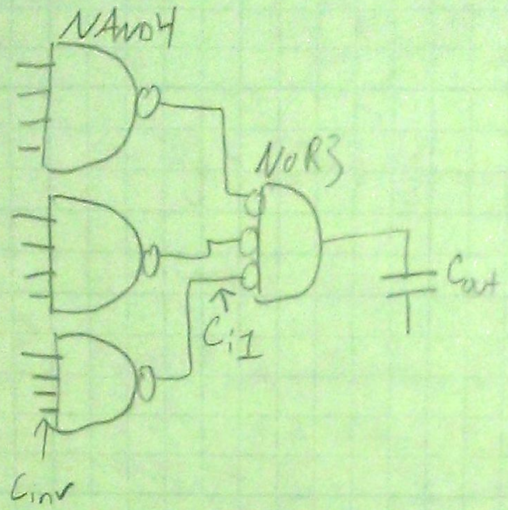
ECE 3060 HW 4

1) a)



$g_{NAND4} = \frac{6}{3} = 2$

$g_{NOR3} = \frac{7}{3}$

$G = \prod g_i = \frac{6}{3} \cdot \frac{7}{3} = \frac{14}{3}$

$C_{out} = 45 C_{inv}$

$H = \frac{C_{out}}{C_{in}} = \frac{45 C_{inv}}{C_{inv}} = 45$

$B = 1$

$F = BGH = 45 \cdot \frac{14}{3} \cdot 1 = 210$ $\quad \hat{f} = (210)^{(1/2)} = 14.491$

$f_i = g_i h_i = g_i \frac{C_{out_i}}{C_{in_i}}$ $\quad C_{in_i} = \frac{g_i C_{out_i}}{\hat{f}} = \frac{7 \cdot 45 C_{inv}}{3 \cdot 14.491}$

I $\boxed{C_{i_1} = 7.2457 C_{inv}}$

Check

$f_i = \frac{\cancel{6} \cdot 7.2457 C_{inv}}{3 \; C_{inv}} = 14.491$ ✓

$D = \sum d_i = (\underbrace{2 \cdot 7.2457 C_{inv}}_{\hat{f}} + \overset{P_1}{4}) + (\underbrace{\frac{7}{3} \cdot 6.2106}_{\hat{f}} + \overset{P_2}{3})$

$2\hat{f} + 7$

I $\boxed{D = 2\sqrt{210} + 7 = 35.98 \, \tau}$

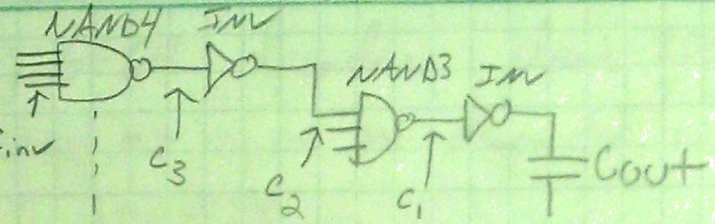
When $C_{out} = 400 C_{inv}$, $H = 400$. $\quad F = 400 \cdot \frac{14}{3} \cdot 1 = 1866.\bar{6}$

$\hat{f} = \sqrt{1866.\bar{6}} = 43.205$.

II $\boxed{D = 2\hat{f} + \sum p = 2 \cdot 43.205 + 7 = 93.41 \tau}$

II $\boxed{C_{i_1} = \frac{7 \cdot 400 C_{inv}}{3 \cdot 43.205} = 21.602 C_{inv}}$

b)



4 stages

$g_i(NAND4) = 2$

$g_i(NAND3) = 5/3$

$G = \Pi g = \left(2 \cdot 1 \cdot \frac{5}{3} \cdot 1\right) = \frac{10}{3}$

$H = \frac{C_{out}}{C_{in}} = \frac{C_{inv}}{C_{inv}} = \underline{1}$ $\quad B = 1. \quad F = BGH = \frac{10}{3}$

$\hat{f} = \left(\frac{10}{3}\right)^{1/4} = 1.3512.$

I $\boxed{C_1 = \frac{C_{out} \cdot g_i}{\hat{f}} = \frac{C_{inv} \cdot 1}{1.3512} = 0.74C_{inv}}$

I $\boxed{C_2 = \frac{C_{out}\, g_i}{\hat{f}} = \frac{0.74C_{inv} \cdot 5}{3 \cdot 1.3512} = 0.9129C_{inv}}$ I $\boxed{C_3 = \frac{0.9129C_{inv} \cdot 1}{1.3512} = 0.6756C_{inv}}$

I $\boxed{D = \Sigma f + p = 4(1.3512) + (4+1+3+1) = 14.4048\ \tau}$

$C_{out} = 250C_{inv}, \quad H = 250. \quad F = \frac{10 \cdot 250 \cdot 1}{3} = 833.\overline{3}$

$\hat{f} = (833.\overline{3})^{1/4} = 5.3728.$

II $\boxed{C_1 = \frac{C_{out} \cdot g_i}{\hat{f}} = \frac{250C_{inv} \cdot 1}{5.3728} = 46.5307C_{inv}}$

II $\boxed{C_2 = \frac{46.5307 \cdot 5}{3 \cdot 5.3728} = 14.4340C_{inv}}$ II $\boxed{C_3 = \frac{14.434 \cdot 1}{5.3728} = 2.6865C_{inv}}$

II $\boxed{D = \Sigma f + \Sigma p = 4(5.3728) + 9 = 30.4912\ \tau}$

2) 6-64 decoder drive 800C_{inv}.

Because our inputs are driven by min inv, $C_{in} = C_{inv}$.

$\therefore$ $H = 800$.

Also, each input controls 1/2 the outputs, so $B = \frac{64}{2} = 32$

Because H is large, I will assume a chain of NAND2 will give the best result.

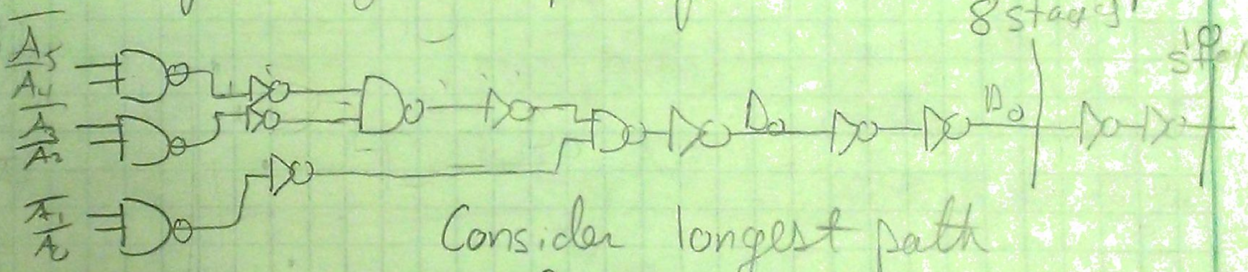
A_5	A_4	A_3	A_2	A_1	A_0	D_{63}	D_{62}	...	D_1	D_0
0	0	0	0	0	0	0	0	...	0	1
0	0	0	0	0	1				1	0

$D_0 \; \overline{A_5} \cdot \overline{A_4} \cdot \ldots \cdot \overline{A_0}$

Initial path effort guess,

assume $G = 1$. $F = 25{,}600$ $\hat{f} = \log_{3.6}(F) = 7.92 \approx 8$

8 stage design (probably > 8 due to assumption)



Consider longest path

$G = \prod g_i = \left(\frac{4}{3}\right)^3 = \frac{64}{27}$

$F = GHB = \frac{64}{27} \cdot 800 \cdot 32 = 56{,}888.\overline{8}$

Consider 8 stage design: $\hat{f} = (56{,}888.\overline{8})^{(1/8)} = 3.9299$

$D = \sum f + p = 8(3.9299) + \underbrace{(2+1+2+1+2+1+1+1)}_{11}$

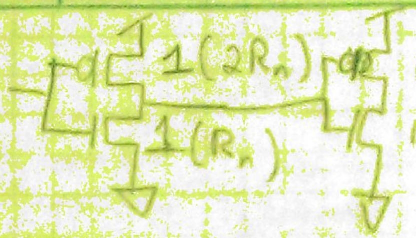
$D = 42.439\tau$

Consider 10 stage design:

$\hat{f} = (56{,}888.\overline{8})^{1/10} = 2.9888$

$D = 10(2.9888) + 11 = 40.888\tau$

3) $\gamma = 1.$



$T = 3R_nC_{set}$

$C_{inv} = 2C_{set}$

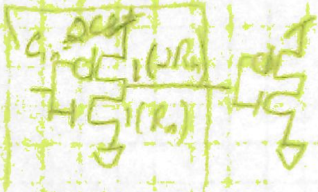
$2R_n \cdot 2C_{set}$

$\tilde{\tau} = 4R_nC_{set}$

$\tilde{\tau} = \frac{4}{3}\tau$

4) Show $f = gh = (t_r + t_f)/2$

Take inv chain

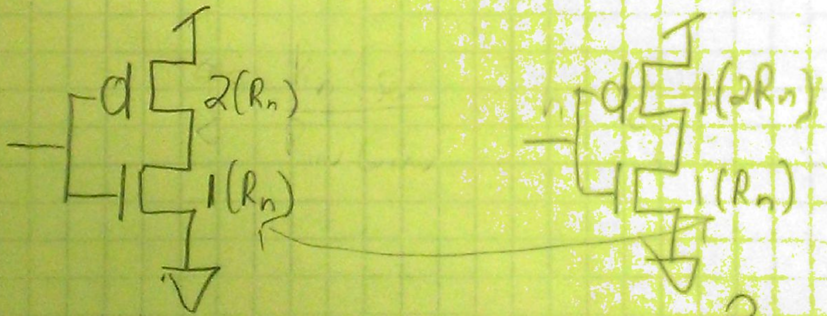


$t_{rise} = 2R_n \cdot 2C_{set} = 4R_nC_{set} = \tilde{\tau}$

$t_{fall} = R_n \cdot 2C_{set} = 2R_nC_{set}. \quad \frac{t_r + t_f}{2} = 3R_nC_{set}$

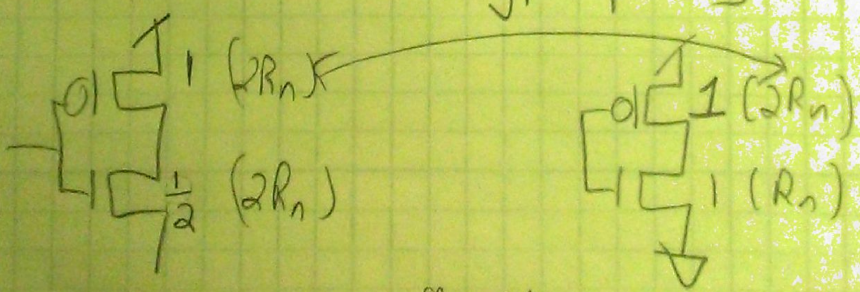
un-skewed

skewed



Get log. eff by comparing to unskewed inv w/ same R on edge

$g_{pullup} = \frac{2}{3}$



$g_{pulldown} = \frac{2}{(\frac{3}{2})} = \frac{4}{3}$

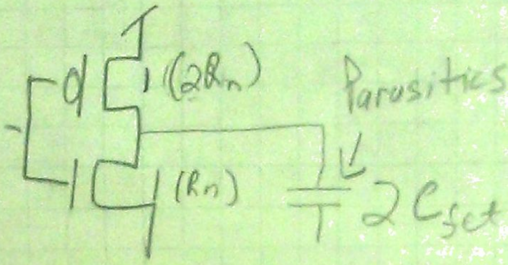
$g = \frac{g_p + g_d}{2} = 1 \therefore \quad g \cdot h = 1\tau.$

$\frac{t_r + t_f}{2} = 3R_nC_f = \tau$

$\therefore \; g \cdot h = \frac{t_f + t_r}{2}.$

5)

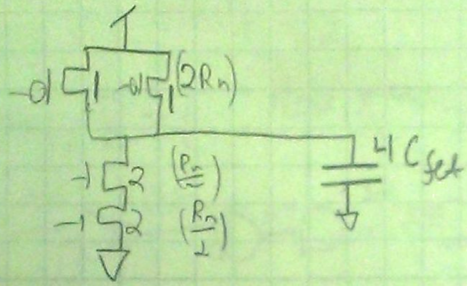
Inv



$\tilde{P}_{inv} = 4C_{set}R_n = \tilde{\tau}$

$\tilde{P}_{inv} = \tilde{\tau} = \frac{4}{3}\tau.$

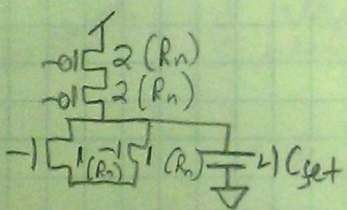
NAND2



$\tilde{P}_{NA2} = 4C_{set}\cdot 2R_n = 8C_{set}R_n = 2\tilde{\tau}$

$\tilde{P}_{NA2} = \frac{4}{3}\cdot 2\tau = \frac{8}{3}\tau$

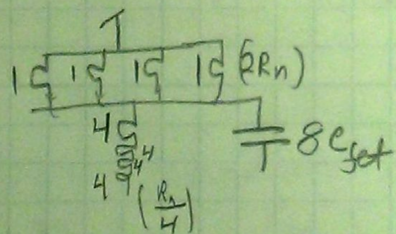
NOR2



$\tilde{P}_{NOR2} = 4C_{set}\cdot 2R_n = 2\tilde{\tau} = \frac{8}{3}\tau.$

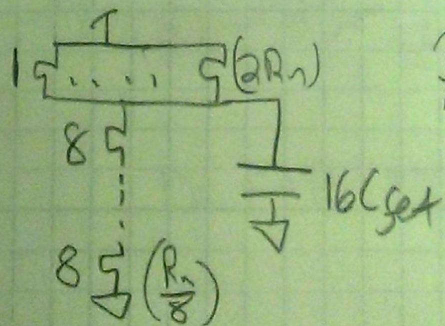
NANDS & NORS have same delay when $\gamma = 1$

NAND4



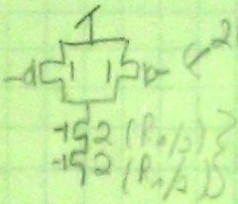
$\tilde{P}_{NAND4} = 8C_f\cdot 2R_n = 4\tilde{\tau} = \frac{16}{3}\tau$

NAND8



$\tilde{P}_{NA8} = 16C_f\cdot 2R_n = 8\tilde{\tau} = \frac{32}{3}\tau$

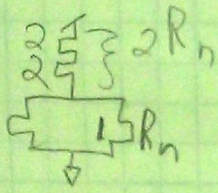
6) NAND. $F = \overline{AB} = \bar{A} + \bar{B}$



$2R_n, w=1$

$C_{inv} = 2.$

$NOR = \overline{A+B} = \overline{\bar{A}\bar{B}}$



$g_{NAND2} = \frac{3}{2}$

$g_{NAND4} = \frac{5}{2}$

$g_{NOR2} = \frac{3}{2}$

4 $\frac{R_n}{4}$

$g_{NA8} = \frac{9}{2}$

8 $\frac{R_n}{4}$ 4 $R_n/2$ 4 4 4

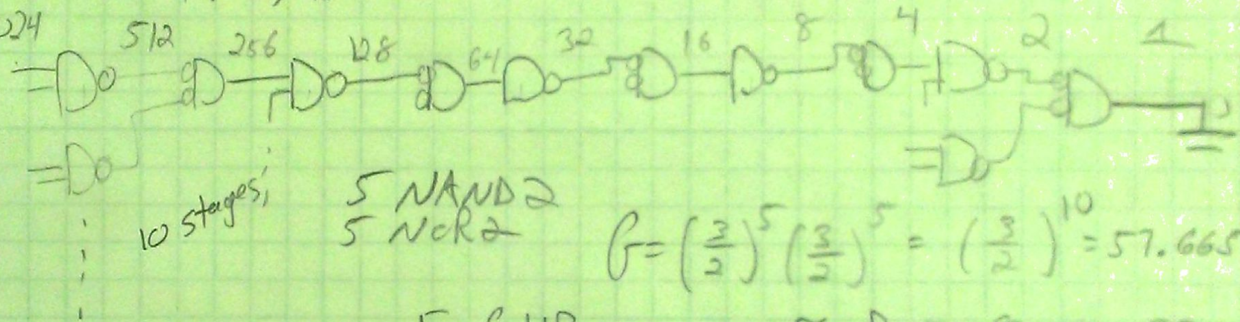
$g_{NOR4} = \frac{5}{2}$

$g_{NOR8} = \frac{9}{2}$

8 $\frac{R_n}{8}$

Radix2

$H=1, B=1$



10 stages;

5 NAND2
5 NOR2

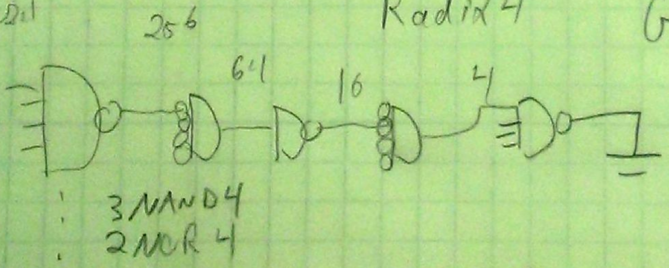
$G = \left(\frac{3}{2}\right)^5 \left(\frac{3}{2}\right)^5 = \left(\frac{3}{2}\right)^{10} = 57.665$

$F = GHB = 57.665\tau \quad P = 10 \cdot \frac{8}{3}\tau = \frac{80}{3}\tau$

$D = F + P = 57.665 \cdot \left(\frac{4}{3}\right)\tau + \frac{80}{3}\tau = 103.55\tau$

Radix4

$G = \left(\frac{5}{2}\right)^5. \quad F = GHB = 97.65625$



3 NAND4
2 NOR4

$P = 5 \cdot \left(\frac{16}{3}\right)\tau$

$D = F + P = 97.6525 \cdot \frac{4}{3}\tau + \frac{5 \cdot 16}{3}\tau =$

156.87τ