi(.)$, $\mathcal{R}(.)$ and $c\pi(.)$: the π-accepted assertions repair as in definition 2, the possibilistic repair as described in section 2 and the Cπ-repair as in definition 4. Consider conflicts as in definition 1 and the positive deductive closure as in definition 3.

Proposition 5. *The unconditional possibilistic inference relations $\models^{\pi}$ and $\models^{c\pi}$ satisfy the following properties (with $s \in \{\pi, c\pi\}$):*

- ***QCE*** *(Query Conjunction Elimination): if $\mathcal{K}_{\triangleright} \models^{s} q_1 \wedge q_2$ then $\mathcal{K}_{\triangleright} \models^{s} q_1$ and $\mathcal{K}_{\triangleright} \models^{s} q_2$.*
- ***QCI*** *(Query Conjunction Introduction): if $\mathcal{K}_{\triangleright} \models^{s} q_1$ and $\mathcal{K}_{\triangleright} \models^{s} q_2$ then $\mathcal{K}_{\triangleright} \models^{s} q_1 \wedge q_2$.*
- ***Cons*** *(Consistency): for any set of assertions $\mathcal{B}$, if $\mathcal{K}_{\triangleright} \models^{s} \mathcal{B}$ then $\langle \mathcal{T}, \mathcal{B} \rangle$ is consistent.*
- ***ConsC*** *(Consistency of conjunction): for any set of assertions $\mathcal{B}$, if $\forall \varphi \in \mathcal{B}$, $\mathcal{K}_{\triangleright} \models^{s} \varphi$ then $\langle \mathcal{T}, \mathcal{B} \rangle$ is consistent.*
- ***ConsS*** *(Consistency of support): for any set of assertions $\mathcal{B}$, if $\mathcal{K} \models^{s} \mathcal{B}$ then there is a maximally consistent sub-base $\mathcal{A}'$ of $\mathcal{A}_{\trianglerighteq}$ s.t. $\langle \mathcal{T}, \mathcal{A}' \rangle \models \mathcal{B}$.*

Proof. Let $\mathcal{K}_{\triangleright} = \langle \mathcal{T}, \mathcal{A}_{\trianglerighteq} \rangle$ be a partially preordered KB and q be a query. Let $\models^{\pi}$ (resp. $\models^{c\pi}$) be the unconditional query answering relations using the π-accepted assertions (resp. Cπ-repair) methods. The proof of Proposition 5 is straightforward. It first follows from the fact that both repairs $\pi(\mathcal{A}_{\trianglerighteq})$ and $c\pi(\mathcal{A}_{\trianglerighteq})$ are consistent (Proposition 3). It also follows from the fact that the unconditional query answering relation $\mathcal{K}_{\triangleright} \models^{\pi} q$ (resp. $\mathcal{K}_{\triangleright} \models^{c\pi} q$) amounts to a standard inference relation from a consistent KB: $\langle \mathcal{T}, \pi(\mathcal{A}_{\trianglerighteq}) \rangle \models q$ (resp. $\langle \mathcal{T}, c\pi(\mathcal{A}_{\trianglerighteq}) \rangle \models q$). As for any consistent and certain KB the standard entailment $\models$ satisfies **QCE**, **QCI**, **Cons**, and **ConsC** [3]. Therefore, since $\pi(\mathcal{A}_{\trianglerighteq})$ and $c\pi(\mathcal{A}_{\trianglerighteq})$ are both consistent, they both satisfy these properties. The proof of **ConsS** is also immediate. As stated in Proposition 3, $\pi(\mathcal{A}_{\trianglerighteq})$ (resp. $c\pi(\mathcal{A}_{\trianglerighteq})$) is consistent, then it is enough to start with $\pi(\mathcal{A}_{\trianglerighteq})$ (resp. $c\pi(\mathcal{A}_{\trianglerighteq})$), and extend it to get a maximally consistent repair.

Proposition 6. *Let $\mathcal{K}_{\triangleright} = \langle \mathcal{T}, \mathcal{A}_{\trianglerighteq} \rangle$ be a partially preordered KB, $\mathcal{O}_i$ be a set of assertions and q be a query. Query answering relations $\mid\!\sim^{\pi}_{\mathcal{K}_{\triangleright}}$ and $\mid\!\sim^{c\pi}_{\mathcal{K}_{\triangleright}}$ satisfy the properties **R**, **LLE**, **RW**, **Cut**, **CM** and **And**.*

Lemma 2. *Query answering relations $\mid\!\sim^{\pi}_{\mathcal{K}_{\triangleright}}$ and $\mid\!\sim^{c\pi}_{\mathcal{K}_{\triangleright}}$ satisfy **R** ($\mathcal{O}_i \mid\!\sim^{\pi}_{\mathcal{K}_{\triangleright}} \mathcal{O}_i$ and $\mathcal{O}_i \mid\!\sim^{c\pi}_{\mathcal{K}_{\triangleright}} \mathcal{O}_i$).*

Proof. Let $\mathcal{O}_i$ be a set of assertions, $\mathcal{K}_{\mathcal{O}_i} = \langle \mathcal{T}, \mathcal{A}_{\trianglerighteq_{\mathcal{O}_i}} \rangle$ is the augmented KB where $\mathcal{A}_{\trianglerighteq_{\mathcal{O}_i}} = (\mathcal{A}_{\trianglerighteq} \cup \mathcal{O}_i)$ results from adding $\mathcal{O}_i$ to $\mathcal{A}_{\trianglerighteq}$ with the highest priority. In order to prove the satisfaction of **R**, we show that, for any observation $\mathcal{O}_i$

(a set of assertions), the construction of $\mathcal{A}_{\trianglerighteq_{\mathcal{O}_i}}$ guarantees that $\mathcal{O}_i \subseteq \pi(\mathcal{A}_{\trianglerighteq_{\mathcal{O}_i}})$ (resp. $\mathcal{O}_i \subseteq c\pi(\mathcal{A}_{\trianglerighteq_{\mathcal{O}_i}})$).
By definition, the resulting partial preorder $\trianglerighteq_{\mathcal{O}_i}$ has: $\forall \varphi_1 \in \mathcal{O}_i$, $\forall \varphi_2 \in \mathcal{A}_{\trianglerighteq} \setminus \mathcal{O}_i$: $\varphi_1 \triangleright_{\mathcal{O}_i} \varphi_2$. Which means, for any totally preordered extension $\mathcal{A}_{\geq_{\mathcal{O}_i}}$ of $\mathcal{A}_{\trianglerighteq_{\mathcal{O}_i}}$, $\forall \varphi_1 \in \mathcal{O}_i$, $\forall \varphi_2 \in \mathcal{A}_{\geq} \setminus \mathcal{O}_i$: $\varphi_1 \geq_{\mathcal{O}_i} \varphi_2$.
Let $\mathcal{S}_1 \cup \cdots \cup \mathcal{S}_n$ be the well-preordered partition associated with $\mathcal{A}_{\geq}$, then $\mathcal{A}_{\geq_{\mathcal{O}_i}} = \mathcal{O}_i \cup \mathcal{S}_1 \cup \cdots \cup \mathcal{S}_n$. Since $\mathcal{O}_i$ is consistent, by definition of $\mathcal{R}(\mathcal{A}_{\geq_{\mathcal{O}_i}})$ the possibilistic repair (see Section 2), $\mathcal{O}_i \subseteq \mathcal{R}(\mathcal{A}_{\geq_{\mathcal{O}_i}})$ and $\mathcal{O}_i \subseteq cl(\mathcal{R}(\mathcal{A}_{\geq_{\mathcal{O}_i}}))$. Therefore, $\mathcal{O}_i \subseteq \pi(\mathcal{A}_{\trianglerighteq_{\mathcal{O}_i}})$ (resp. $\mathcal{O}_i \subseteq c\pi(\mathcal{A}_{\trianglerighteq_{\mathcal{O}_i}})$).

$\mathcal{O}_i \subseteq \pi(\mathcal{A}_{\trianglerighteq_{\mathcal{O}_i}})$ (resp. $\mathcal{O}_i \subseteq c\pi(\mathcal{A}_{\trianglerighteq_{\mathcal{O}_i}})$) means that $\langle \mathcal{T}, \pi(\mathcal{A}_{\trianglerighteq_{\mathcal{O}_i}}\rangle \models \mathcal{O}_i$ (resp. $\langle \mathcal{T}, c\pi(\mathcal{A}_{\trianglerighteq_{\mathcal{O}_i}}\rangle \models \mathcal{O}_i$). Hence, $\mathcal{K}_{\mathcal{O}_i} \models^{\pi} \mathcal{O}_i$ (resp. $\mathcal{K}_{\mathcal{O}_i} \models^{c\pi} \mathcal{O}_i$). Therefore, by Definition 7, $\mathcal{O}_i \mid\!\sim^{\pi}_{\mathcal{K}_{\triangleright}} \mathcal{O}_i$ (resp. $\mathcal{O}_i \mid\!\sim^{c\pi}_{\mathcal{K}_{\triangleright}} \mathcal{O}_i$).

Lemma 3. *Query answering relations $\mid\!\sim^{\pi}_{\mathcal{K}_{\triangleright}}$ and $\mid\!\sim^{c\pi}_{\mathcal{K}_{\triangleright}}$ satisfy **LLE**. Formally, if $\langle \mathcal{T}, \mathcal{O}_1\rangle \equiv \langle \mathcal{T}, \mathcal{O}_2\rangle$ and $\mathcal{O}_1 \mid\!\sim^{s}_{\mathcal{K}_{\triangleright}} \mathcal{O}_3$ then $\mathcal{O}_2 \mid\!\sim^{s}_{\mathcal{K}_{\triangleright}} \mathcal{O}_3$ (in our case $s = \pi$ or $s = c\pi$).*

Proof. Let $\mathcal{O}_1$ and $\mathcal{O}_2$ be two consistent sets of assertions and $\mathcal{K}_{\triangleright} = \langle \mathcal{T}, \mathcal{A}_{\trianglerighteq}\rangle$ be a partially preordered KB. For $(i = 1, 2)$ $\mathcal{K}_{\mathcal{O}_i} = \langle \mathcal{T}, \mathcal{A}_{\trianglerighteq_{\mathcal{O}_i}}\rangle$ is the augmented KB where $\mathcal{A}_{\trianglerighteq_{\mathcal{O}_i}} = (\mathcal{A}_{\trianglerighteq} \cup \mathcal{O}_i)$ results from adding $\mathcal{O}_i$ to $\mathcal{A}_{\trianglerighteq}$ with the highest priority. In order to prove the satisfaction of **LLE**, we show that if $\langle \mathcal{T}, \mathcal{O}_1\rangle \equiv \langle \mathcal{T}, \mathcal{O}_2\rangle$ then $\langle \mathcal{T}, \pi(\mathcal{A}_{\trianglerighteq_{\mathcal{O}_1}})\rangle \equiv \langle \mathcal{T}, \pi(\mathcal{A}_{\trianglerighteq_{\mathcal{O}_2}})\rangle$ (resp. $\langle \mathcal{T}, c\pi(\mathcal{A}_{\trianglerighteq_{\mathcal{O}_1}})\rangle \equiv \langle \mathcal{T}, c\pi(\mathcal{A}_{\trianglerighteq_{\mathcal{O}_2}})\rangle$).

Assume that $\langle \mathcal{T}, \mathcal{O}_1\rangle \equiv \langle \mathcal{T}, \mathcal{O}_2\rangle$ and let us show that we have indeed:
$\langle \mathcal{T}, \pi(\mathcal{A}_{\trianglerighteq_{\mathcal{O}_1}})\rangle \equiv \langle \mathcal{T}, \pi(\mathcal{A}_{\trianglerighteq_{\mathcal{O}_2}})\rangle$:
this means that for any given subset of assertions $\mathcal{S}_i$, $\langle \mathcal{T}, \mathcal{O}_1 \cup \mathcal{S}_i\rangle$ is consistent if and only if $\langle \mathcal{T}, \mathcal{O}_2 \cup \mathcal{S}_i\rangle$ is consistent and $\langle \mathcal{T}, \mathcal{O}_1 \cup \mathcal{S}_i\rangle$ is inconsistent if and only if $\langle \mathcal{T}, \mathcal{O}_2 \cup \mathcal{S}_i\rangle$ is inconsistent ... (*).
Now assume that $\langle \mathcal{T}, \pi(\mathcal{A}_{\trianglerighteq_{\mathcal{O}_1}})\rangle \equiv \langle \mathcal{T}, \pi(\mathcal{A}_{\trianglerighteq_{\mathcal{O}_2}})\rangle$ does not hold, this means there exists at least one totally preordered extension $\mathcal{A}_{\geq}$ of $\mathcal{A}_{\trianglerighteq}$ s.t. $\langle \mathcal{T}, \mathcal{R}(\mathcal{A}_{\geq_{\mathcal{O}_1}})\rangle \not\equiv \langle \mathcal{T}, \mathcal{R}(\mathcal{A}_{\geq_{\mathcal{O}_2}})\rangle$ ... (**) (with $\mathcal{A}_{\geq_{\mathcal{O}_1}}$ and $\mathcal{A}_{\geq_{\mathcal{O}_2}}$ are respectively the associated totally preordered extensions of $\mathcal{A}_{\trianglerighteq_{\mathcal{O}_1}}$ and $\mathcal{A}_{\trianglerighteq_{\mathcal{O}_2}}$).

Let $(\mathcal{O}_1 \cup \mathcal{S}_1 \cup \cdots \cup \mathcal{S}_n)$ be the well-ordered partition associated with $\mathcal{A}_{\geq_{\mathcal{O}_1}}$. The possibilistic repair in this case is $\mathcal{R}(\mathcal{A}_{\geq_{\mathcal{O}_1}}) = \mathcal{O}_1 \cup \mathcal{S}_1 \cup \cdots \cup \mathcal{S}_i$ s.t.
$\langle \mathcal{T}, \mathcal{R}(\mathcal{A}_{\geq_{\mathcal{O}_1}})\rangle$ is consistent and $\langle \mathcal{T}, \mathcal{R}(\mathcal{A}_{\geq_{\mathcal{O}_1}}) \cup \mathcal{S}_{i+1}\rangle$ is inconsistent. $\mathcal{R}(\mathcal{A}_{\geq_{\mathcal{O}_2}})$ is computed in the same way, in fact, based on the result of (*), since $\mathcal{O}_1 \cup \mathcal{S}_1 \cup \cdots \cup \mathcal{S}_i$ is consistent, $\mathcal{O}_2 \cup \mathcal{S}_1 \cup \cdots \cup \mathcal{S}_i$ is consistent and since $\mathcal{O}_1 \cup \mathcal{S}_1 \cup \cdots \cup \mathcal{S}_{i+1}$ is inconsistent then $\mathcal{O}_2 \cup \mathcal{S}_1 \cup \cdots \cup \mathcal{S}_{i+1}$ is inconsistent. Hence, $\mathcal{R}(\mathcal{A}_{\geq_{\mathcal{O}_2}}) = \mathcal{O}_2 \cup \mathcal{S}_1 \cup \cdots \cup \mathcal{S}_i$. Therefore, the repairs $\mathcal{R}(\mathcal{A}_{\geq_{\mathcal{O}_1}})$ and $\mathcal{R}(\mathcal{A}_{\geq_{\mathcal{O}_2}})$ are equivalent. This contradicts our assumption in (**). Which means for all totally preordered extension $\mathcal{A}_{\geq}$ of $\mathcal{A}_{\trianglerighteq}$, $\langle \mathcal{T}, \mathcal{R}(\mathcal{A}_{\geq_{\mathcal{O}_1}})\rangle \equiv \langle \mathcal{T}, \mathcal{R}(\mathcal{A}_{\geq_{\mathcal{O}_2}})\rangle$. Thus, $\langle \mathcal{T}, \pi(\mathcal{A}_{\trianglerighteq_{\mathcal{O}_1}})\rangle \equiv \langle \mathcal{T}, \pi(\mathcal{A}_{\trianglerighteq_{\mathcal{O}_2}})\rangle$. Similarly for the case of Cπ-repair, applying the deductive closure on equivalent consistent bases gives equivalent bases. Hence, $\langle \mathcal{T}, cl(\mathcal{R}(\mathcal{A}_{\geq_{\mathcal{O}_1}}))\rangle \equiv \langle \mathcal{T}, cl(\mathcal{R}(\mathcal{A}_{\geq_{\mathcal{O}_2}}))\rangle$. Therefore, we also have $\langle \mathcal{T}, c\pi(\mathcal{A}_{\trianglerighteq_{\mathcal{O}_1}})\rangle \equiv \langle \mathcal{T}, c\pi(\mathcal{A}_{\trianglerighteq_{\mathcal{O}_2}})\rangle$.

Based on these results, $\pi(\mathcal{A}_{\trianglerighteq_{\mathcal{O}_1}})$ and $\pi(\mathcal{A}_{\trianglerighteq_{\mathcal{O}_2}})$ have the same consequences (resp. $c\pi(\mathcal{A}_{\trianglerighteq_{\mathcal{O}_1}})$ and $c\pi(\mathcal{A}_{\trianglerighteq_{\mathcal{O}_2}})$). Thus, for a given subset of assertions $\mathcal{O}_3$, if $\langle \mathcal{T}, \pi(\mathcal{A}_{\trianglerighteq_{\mathcal{O}_1}})\rangle \models \mathcal{O}_3$ then $\langle \mathcal{T}, \pi(\mathcal{A}_{\trianglerighteq_{\mathcal{O}_2}})\rangle \models \mathcal{O}_3$. This amounts to if $\mathcal{K}_{\mathcal{O}_1} \models^{\pi} \mathcal{O}_3$ then

$\mathcal{K}_{\mathcal{O}_2} \models^{\pi} \mathcal{O}_3$, therefore, by Definition 7, if $\mathcal{O}_1 \mid\!\sim^{\pi}_{\mathcal{K}_{\rhd}} \mathcal{O}_3$ then $\mathcal{O}_2 \mid\!\sim^{\pi}_{\mathcal{K}_{\rhd}} \mathcal{O}_3$. Respectively, if $\langle \mathcal{T}, c\pi(\mathcal{A}_{\trianglerighteq_{\mathcal{O}_1}})\rangle \models \mathcal{O}_3$ then $\langle \mathcal{T}, c\pi(\mathcal{A}_{\trianglerighteq_{\mathcal{O}_2}})\rangle \models \mathcal{O}_3$. Which also amounts to if $\mathcal{O}_1 \mid\!\sim^{c\pi}_{\mathcal{K}_{\rhd}} \mathcal{O}_3$ then $\mathcal{O}_2 \mid\!\sim^{c\pi}_{\mathcal{K}_{\rhd}} \mathcal{O}_3$.

Lemma 4. *Query answering relations $\mid\!\sim^{\pi}_{\mathcal{K}_{\rhd}}$ and $\mid\!\sim^{c\pi}_{\mathcal{K}_{\rhd}}$ satisfy **RW**. Formally, if $\langle \mathcal{T}, \mathcal{O}_1\rangle \models \langle \mathcal{T}, \mathcal{O}_2\rangle$ and $\mathcal{O}_3 \mid\!\sim^{s}_{\mathcal{K}_{\rhd}} \mathcal{O}_1$ then $\mathcal{O}_3 \mid\!\sim^{s}_{\mathcal{K}_{\rhd}} \mathcal{O}_2$ (in our case $s = \pi$ or $s = c\pi$).*

Proof. Let $\mathcal{O}_3$ be a set of assertions and $\mathcal{K}_{\rhd} = \langle \mathcal{T}, \mathcal{A}_{\trianglerighteq}\rangle$ be a partially preordered KB. $\mathcal{K}_{\mathcal{O}_3} = \langle \mathcal{T}, \mathcal{A}_{\trianglerighteq_{\mathcal{O}_3}}\rangle$ is the augmented KB where $\mathcal{A}_{\trianglerighteq_{\mathcal{O}_3}} = (\mathcal{A}_{\trianglerighteq} \cup \mathcal{O}_3)$ results from adding $\mathcal{O}_3$ to $\mathcal{A}_{\trianglerighteq}$ with the highest priority.

By Definition 7, $\mathcal{O}_3 \mid\!\sim^{\pi}_{\mathcal{K}_{\rhd}} \mathcal{O}_1$ (resp. $\mathcal{O}_3 \mid\!\sim^{c\pi}_{\mathcal{K}_{\rhd}} \mathcal{O}_1$) amounts to $\mathcal{K}_{\mathcal{O}_3} \models^{\pi} \mathcal{O}_1$ (resp. $\mathcal{K}_{\mathcal{O}_3} \models^{c\pi} \mathcal{O}_1$). Which then amounts to a standard entailment relation $\langle \mathcal{T}, \pi(\mathcal{A}_{\trianglerighteq_{\mathcal{O}_3}})\rangle \models \mathcal{O}_1$ (resp. $\langle \mathcal{T}, c\pi(\mathcal{A}_{\trianglerighteq_{\mathcal{O}_3}})\rangle \models \mathcal{O}_1$).
By definition of a repair, $\pi(\mathcal{A}_{\trianglerighteq_{\mathcal{O}_3}})$ (resp. $c\pi(\mathcal{A}_{\trianglerighteq_{\mathcal{O}_3}})$) is a consistent base. Therefore, one can directly infer $\langle \mathcal{T}, \pi(\mathcal{A}_{\trianglerighteq_{\mathcal{O}_3}})\rangle \models \mathcal{O}_2$ (resp. $\langle \mathcal{T}, \pi(\mathcal{A}_{\trianglerighteq_{\mathcal{O}_3}})\rangle \models \mathcal{O}_2$) having $\langle \mathcal{T}, \mathcal{O}_1\rangle \models \langle \mathcal{T}, \mathcal{O}_2\rangle$.

Lemma 5. *Query answering relations $\mid\!\sim^{\pi}_{\mathcal{K}_{\rhd}}$ and $\mid\!\sim^{c\pi}_{\mathcal{K}_{\rhd}}$ satisfy **Cut**. Formally, if $\mathcal{O}_1 \mid\!\sim^{s}_{\mathcal{K}_{\rhd}} \mathcal{O}_2$ and $\mathcal{O}_1 \cup \mathcal{O}_2 \mid\!\sim^{s}_{\mathcal{K}_{\rhd}} \mathcal{O}_3$ then $\mathcal{O}_1 \mid\!\sim^{s}_{\mathcal{K}_{\rhd}} \mathcal{O}_3$ (in our case $s = \pi$ or $s = c\pi$).*

Proof. Let $\mathcal{O}_1$, $\mathcal{O}_2$ and $\mathcal{O}_3$ be sets of assertions and $\mathcal{K}_{\rhd} = \langle \mathcal{T}, \mathcal{A}_{\trianglerighteq}\rangle$ be a partially preordered KB. $\mathcal{K}_{\mathcal{O}_1} = \langle \mathcal{T}, \mathcal{A}_{\trianglerighteq_{\mathcal{O}_1}}\rangle$ is the augmented KB where $\mathcal{A}_{\trianglerighteq_{\mathcal{O}_1}} = (\mathcal{A}_{\trianglerighteq} \cup \mathcal{O}_1)$ results from adding $\mathcal{O}_1$ to $\mathcal{A}_{\trianglerighteq}$ with the highest priority. And, $\mathcal{K}_{\mathcal{O}_{1,2}} = \langle \mathcal{T}, \mathcal{A}_{\trianglerighteq_{\mathcal{O}_{1,2}}}\rangle$ is the augmented KB where $\mathcal{A}_{\trianglerighteq_{\mathcal{O}_{1,2}}} = (\mathcal{A}_{\trianglerighteq} \cup \mathcal{O}_1 \cup \mathcal{O}_2)$ results from adding $\mathcal{O}_1$ and $\mathcal{O}_2$ to $\mathcal{A}_{\trianglerighteq}$ with the highest priority. By Definition 7, $\mathcal{O}_1 \mid\!\sim^{s}_{\mathcal{K}_{\rhd}} \mathcal{O}_2$ amounts to $\mathcal{K}_{\mathcal{O}_1} \models^{s} \mathcal{O}_2$ which amounts to a standard entailment relation $\langle \mathcal{T}, s(\mathcal{A}_{\trianglerighteq_{\mathcal{O}_1}})\rangle \models \mathcal{O}_2$. In the same way, $\mathcal{O}_1 \cup \mathcal{O}_2 \mid\!\sim^{s}_{\mathcal{K}_{\rhd}} \mathcal{O}_3$ amounts to $\mathcal{K}_{\mathcal{O}_{1,2}} \models^{s} \mathcal{O}_3$ which amounts to a standard entailment relation $\langle \mathcal{T}, s(\mathcal{A}_{\trianglerighteq_{\mathcal{O}_{1,2}}})\rangle \models \mathcal{O}_3$ (here $s = \pi$ or $s = c\pi$).

In order to prove **Cut** for the case of $\mid\!\sim^{\pi}_{\mathcal{K}_{\rhd}}$, we start by showing that adding a plausible consequence as an observation to an ABox keeps the same part of the ABox taken into the π-accepted repair, formally, if $\langle \mathcal{T}, \pi(\mathcal{A}_{\trianglerighteq})\rangle \models \mathcal{O}_1$ then $\pi(\mathcal{A}_{\trianglerighteq_{\mathcal{O}_1}}) = \mathcal{O}_1 \cup \pi(\mathcal{A}_{\trianglerighteq}) \ldots (*)$.

$\langle \mathcal{T}, \pi(\mathcal{A}_{\trianglerighteq})\rangle \models \mathcal{O}_1$ means that for all totally preordered extension $\mathcal{A}_{\geq}$ of $\mathcal{A}_{\trianglerighteq}$, $\langle \mathcal{T}, \mathcal{R}(\mathcal{A}_{\geq})\rangle \models \mathcal{O}_1$. We show that for any $\mathcal{A}_{\geq}$ if $\langle \mathcal{T}, \mathcal{R}(\mathcal{A}_{\geq})\rangle \models \mathcal{O}_1$ then $\mathcal{R}(\mathcal{A}_{\geq_{\mathcal{O}_1}}) = \mathcal{O}_1 \cup \mathcal{R}(\mathcal{A}_{\geq})$ (where $\mathcal{A}_{\geq_{\mathcal{O}_1}}$ is the totally preordered extension of $\mathcal{A}_{\trianglerighteq_{\mathcal{O}_1}}$). Let $\mathcal{S}_1 \cup \cdots \cup \mathcal{S}_n$ be the well-ordered partition associated with $\mathcal{A}_{\geq}$ and $\mathcal{R}(\mathcal{A}_{\geq}) = (\mathcal{S}_1 \cup \cdots \cup \mathcal{S}_i)$ s.t. $\langle \mathcal{T}, \mathcal{R}(\mathcal{A}_{\geq})\rangle$ is consistent and $\langle \mathcal{T}, \mathcal{R}(\mathcal{A}_{\geq}) \cup \mathcal{S}_{i+1}\rangle$ is inconsistent. Having $\langle \mathcal{T}, \mathcal{R}(\mathcal{A}_{\geq})\rangle \models \mathcal{O}_1$ means that $\langle \mathcal{T}, \mathcal{S}_1 \cup \cdots \cup \mathcal{S}_i\rangle \models \mathcal{O}_1$, one can infer that $\langle \mathcal{T}, \mathcal{O}_1 \cup \mathcal{S}_1 \cup \cdots \cup \mathcal{S}_i\rangle$ is consistent and $\langle \mathcal{T}, \mathcal{O}_1 \cup \mathcal{S}_1 \cup \cdots \cup \mathcal{S}_{i+1}\rangle$ is inconsistent. This collapses with the possibilistic repair of $\mathcal{R}(\mathcal{A}_{\geq_{\mathcal{O}_1}}) = (\mathcal{O}_1 \cup \mathcal{S}_1 \cup \cdots \cup \mathcal{S}_i)$ (see Section 2). Hence, $\mathcal{R}(\mathcal{A}_{\geq_{\mathcal{O}_1}}) = \mathcal{O}_1 \cup \mathcal{R}(\mathcal{A}_{\geq})$. This result is also valid for $\pi(\mathcal{A}_{\trianglerighteq_{\mathcal{O}_1}})$ since it takes the intersection of the repairs of the totally preordered extensions: $\pi(\mathcal{A}_{\trianglerighteq_{\mathcal{O}_1}}) = \bigcap\{\mathcal{R}(\mathcal{A}_{\geq_{\mathcal{O}_1}})\} = \bigcap\{\mathcal{O}_1 \cup \mathcal{R}(\mathcal{A}_{\geq})\} = \mathcal{O}_1 \cup \pi(\mathcal{A}_{\trianglerighteq})$. Thus,

(*) is verified. Based on this result, one can derive that if $\langle\mathcal{T},\pi(\mathcal{A}_{\trianglerighteq\mathcal{O}_1})\rangle\vDash\mathcal{O}_2$ then $\pi(\mathcal{A}_{\trianglerighteq\mathcal{O}_{1,2}})=\mathcal{O}_2\cup\pi(\mathcal{A}_{\trianglerighteq\mathcal{O}_1})$.

Now, having $\pi(\mathcal{A}_{\trianglerighteq\mathcal{O}_{1,2}})=\mathcal{O}_2\cup\pi(\mathcal{A}_{\trianglerighteq\mathcal{O}_1})$ means that $\langle\mathcal{T},\pi(\mathcal{A}_{\trianglerighteq\mathcal{O}_{1,2}})\rangle\equiv\langle\mathcal{T},\mathcal{O}_2\cup\pi(\mathcal{A}_{\trianglerighteq\mathcal{O}_1})\rangle$ and since $\langle\mathcal{T},\pi(\mathcal{A}_{\trianglerighteq\mathcal{O}_1})\rangle\vDash\mathcal{O}_2$ we obtain $\langle\mathcal{T},\pi(\mathcal{A}_{\trianglerighteq\mathcal{O}_1})\rangle\equiv\langle\mathcal{T},\pi(\mathcal{A}_{\trianglerighteq\mathcal{O}_{1,2}})\rangle$. Therefore, for a set of assertions $\mathcal{O}_3$, if $\langle\mathcal{T},\pi(\mathcal{A}_{\trianglerighteq\mathcal{O}_{1,2}})\rangle\vDash\mathcal{O}_3$ then $\langle\mathcal{T},\pi(\mathcal{A}_{\trianglerighteq\mathcal{O}_1})\rangle\vDash\mathcal{O}_3$.

For the case of $\mid\!\sim^{c\pi}_{\mathcal{K}_\triangleright}$, showing that if $\langle\mathcal{T},c\pi(\mathcal{A}_{\trianglerighteq\mathcal{O}_1})\rangle\vDash\mathcal{O}_2$ then $\langle\mathcal{T},c\pi(\mathcal{A}_{\trianglerighteq\mathcal{O}_1})\rangle\equiv\langle\mathcal{T},c\pi(\mathcal{A}_{\trianglerighteq\mathcal{O}_{1,2}})\rangle$ can be done directly. In fact, $\langle\mathcal{T},c\pi(\mathcal{A}_{\trianglerighteq\mathcal{O}_1})\rangle\vDash\mathcal{O}_2$ means that for all totally preordered extension $\mathcal{A}_{\geq\mathcal{O}_1}$ of $\mathcal{A}_{\trianglerighteq\mathcal{O}_1}$, $\langle\mathcal{T},cl(\mathcal{R}(\mathcal{A}_{\geq\mathcal{O}_1}))\rangle\vDash\mathcal{O}_2$. Which means, by Definition 3 of the deductive closure that $\mathcal{O}_2\subseteq cl(\mathcal{R}(\mathcal{A}_{\geq\mathcal{O}_1}))$, hence, $\mathcal{O}_2\subseteq c\pi(\mathcal{A}_{\trianglerighteq\mathcal{O}_1})$. Therefore, $\langle\mathcal{T},c\pi(\mathcal{A}_{\trianglerighteq\mathcal{O}_1})\rangle\equiv\langle\mathcal{T},c\pi(\mathcal{A}_{\trianglerighteq\mathcal{O}_{1,2}})\rangle$ is verified. Having a set of assertions $\mathcal{O}_3$, if $\langle\mathcal{T},c\pi(\mathcal{A}_{\trianglerighteq\mathcal{O}_{1,2}})\rangle\vDash\mathcal{O}_3$ then $\langle\mathcal{T},c\pi(\mathcal{A}_{\trianglerighteq\mathcal{O}_1})\rangle\vDash\mathcal{O}_3$.

Lemma 6. *Query answering relations $\mid\!\sim^{\pi}_{\mathcal{K}_\triangleright}$ and $\mid\!\sim^{c\pi}_{\mathcal{K}_\triangleright}$ satisfy* ***CM****. Formally, if $\mathcal{O}_1\mid\!\sim^{s}_{\mathcal{K}_\triangleright}\mathcal{O}_2$ and $\mathcal{O}_1\mid\!\sim^{s}_{\mathcal{K}_\triangleright}\mathcal{O}_3$ then $\mathcal{O}_1\cup\mathcal{O}_2\mid\!\sim^{s}_{\mathcal{K}_\triangleright}\mathcal{O}_3$ (in our case $s=\pi$ or $s=c\pi$).*

Proof. Let $\mathcal{O}_1$, $\mathcal{O}_2$ and $\mathcal{O}_3$ be sets of assertions and $\mathcal{K}_\triangleright=\langle\mathcal{T},\mathcal{A}_\trianglerighteq\rangle$ be a partially preordered KB. $\mathcal{K}_{\mathcal{O}_1}=\langle\mathcal{T},\mathcal{A}_{\trianglerighteq\mathcal{O}_1}\rangle$ is the augmented KB where $\mathcal{A}_{\trianglerighteq\mathcal{O}_1}=(\mathcal{A}_\trianglerighteq\cup\mathcal{O}_1)$ results from adding $\mathcal{O}_1$ to $\mathcal{A}_\trianglerighteq$ with the highest priority. $\mathcal{K}_{\mathcal{O}_{1,2}}=\langle\mathcal{T},\mathcal{A}_{\trianglerighteq\mathcal{O}_{1,2}}\rangle$ is the augmented KB where $\mathcal{A}_{\trianglerighteq\mathcal{O}_{1,2}}=(\mathcal{A}_\trianglerighteq\cup\mathcal{O}_1\cup\mathcal{O}_2)$ results from adding $\mathcal{O}_1$ and $\mathcal{O}_2$ to $\mathcal{A}_\trianglerighteq$ with the highest priority. By Definition 7, $\mathcal{O}_1\mid\!\sim^{s}_{\mathcal{K}_\triangleright}\mathcal{O}_2$ (resp. $\mathcal{O}_1\mid\!\sim^{s}_{\mathcal{K}_\triangleright}\mathcal{O}_3$) amounts to $\mathcal{K}_{\mathcal{O}_1}\vDash^s\mathcal{O}_2$ (resp. $\mathcal{K}_{\mathcal{O}_1}\vDash^s\mathcal{O}_3$) which amounts to a standard entailment relation $\langle\mathcal{T},s(\mathcal{A}_{\trianglerighteq\mathcal{O}_1})\rangle\vDash\mathcal{O}_2$ (resp. $\langle\mathcal{T},s(\mathcal{A}_{\trianglerighteq\mathcal{O}_1})\rangle\vDash\mathcal{O}_3$).

In order to prove the satisfaction of **CM** for both $\mid\!\sim^{\pi}_{\mathcal{K}_\triangleright}$ and $\mid\!\sim^{c\pi}_{\mathcal{K}_\triangleright}$, we start by showing that adding already inferred consequences of a repair subsumes this one. Formally, if $\langle\mathcal{T},\pi(\mathcal{A}_{\trianglerighteq\mathcal{O}_1})\rangle\vDash\mathcal{O}_2$ (resp. $\langle\mathcal{T},c\pi(\mathcal{A}_{\trianglerighteq\mathcal{O}_1})\rangle\vDash\mathcal{O}_2$) then $\pi(\mathcal{A}_{\trianglerighteq\mathcal{O}_1})\subseteq\pi(\mathcal{A}_{\trianglerighteq\mathcal{O}_{1,2}})\ldots(*)$ (resp. $c\pi(\mathcal{A}_{\trianglerighteq\mathcal{O}_1})\subseteq c\pi(\mathcal{A}_{\trianglerighteq\mathcal{O}_{1,2}})\ldots(**)$).

For $\mid\!\sim^{\pi}_{\mathcal{K}_\triangleright}$, $\langle\mathcal{T},\pi(\mathcal{A}_{\trianglerighteq\mathcal{O}_1})\rangle\vDash\mathcal{O}_2$ means that for all totally preordered extension $\mathcal{A}_{\geq\mathcal{O}_1}$ of $\mathcal{A}_{\trianglerighteq\mathcal{O}_1}$: $\langle\mathcal{T},\mathcal{R}(\mathcal{A}_{\geq\mathcal{O}_1})\rangle\vDash\mathcal{O}_2$. We start by showing that for any totally preordered extension $\mathcal{A}_{\geq\mathcal{O}_1}$ of $\mathcal{A}_{\trianglerighteq\mathcal{O}_1}$, adding $\mathcal{O}_2$ on top of the possibilistic repair: $\mathcal{O}_2\cup\mathcal{R}(\mathcal{A}_{\geq\mathcal{O}_1})$ conducts to the possibilistic repair of the associated totally preordered extension $\mathcal{A}_{\geq\mathcal{O}_{1,2}}$ of $\mathcal{A}_{\trianglerighteq\mathcal{O}_{1,2}}$. Let $\mathcal{O}_1\cup\mathcal{S}_1\cup\cdots\cup\mathcal{S}_n$ be the well-ordered partition of $\mathcal{A}_{\geq\mathcal{O}_1}$. By definition of possibilistic repair $\mathcal{R}(\mathcal{A}_{\geq\mathcal{O}_1})=(\mathcal{O}_1\cup\mathcal{S}_1\cup\cdots\cup\mathcal{S}_i)$ s.t. $\langle\mathcal{T},\mathcal{R}(\mathcal{A}_{\geq\mathcal{O}_1})\rangle$ is consistent and $\langle\mathcal{T},\mathcal{R}(\mathcal{A}_{\geq\mathcal{O}_1})\cup\mathcal{S}_{i+1}\rangle$.

Having $\langle\mathcal{T},\mathcal{R}(\mathcal{A}_{\geq\mathcal{O}_1})\rangle\vDash\mathcal{O}_2$ means that $\mathcal{O}_2$ is consistent with $\mathcal{R}(\mathcal{A}_{\geq\mathcal{O}_1})$, then, adding $\mathcal{O}_2$ on top of $\mathcal{R}(\mathcal{A}_{\geq\mathcal{O}_1})$ gives: $\langle\mathcal{T},\mathcal{O}_2\cup\mathcal{R}(\mathcal{A}_{\geq\mathcal{O}_1})\rangle$ is consistent and $\langle\mathcal{T},\mathcal{O}_2\cup\mathcal{R}(\mathcal{A}_{\geq\mathcal{O}_1})\cup\mathcal{S}_{i+1}\rangle$ is inconsistent. This amounts to the possibilistic repair of $\mathcal{R}(\mathcal{A}_{\geq\mathcal{O}_{1,2}})$ which equals to: $(\mathcal{O}_2\cup\mathcal{O}_1\cup\mathcal{S}_1\cup\cdots\cup\mathcal{S}_i)$ (see Section 2). Therefore, $\mathcal{R}(\mathcal{A}_{\geq\mathcal{O}_{1,2}})=\mathcal{O}_2\cup\mathcal{R}(\mathcal{A}_{\geq\mathcal{O}_1})$ which means that $\mathcal{R}(\mathcal{A}_{\geq\mathcal{O}_1})\subseteq\mathcal{R}(\mathcal{A}_{\geq\mathcal{O}_{1,2}})$.

As a result, having for any totally preordered extension:
$\mathcal{R}(\mathcal{A}_{\geq\mathcal{O}_1})\subseteq\mathcal{R}(\mathcal{A}_{\geq\mathcal{O}_{1,2}})$ means that (*) is verified. Now, one can infer directly that if $\langle\mathcal{T},\pi(\mathcal{A}_{\trianglerighteq\mathcal{O}_1})\rangle\vDash\mathcal{O}_3$ then $\langle\mathcal{T},\pi(\mathcal{A}_{\trianglerighteq\mathcal{O}_{1,2}})\rangle\vDash\mathcal{O}_3$ (plausible consequences of $\pi(\mathcal{A}_{\trianglerighteq\mathcal{O}_1})$ are also plausible consequences of $\pi(\mathcal{A}_{\trianglerighteq\mathcal{O}_{1,2}})$).

In the same manner, for $\mathrel{|}\!\sim^{c\pi}_{\mathcal{K}_{\triangleright}}$, since for any totally preordered extension: $\mathcal{R}(\mathcal{A}_{\geq_{\mathcal{O}_1}}) \subseteq \mathcal{R}(\mathcal{A}_{\geq_{\mathcal{O}_{1,2}}})$ introducing the closure gives also $cl(\mathcal{R}(\mathcal{A}_{\geq_{\mathcal{O}_1}})) \subseteq cl(\mathcal{R}(\mathcal{A}_{\geq_{\mathcal{O}_{1,2}}}))$ (see Definition 3 of the deductive closure). Hence, (**) is verified too. Therefore, one can also infer directly that if $\langle \mathcal{T}, c\pi(\mathcal{A}_{\trianglerighteq_{\mathcal{O}_1}}) \rangle \models \mathcal{O}_3$ then $\langle \mathcal{T}, c\pi(\mathcal{A}_{\trianglerighteq_{\mathcal{O}_{1,2}}}) \rangle \models \mathcal{O}_3$ (plausible consequences of $c\pi(\mathcal{A}_{\trianglerighteq_{\mathcal{O}_1}})$ are also plausible consequences of $c\pi(\mathcal{A}_{\trianglerighteq_{\mathcal{O}_{1,2}}})$).

Lemma 7. *Query answering relations $\mathrel{|}\!\sim^{\pi}_{\mathcal{K}_{\triangleright}}$ and $\mathrel{|}\!\sim^{c\pi}_{\mathcal{K}_{\triangleright}}$ satisfy* ***AND****. Formally, if $\mathcal{O}_1 \mathrel{|}\!\sim^{s}_{\mathcal{K}_{\triangleright}} \mathcal{O}_2$ and $\mathcal{O}_1 \mathrel{|}\!\sim^{s}_{\mathcal{K}_{\triangleright}} \mathcal{O}_3$ then $\mathcal{O}_1 \mathrel{|}\!\sim^{s}_{\mathcal{K}_{\triangleright}} \mathcal{O}_2 \cup \mathcal{O}_3$ (in our case $s = \pi$ or $s = c\pi$).*

Proof. Let $\mathcal{O}_1$, $\mathcal{O}_2$ and $\mathcal{O}_3$ be sets of assertions and $\mathcal{K}_{\triangleright} = \langle \mathcal{T}, \mathcal{A}_{\trianglerighteq} \rangle$ be a partially preordered KB. $\mathcal{K}_{\mathcal{O}_1} = \langle \mathcal{T}, \mathcal{A}_{\trianglerighteq_{\mathcal{O}_1}} \rangle$ is the augmented KB where $\mathcal{A}_{\trianglerighteq_{\mathcal{O}_1}} = (\mathcal{A}_{\trianglerighteq} \cup \mathcal{O}_1)$ results from adding $\mathcal{O}_1$ to $\mathcal{A}_{\trianglerighteq}$ with the highest priority.

By Definition 7, $\mathcal{O}_1 \mathrel{|}\!\sim^{\pi}_{\mathcal{K}_{\triangleright}} \mathcal{O}_2$ (resp. $\mathcal{O}_1 \mathrel{|}\!\sim^{c\pi}_{\mathcal{K}_{\triangleright}} \mathcal{O}_2$) amounts to $\mathcal{K}_{\mathcal{O}_1} \models^{\pi} \mathcal{O}_2$ (resp. $\mathcal{K}_{\mathcal{O}_1} \models^{c\pi} \mathcal{O}_2$) and $\mathcal{O}_1 \mathrel{|}\!\sim^{\pi}_{\mathcal{K}_{\triangleright}} \mathcal{O}_3$ (resp. $\mathcal{O}_1 \mathrel{|}\!\sim^{c\pi}_{\mathcal{K}_{\triangleright}} \mathcal{O}_3$) amounts to $\mathcal{K}_{\mathcal{O}_1} \models^{\pi} \mathcal{O}_3$ (resp. $\mathcal{K}_{\mathcal{O}_1} \models^{c\pi} \mathcal{O}_3$).

From Proposition 5, both $\models^{\pi}$ and $\models^{c\pi}$ satisfy the **QCI** property which states that if $\mathcal{K}_{\triangleright} \models^{s} q_1$ and $\mathcal{K}_{\triangleright} \models^{s} q_2$ then $\mathcal{K}_{\triangleright} \models^{s} q_1 \wedge q_2$ ($s = \pi$ or $s = c\pi$). In our case we have, $\mathcal{K}_{\mathcal{O}_1} \models^{\pi} \mathcal{O}_2$ and $\mathcal{K}_{\mathcal{O}_1} \models^{\pi} \mathcal{O}_3$ (resp. $\mathcal{K}_{\mathcal{O}_1} \models^{c\pi} \mathcal{O}_2$ and $\mathcal{K}_{\mathcal{O}_1} \models^{c\pi} \mathcal{O}_3$). Hence, $\mathcal{K}_{\mathcal{O}_1} \models^{\pi} \mathcal{O}_2 \cup \mathcal{O}_3$ (resp. $\mathcal{K}_{\mathcal{O}_1} \models^{c\pi} \mathcal{O}_2 \cup \mathcal{O}_3$), which amount to $\mathcal{O}_1 \mathrel{|}\!\sim^{\pi}_{\mathcal{K}_{\triangleright}} \mathcal{O}_2 \cup \mathcal{O}_3$ (resp. $\mathcal{O}_1 \mathrel{|}\!\sim^{c\pi}_{\mathcal{K}_{\triangleright}} \mathcal{O}_2 \cup \mathcal{O}_3$).

Proposition 7. *Let $\mathcal{K}_{\triangleright} = \langle \mathcal{T}, \mathcal{A}_{\trianglerighteq} \rangle$ be a totally preordered KB (with $\trianglerighteq$ being a total preorder), $\mathcal{O}$ be a set of assertions and q be a query. Query answering relations $\mathrel{|}\!\sim^{\pi}_{\mathcal{K}_{\triangleright}}$ and $\mathrel{|}\!\sim^{c\pi}_{\mathcal{K}_{\triangleright}}$ satisfy* ***RM****.*

Proof. Assume that for a given two sets of assertions $\mathcal{O}_2$ and $\mathcal{O}_1$, $\mathcal{O}_1 \mathrel{|}\!\sim^{s}_{\mathcal{K}_{\triangleright}} \mathcal{O}_3$ holds, but neither of $\mathcal{O}_1 \cup \mathcal{O}_2 \mathrel{|}\!\sim^{s}_{\mathcal{K}_{\triangleright}} \mathcal{O}_3$ or $\langle \mathcal{T}, \mathcal{O}_1 \cup \mathcal{O}_2 s(\mathcal{A}_{\trianglerighteq}) \rangle$ is inconsistent holds (with $s \in \pi, c\pi$).

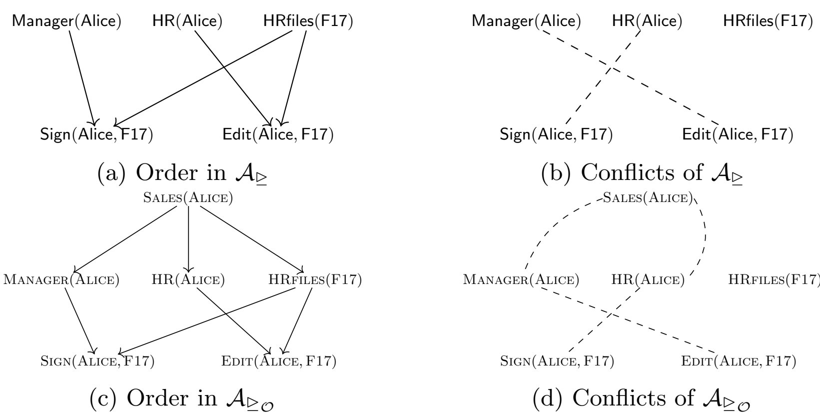
This means, using Definition 7 of conditional inference, that $\langle \mathcal{T}, s(\mathcal{A}_{\trianglerighteq_{\mathcal{O}_1}}) \rangle \models \mathcal{O}_3$ holds, but $\langle \mathcal{T}, s(\mathcal{A}_{\trianglerighteq_{\mathcal{O}_1 \cup \mathcal{O}_2}}) \rangle \not\models \mathcal{O}_3$ and $\langle \mathcal{T}, \mathcal{O}_1 \cup \mathcal{O}_2 \cup s(\mathcal{A}_{\trianglerighteq}) \rangle$ is consistent ... (*).

$\trianglerighteq$ being a total preorder relation means that the relations $\trianglerighteq_{\mathcal{O}_1}$ and $\trianglerighteq_{\mathcal{O}_1 \cup \mathcal{O}_2}$ which are obtained after observing $\mathcal{O}_1$ and $\mathcal{O}_1 \cup \mathcal{O}_2$ respectively are also total preorders.

The π-accepted assertions repair in the case of total preorders amounts to possibilistic repair $\mathcal{R}(.)$ given in Section 2. Now, assume that $\mathcal{A}_{\trianglerighteq_{\mathcal{O}_1}} = \mathcal{O}_1 \cup \mathcal{S}_1 \cup \cdots \cup \mathcal{S}_n$ is the well-ordered partition of $\mathcal{A}_{\trianglerighteq_{\mathcal{O}_1}}$ and $\mathcal{R}(\mathcal{A}_{\trianglerighteq_{\mathcal{O}_1}} = \mathcal{O}_1 \cup \mathcal{S}_1 \cup \cdots \cup \mathcal{S}_i$ its associated possibilistic repair. One can easily check that by adding $\mathcal{O}_2$ as an observation, we distinguish two possible cases, either $\mathcal{R}(\mathcal{A}_{\trianglerighteq_{\mathcal{O}_1 \cup \mathcal{O}_2}} = \mathcal{O}_1 \cup \mathcal{O}_2 \cup \mathcal{S}_1 \cup \cdots \cup \mathcal{S}_i$ or $\langle \mathcal{T}, \mathcal{O}_1 \cup \mathcal{O}_2 \cup \mathcal{S}_1 \cup \cdots \cup \mathcal{S}_i \rangle$ is inconsistent and $\mathcal{R}(\mathcal{A}_{\trianglerighteq_{\mathcal{O}_1 \cup \mathcal{O}_2}} = \mathcal{O}_1 \cup \mathcal{O}_2 \cup \mathcal{S}_1 \cup \cdots \cup \mathcal{S}_j$ (with $j < i$). In the first case, $\forall \mathcal{O}_3$ if $\langle \mathcal{T}, \mathcal{R}(\mathcal{A}_{\trianglerighteq_{\mathcal{O}_1}} \rangle \models \mathcal{O}_3$ then $\langle \mathcal{T}, \mathcal{R}(\mathcal{A}_{\trianglerighteq_{\mathcal{O}_1 \cup \mathcal{O}_2}} \rangle \models \mathcal{O}_3$. In the second, clearly $\mathcal{O}_2$ and $\mathcal{R}(\mathcal{A}_{\trianglerighteq_{\mathcal{O}_1}}$ are inconsistent w.r.t. $\mathcal{T}$ since $\langle \mathcal{T}, \mathcal{O}_1 \cup \mathcal{O}_2 \cup \mathcal{S}_1 \cup \cdots \cup \mathcal{S}_i \rangle$ is inconsistent. Which contradicts our first assumption in (*). Therefore, **RM** is satisfied if $\trianglerighteq$ is a total preorder relation.

C Counterexample of RM and Comp

The following example shows a case where the properties of rational monotony and completeness are not satisfied by the inference relations of partially preordered possibilistic repair and its closure, namely $\mid\!\sim^{\pi}_{\mathcal{K}_{\rhd}}$ and $\mid\!\sim^{c\pi}_{\mathcal{K}_{\rhd}}$.



(a) Order in $\mathcal{A}_{\unrhd}$ (b) Conflicts of $\mathcal{A}_{\unrhd}$

(c) Order in $\mathcal{A}_{\unrhd_{\mathcal{O}}}$ (d) Conflicts of $\mathcal{A}_{\unrhd_{\mathcal{O}}}$

Fig. 1: $\longrightarrow$: strict preference. $---$: conflict.

Example 1. Let $\mathcal{K}_{\rhd} = \langle \mathcal{T}, \mathcal{A}_{\unrhd} \rangle$ be a partially preordered KB given by:

$$\mathcal{T} = \begin{Bmatrix} \mathsf{Manager} \sqsubseteq \neg\, \exists \mathsf{Edit} \\ \mathsf{HR} \sqsubseteq \neg\, \exists \mathsf{Sign} \\ \mathsf{Sales} \sqsubseteq \neg\, \mathsf{Manager} \\ \mathsf{Sales} \sqsubseteq \neg\, \mathsf{HR} \end{Bmatrix} \qquad \mathcal{A}_{\unrhd} = \begin{Bmatrix} \mathsf{HRfiles(F17)}, \mathsf{Manager(Alice)}, \\ \mathsf{HR(Alice)}, \mathsf{Sign(Alice, F17)}, \\ \mathsf{Edit(Alice, F17)} \end{Bmatrix}$$

With $\unrhd$ as shown in Figure 1a and $\mathsf{Cf}(\mathcal{A}_{\unrhd})$ as shown in Figure 1b.

We provide a counterexample to **RM**, namely: If $\mathcal{O}_1 \mid\!\sim^{s}_{\mathcal{K}_{\rhd}} \mathcal{O}_3$, then $\mathcal{O}_1 \cup \mathcal{O}_2 \mid\!\sim^{s}_{\mathcal{K}_{\rhd}} \mathcal{O}_3$ or $(\mathcal{O}_2 \cup \mathcal{O}_1 \cup \mathcal{A}_{\unrhd})$ is inconsistent w.r.t. $\mathcal{T}$.

We let $\mathcal{O}_1 = \emptyset$, $\mathcal{O}_2 = \{\mathsf{Sales(Alice)}\}$ and $\mathcal{O}_3 = \{\mathsf{HRfiles(F17)}\}$.

Having $\pi(\mathcal{A}_{\unrhd}) = \{\mathsf{HRfiles(F17)}\}$, one can check that $\langle \mathcal{T}, \mathcal{A}_{\unrhd} \rangle \models^{\pi} \mathsf{HRfiles(F17)}$, hence by definition of the conditional inference: $\mathcal{O}_1 \mid\!\sim^{\pi}_{\mathcal{K}_{\rhd}} \mathsf{HRfiles(F17)}$.

Now, let us consider $\mathcal{O}_2 = \{\mathsf{Sales(Alice)}\}$ which is added to $\mathcal{A}_{\unrhd}$ with the highest preference; namely $\mathcal{A}_{\unrhd_{\mathcal{O}}} = (\mathcal{A}_{\unrhd} \cup \mathcal{O}_2)$ where $\unrhd_{\mathcal{O}}$ is a partial preorder obtained by:

(i) adding strict preference relations between $\mathsf{Sales(Alice)}$ and each assertion of $\mathcal{A}_{\unrhd}$, and;

(ii) preserving the other relations of $\unrhd$ (the resulting base is shown in Figure 1c).

After observing $\mathcal{O}_2$, we get $\pi(\mathcal{A}_{\unrhd_{\mathcal{O}}}) = \{\mathsf{Sales(Alice)}\}$. One can check that $\langle \mathcal{T}, \mathcal{A}_{\unrhd_{\mathcal{O}}} \rangle \not\models^{\pi} \mathsf{HRfiles(F17)}$, hence we cannot infer $\mathsf{HRfiles(F17)}$ anymore: $\mathcal{O}_1 \cup \mathcal{O}_2 \not\mid\!\sim^{\pi}_{\mathcal{K}_{\rhd}} \mathsf{HRfiles(F17)}$.

On the other hand, $\pi(\mathcal{A}_{\trianglerighteq})$ and $\mathsf{Sales}(\mathsf{Alice})$ are consistent w.r.t. $\mathcal{T}$. This contradicts the definition of **RM**.

Similarly for **Comp**, not inferring $\mathsf{HRfiles}(\mathsf{F17})$ anymore after the introduction of $\mathcal{O}_2$: $\mathcal{O}_1 \cup \mathcal{O}_2 \not\models^{\pi}_{\mathcal{K}_{\triangleright}} \mathsf{HRfiles}(\mathsf{F17})$, while $\{\mathsf{HRfiles}(\mathsf{F17}), \mathsf{Sales}(\mathsf{Alice})\}$, and $\pi(\mathcal{A}_{\trianglerighteq_{\mathcal{O}}})$ are consistent w.r.t. $\mathcal{T}$; hence **Comp** is falsified.

For the Cπ-repair, the same counterexample applies: $\mathcal{O}_1 \cup \mathcal{O}_2 \not\models^{c\pi}_{\mathcal{K}_{\triangleright}} \mathsf{HRfiles}(\mathsf{F17})$ but $c\pi(\mathcal{A}_{\trianglerighteq})$ and $\mathsf{Sales}(\mathsf{Alice})$ are consistent (resp. $\{\mathsf{HRfiles}(\mathsf{F17}),\ \mathsf{Sales}(\mathsf{Alice})\}$ and $c\pi(\mathcal{A}_{\trianglerighteq_{\mathcal{O}}})$ are consistent). Therefore, **RM** does not hold (resp. **Comp** does not hold).

D Counterexample of tractability beyond DL-Lite$_\mathcal{R}$

R_1	R_2	R_3		R_n
(a, b_1)	(a, b_1)	(a, b_1)		(a, b_1)
(a, b_2)	(a, b_2)	(a, b_2)		(a, b_2)
(a, b_3)	(a, b_3)	(a, b_3)	...	(a, b_3)
...	...	...		...
...	...	...		...
(a, b_m)	(a, b_m)	(a, b_m)		(a, b_m)

Fig. 2: The ABox of Example 2

The following example illustrates how taking into account languages richer than DL-Lite$_\mathcal{R}$ leads to a loss of tractability. Recall that the approach proposed in this paper, given by the Propositions 1 and 2, is tractable thanks to the fact that the computation of the set of conflicts is done in polynomial time and that its size is polynomial w.r.t. the size of the ABox, in DL-Lite$_\mathcal{R}$. In this example, we face a situation where the KB have exponentially many conflicts. Formally:

Example 2. Let $\mathcal{K} = \langle \mathcal{T}, \mathcal{A} \rangle$ be a KB such that:

$$\mathcal{T} = \{\exists R_1 \sqcap \exists R_2 \sqcap \ldots \sqcap \exists R_n \sqsubseteq \perp\},$$

where $R_i, i = 1, \ldots, n$, are roles. And:

$$\mathcal{A} = \{R_1(a, b_1), \ldots, R_1(a, b_m)\} \cup \ldots \cup \{R_n(a, b_1), \ldots, R_n(a, b_m)\}.$$

where $\{a, b_1, \ldots, b_m\}$ are individuals. The content of the ABox $\mathcal{A}$ is illustrated by Figure 2. One can easily check that the size of the ABox $\mathcal{A}$ is equal to $m * n$ assertions.

Let us now show that the size of the set of conflicts $\mathsf{Cf}(\mathcal{A})$ is equal to: $\mathcal{O}(m^n)$. This is based on the two following facts:

- The size of the following Cartesian product:

$$\begin{aligned}\mathsf{Cf}(\mathcal{A}) = &\{R_1(a, b_1), ..., R_1(a, b_m)\} \times \{R_2(a, b_1), ..., R_2(a, b_m)\} \\ &\times \ldots \times \{R_n(a, b_1), ..., R_n(a, b_m)\}\end{aligned}$$

  is exactly: $\mathcal{O}(m^n)$, where the operator $\times$ denotes the Cartesian product of sets.
  We recall that for each, $i = 1, \ldots, n$ and $j = 1, \ldots, n$, we have:

$$|\{R_i(a, b_1), ..., R_i(a, b_m\}| = |\{R_j(a, b_1), ..., R_j(a, b_m\}| = m.$$

- Each m-uple of this Cartesian product:

$$\mathcal{C} = \{R_1(a, b_{j_1}), R_2(a, b_{j_2}), \ldots, R_n(a, b_{j_n})\}.$$

is a conflict. Indeed:

  – We have $\mathcal{C}$ is inconsistent with $\mathcal{T} = \{\exists R_1 \sqcap \exists R_2 \sqcap \ldots \sqcap \exists R_n \sqsubseteq \bot\}$. Indeed, the conflict contains exactly one element from each role. And since, whatever is the considered assertional role of the ABox, it is of the form $R_i(a, b_j)$, and hence $\exists R_i(a, b_j) = \{a\}$ and hence the $\exists R_1 \sqcap \exists R_2 \sqcap \ldots \sqcap \exists R_n \sqsubseteq \bot$ cannot be satisfied.
  – We can also easily check that $\mathcal{C}$ is minimally inconsistent with $\mathcal{T}$; namely any strict subset of $\mathcal{C}' \subsetneq \mathcal{C}$ is consistent with $\mathcal{T}$ (in this case, it is easy to find a model of $\mathcal{T}$ and $\mathcal{C}'$).
  – Lastly, one can also show that $\mathsf{Cf}(\mathcal{A})$ contains all possible conflicts of $\mathcal{A}$ w.r.t. $\mathcal{T}$. Indeed, let X be a subset of $\mathcal{A}$ such that $X \notin \mathsf{Cf}(\mathcal{A})$. We have two cases to consider:
    * $\exists \mathcal{C} \in \mathsf{Cf}(\mathcal{A})$ such that $\mathcal{C} \subsetneq X$. Then X cannot be a conflict because it is not minimally inconsistent.
    * $\forall \mathcal{C} \in \mathsf{Cf}(\mathcal{A})$ we have $\mathcal{C} \not\subset X$. This means that there exists an $i = 1, .., n$ such that for all $j = 1, ..., m$, $R_i(a, bj) \notin X$. This means that X is consistent with $\mathcal{T}$ and hence it cannot be a conflict.

E Tables of results for the study of rationality properties

In the following, Tables 1 and 2 summarize the results of the study of rationality properties done in Section 5 and given by Propositions 5 and 6, for both the π-accepted assertions and the Cπ-repair methods.

Properties	**QCE**	**QCI**	**Cons**	**ConsC**	**ConsS**
π-accepted	√	√	√	√	√
Cπ-repair	√	√	√	√	√

Table 1: Properties of unconditional inference.

Properties	**R**	**LLE**	**RW**	**Cut**	**CM**	**And**	**RM**	**Comp**
π-accepted	√	√	√	√	√	√	✗	✗
Cπ-repair	√	√	√	√	√	√	✗	✗

Table 2: Properties of conditional inference.

F Algorithm to check if an assertion belongs to Cπ-repair

In this section, we provide a polynomial algorithm to check if an assertion belongs to the Cπ-repair of a KB given the TBox and the ABox.

Algorithm 1 Check in Cπ-repair

Inputs: φ : a given assertion to be checked if in Cπ-repair,
$\mathcal{T}$: The TBox,
$\mathcal{A}_{\trianglerighteq}$: The partially preordered ABox.

Output: *member*: Boolean value indicating if the assertion is in the Cπ-repair of $\mathcal{A}_{\trianglerighteq}$

supports ← ComputeSupports($\varphi, \mathcal{T}, \mathcal{A}_{\trianglerighteq}$) /* Compute the supports of φ */
for each *negative_axiom* in *cln*($\mathcal{T}$) **do**
  $Confs$ ← ComputeConflicts(negative_axiom, $\mathcal{A}_{\trianglerighteq}$) /* Compute the conflicts arising from the negative axiom */
  for each $\mathcal{C}$ in $Confs$ **do**
    member ← False /* Flag to track if a supporting assertion dominates the conflict */
    for each s in *supports* **do**
      if $s \triangleright^{dom} \mathcal{C}$ **then**
        member ← True /* Set the flag to indicate that a dominating support is found for $\mathcal{C}$ */
        break /* Exit the loop for s, as a dominating support was found */
      end if
    end for
    if *member* is False **then**
      return False /* If no dominating support is found, exit the algorithm */
    end if
  end for
end for
return True /* All conflicts are dominated by a support, the assertion is in the Cπ-repair of $\mathcal{A}_{\trianglerighteq}$ */

In practice, the methods ComputeSupports() and ComputeConflicts() from Algorithm 1, translate to a set of SQL queries on a relational database containing the ABox assertions. For instance, let $\mathsf{HR} \sqsubseteq \neg \exists \mathsf{Sign}$ be a negative axiom from the TBox. In order to get the conflicts induced by this axiom, we use the following query:

```sql
SELECT t1.*, t2.* FROM HR t1 INNER JOIN Sign t2
ON t1.individual_1 = t2.individual_1;
```

This algorithm avoids generating and storing beforehand the whole set of conflicts, instead, it examines progressively the conflicts generated by one negative axiom at a time. This practice is beneficial for both massive and high-conflicting ABoxes, this helps keep a reasonable spatial complexity of the method. The use of SQL queries ensures the same for time complexity.

One can extend the given algorithm, which checks if an assertion belongs to the Cπ-repair of an ABox, to compute the whole repair by running it for each possible assertion from the KB. This can be achieved in polynomial time since the number of possible assertions is finite.

In addition to the algorithm given in this part, an implementation of the method has been made in order to test its applicability. This implementation is open to access and can be viewed at the GitHub repository:

`https://github.com/ahmedlaouar/py_reasoner`.

We intend to carry out an in-depth experimental study of the method in future work.