Schema Theory

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Why do Genetic Algorithms work?

In this section we take an in-depth look at the working of the standard genetic algorithm, explaining why GA constitutes an effective search procedure

For simplicity we discuss binary string representation of individuals

Schema Theory

- Schema theory seeks to give a theoretical justification for the efficacy of the field of genetic algorithms.

- What is a Schema:
  - a template
    - the new gene alphabet $\Rightarrow$ {0,1,*} where * is a don't care

*** * 1 1 0 * * 1 1 * * ***

  - Schema is favorable traits in a solution, where a favorable schema is called an above average schema
  - allows exploration of similarities among chromosomes
  - represents all matching strings

Schema Theory

Example:

the schema (* 1 1 1 1 0 0 1 0 0) matches two strings:

(0 1 1 1 1 0 0 1 0 0) and

(1 1 1 1 1 0 0 1 0 0)

Q. which strings does this schema match?

(0 1 1 * 1 0 1 1 * *)

$2^3 = 8$

011 0 1011 00
011 0 1011 01
011 0 1011 10
⋮
011 1 1011 11

- A schema matches 2^r strings

r: # of (*) in the schema

Schema Properties

- **Order of schema S: o(S)**
  - number of **fixed** (non-*) positions
    S1 = **11**101**
    12 345
    o(S1) = 5
  - The order of a schema is useful to calculate survival probability of the schema for mutations
- **Defining length of schema S: d(S)**
  - distance between first and last **fixed** string positions
    S1 = **11**101**
    3 9
    above d(S1) = 9-3= 6
    S2= **101***
    3 5
    d(S2)=?
    5-3=2
  - The defining length of a schema is useful to calculate survival probability of the schema for crossovers

Effect of Operators on Schema
1-Selection

- Assume total number of chromosomes is PopSize.
- The number of above average individuals abiding by the above average schema S is P.
- f_i : fitness of chromosome *i* in the population of all chromosomes.
- Average fitness of population = $\frac{\sum_{i=1}^{PopSize} f_i}{PopSize} = \bar{F}$

Effect of Operators on Schema
1-Selection

- The average fitness of above average individuals abiding with favorable schema S=

$$F_{av}(S) = \frac{\sum_{j=1}^{P} f_j}{P}$$

  – **where P is the number of individuals abiding by S**

- Let :
  – **m(S, t)** denote the expected no. of individuals matched by schema S at time t.
  – **F** = total fitness of population, F=$\sum_{i=1}^{PopSize} f_i$
  – **m(S, t+1)** denote the expected no. of individuals matched by schema S at time t+1(next iteration).

Effect of Operators on Schema
1-Selection

- $m(S, t+1) = m(S,t) * PopSize * \frac{f_{av}(S)}{F}$ 1

- But F = total fitness
 where $\bar{F} = \frac{F}{PopSize}$, so F= $\bar{F} * PopSize$

- Substitute in (1):
 $m(S, t+1) = m(S,t) * PopSize * \frac{f_{av}(S)}{\bar{F} * PopSize}$
 $m(S, t+1) = m(S,t) * \frac{f_{av}(S)}{\bar{F}}$ 2

Effect of Operators on Schema
1-Selection

$$m(S, t+1) = m(S,t) * \frac{f_{av}(S)}{\overline{F}} \qquad 2$$

Note that $\frac{f_{av}(S)}{\overline{F}} > 1$

$6 > 4 \Rightarrow 6 = 4 + \frac{1}{2} 4$

$f_{av}(s) > \overline{F}$

$f_{av}(s) = \overline{F} + \in \overline{F} \quad \rightarrow \quad f_{av}(s) = (1+\in)\overline{F}$

<u>Substitute in (2):</u>

$m(s,1) = m(s,0) * (1+\epsilon)$

$m(s,2) = m(s,1) * (1+\epsilon)$

$m(s,2) = m(s,0) * (1+\epsilon) * (1+\epsilon)$

$= m(s,0) * (1+\epsilon)^2$

- $m(S, t+1) = m(S,t) * \frac{(1+\in)\overline{F}}{\overline{F}}$

  $= m(S,t) * (1+\in) \text{ ---- } (1+\in) > 1$
- $m(S,t) = m(S,0) * (1+\in)^t$ Geometric Series!

Effect of Operators on Schema
2-Crossover

<u>Remember:</u> Effect of selection on schema:

$$m(S, t+1) = m(S, t) * \frac{f_{av}(S)}{\overline{F}} \quad 1$$

<u>Effect of Crossover:</u>

What is the probability of **<u>destruction</u>** of a schema by crossover?

$1 \rightarrow l-1$

$1 \rightarrow 6$

*	*	1	*	0	1	*

$\underbrace{\quad}_{d(S)=6-3}$

$\underbrace{\quad}_{l=7}$

$$P_d = \frac{d(S)}{l-1} \rightarrow$$

$\frac{3}{6} = \frac{1}{2}$

where ***d(S)*** is the defining length of the schema, and ***l*** is length of chromosome.

Effect of Operators on Schema
2-Crossover

What is the probability of **survival** of a schema after crossover?

$P_s = 1 - P_d$

P_c

-->It should be (1-probability of destruction), but... destruction only happens if crossover will occur, and crossover occurs with a probability of **P_c**. So:

$$P_s = 1 - P_c \frac{d(S)}{l-1}$$

P_d

Substitute in equation (1):

P_s

$$m(S, t+1) = m(S,t) * \frac{f_{av}(S)}{\bar{F}} * \left(1 - P_c \frac{d(S)}{l-1}\right) \qquad (2)$$

Equation (2) represents the combined effects of selection and crossover.

Effect of Operators on Schema
3-Mutation

What is the probability of **survival** of a schema after mutation?
i.e. all bits survive mutation without any changes

→probability of destruction of 1 bit: $\mathbf{P_d = P_m}$

o(S) = 3

*	*	~~1~~ 0	*	0	1	*

→probability of survival of 1 bit:

$$P_s = 1 - P_d = 1 - P_m$$

$(1-P_m)^3$

→probability of survival of schema: [given order of schema o(S)]

$$P_s(S) = (1 - P_m) * (1 - P_m) * \cdots \text{[for o(S) times]}$$

$$P_s(S) = (1 - P_m)^{o(S)}$$

-- but Pm is a small number

$$P_s(S) \simeq 1 - o(S)P_m$$

Effect of Operators on Schema
3-Mutation

So probability of **survival** of a schema after crossover and mutation is:

crossover P_c mutation

$$P_s = 1 - P_c \frac{d(S)}{l-1} - o(S)P_m$$

→Substitute in (2):

$$\mathbf{m}(S, t+1) = m(S,t) * \frac{f_{av}(S)}{\bar{F}} * \left(1 - P_c \frac{d(S)}{l-1} - o(S)P_m\right)$$

which is called the **Reproductive Schema Growth** Equation **RSG**

Conclusion:

'Short', 'low-order' & 'above-average' schema are liable to grow, increase and flourish in population

Exercise

Given the following schema 'S':

1				5			
1	**0**	**0**	*****	**1**	*****	*****	*****

A- What is the probability of schema S survival after crossover knowing that Pc = 0.8?

$$P_s = 1 - \overset{0.8}{P_c} * \frac{d(s)}{L-1} \quad \frac{4}{7}$$

B- What is the probability of schema S survival after mutation knowing that Pm = 0.1?

$$P_s = 1 - \underset{0.1}{P_m} * O(s)$$
$$* \; 4$$