

Lab 1 – Canonical Genetic Algorithm

What is the “Genetic Algorithm”?

The genetic algorithm (GA) is a **search heuristic** and a mathematical **simulation** of the process of **natural evolution**. It uses approximate calculations to solve both constrained and unconstrained **optimization** problems.

Genetic algorithms are commonly used to generate high-quality solutions (not necessarily optimal solutions) to optimization and search problems using biologically inspired **operators** such as mutation, crossover and selection.

The basic steps performed by the canonical GA are:

- Initialization (of the population)
- Looping over generations and performing:
 - Fitness evaluation
 - Selection (of parents for reproduction)
 - Crossover
 - Mutation
 - Replacement

Some terminologies used in GA:

Before you understand how to solve optimization or search problems using GA, there are a few terminologies that you need to know:

<i>Chromosome</i>	an individual in the population and consists of an array of genes. It models one particular solution to the problem we’re trying to solve.
<i>Gene</i>	a particular bit in the chromosome. It models a particular trait in the solution.
<i>Population</i>	a constant number of chromosomes. It represents the pool of solutions .
<i>Genotype</i>	the genetic structure (how bits are organized) of a particular solution.
<i>Phenotype</i>	the actual solution itself. It can be encoded to or decoded from genotype.

How to solve problems using GA:

Firstly, you need to think how you can appropriately map solutions to chromosomes. Once you're done with that, you can easily apply the canonical GA. (*We will explore each step in detail*)

- **Initialization:**

The initial population is generated **randomly**, allowing the entire range of possible solutions (the search space). Occasionally, the solutions may be "**seeded**" in areas where optimal solutions are likely to be found. The population size depends on the nature of the problem, but typically contains several hundreds or thousands of possible solutions.

- **Fitness Evaluation:**

The fitness function can be the objective function you're trying to maximize (or its inverse if your objective is to minimize a function). The fitness function is evaluated for every chromosome and stored to be used in the selection process.

- **Selection:**

In this step, we are trying to select some fit parents to breed. A lot of different selection methods exist like "**Roulette Wheel**" which is one of the popular selection methods used in GA. In the roulette wheel selection, the probability of choosing an individual for breeding is proportional to its fitness; the better the fitness is, the higher chance for that individual to be chosen.

- **Crossover:**

Crossover is a process of **recombination**. This genetic operator is used to combine the genetic information of two parents to generate new offspring. Crossover is performed with a certain probability (P_c) and at a single random crossover point. (Note: We can also perform **k-point** crossover.)

- **Mutation:**

Mutation alters one or more gene values in a chromosome from its initial state to maintain diversity. It is also performed according to a certain probability (P_m).

- **Replacement:**

There are several different replacement schemes such as:

- **Generational replacement:** reproduce enough offspring to replace all population.
Drawbacks: possibility of losing good genes.
- **Steady-state replacement:** a number of individuals are selected to reproduce, and the offspring replace their parents.
Drawbacks: possibility of losing of good chromosomes.
- **Elitist Strategy:** like steady-state replacement but allows keeping the best so far.
Drawbacks: after some time, chromosomes will become a copy of each other.

Exercise:

How can we use the GA to solve the “Knapsack Problem”?

Given:

- A knapsack that can carry weights up to **W**
- **N** items; each item (x_i) has a weight (w_i) and a value (v_i)

Objective:

- Select the items to carry in the knapsack in order to maximize the total value.

$$(\max. \sum_{i=1}^N x_i \cdot v_i)$$

Constraints:

- The selected items must fit inside the knapsack.
- The entire item is either selected or not.

$$(\sum_{i=1}^N x_i \cdot w_i \leq W \text{ and } x_i = 0 \text{ or } 1)$$

Sol.:

We can represent a solution (the items selected) as a chromosome containing **N bits**; each bit (i) corresponds to item (i) where 1 means that the item is selected and 0 means that it is not selected.

Assume $W = 10$, $N = 5$ and we are given a list of pairs containing each items weight and value: [(5,4), (4,4), (2,1), (2,7), (4,6)]

Step 1: Let's say we started with following randomly initialized chromosomes:

0	1	0	1
1	1	0	0
1	0	1	0
0	0	0	0
1	0	0	0
C1	C2	C3	C4

Note: To make sure your mapping works, you can easily decode these genotypes into phenotypes. For example:

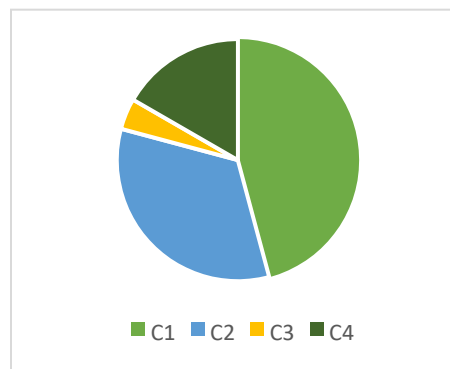
C1: $C1[0]=0$ -> item 1 is not selected, $C1[1]=1$ -> item 2 is selected, $C1[2]=1$ -> item 3 is selected, $C1[3]=0$ -> item 4 is not selected and $C1[4]=1$ -> item 5 is selected.

Step 2: Let's evaluate the fitness of each chromosome using the objective function:

<i>Chromosome</i>	<i>Fitness</i>
C1	11
C2	8
C3	1
C4	4

Step 3: Let's select the parents! First, we need to calculate the cumulative fitness function:

<i>Chromosome</i>	<i>Fitness</i>	<i>Cumulative Fitness</i>
C1	11	11
C2	8	19
C3	1	20
C4	4	24



Note: Another option is to **normalize** the fitness first by dividing it over the total fitness sum to make it range between 0 and 1 (representing the **probability** of selection) and then calculate the cumulative probabilities.

Second, generate a random number (**r1**) between 0 and 24:

If $0 \leq r < 11$, choose **C1**

If $11 \leq r < 19$, choose **C2**

If $19 \leq r < 20$, choose **C3**

If $20 \leq r < 24$, choose **C4**

Assume **r1** = 2.32, therefore C1 is the first parent selected. Generate a random number (**r2**) between 0 and 24 and assume **r2** = 16, so select C2 as the second parent.

Step 4: Let's perform crossover between C1 and C2:

First, generate a random integer (**Xc**) between **1** and **len(C)-1** to be the crossover point.

Second, generate a random number (**rc**) between 0 and 1:

If $r_c \leq P_c$, perform crossover at **Xc**.

If $r_c > P_c$, no crossover. (O1 = C1 and O2 = C2)

Assume $X_c = 2$, $r_c = 0.5$ and $P_c = 0.6$, so this means that we will perform crossover as follows:

0	1	1	0	1
---	---	---	---	---

1	1	0	0	0
---	---	---	---	---

0	1	0	0	0
---	---	---	---	---

O1

1	1	1	0	1
---	---	---	---	---

O2

Step 5: Let's perform mutation on the offspring:

Iterate over **each bit** in **each offspring** chromosome and:

- Generate a random number (r) between 0 and 1.
- If $r \leq P_m$, flip that bit.

Question: What will happen if we never performed mutation?

Step 6: Replace the current generation with the new offspring using any of the replacement strategies explained earlier, go to step 2 and repeat the process.