BINARY GENETIC ALGORITHM EXAMPLE

Soft Computing

PROBLEM

Find the minimum value of this function

$$f(x) = x^2 - 4x + 10$$

How to solve it using Genetic Algorithm (*GA*)?

GIVENS & REQUIREMENTS

Givens:

- $x \in [0,127]$
- Population size (pS)= 6
- Probability of Crossover (P_c) = 0.7
- Probability of Mutation (P_m) = 0.02
- Number of selected Chromosomes (N) = 4

Requirements:

- Use Rank Selection
- Use 2-point Crossover
- Use Elitist Replacement

Objective:

→ Minimize the value of the function

→ $\min. f(x), where\ f(x) = x^2 - 4x + 10$

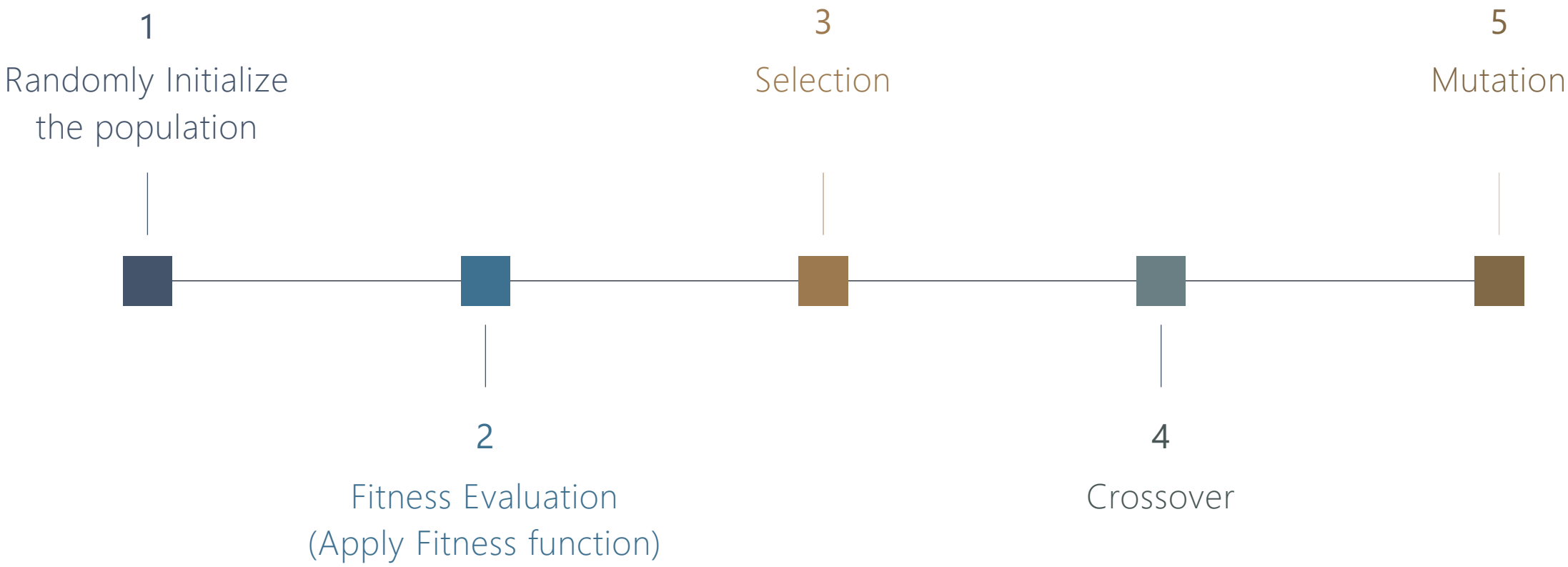
SOLUTION STEPS

→You must decide the representation of the Chromosomes first and determine the problem objective

Initial steps to solve any problem with GA:

1. Find the representation/encoding of the chromosomes
2. Decide the Objective & Fitness Function
3. Apply GA

SOLUTION STEPS



SOLUTION STEPS

6

Replacement

7

Repeat strep from 2 → 6

Until reach your stopping condition

SOLUTION

1. Solution Representation/encoding:

Choose Binary representation of the x value

→ Each chromosome represented as an array of binary bits, with size = 7

Chromosome:

1	0	..	...	...	1	1

2. Find Objective:

→ Minimize the function value. *(already specified in the problem)*

SOLUTION (3. APPLY GA)

1. Randomly Initialize the population:

Population size = 6 → generate 6 random chromosomes:

C1:	0	0	0	0	1	1	1
C2:	0	0	1	1	0	0	0
C3:	0	1	1	0	0	1	1
C4:	1	0	0	0	0	0	0
C5:	1	0	0	0	1	1	1
C6:	0	0	1	1	0	1	1

SOLUTION (3. APPLY GA)

2. Fitness Evaluation: $f(x) = x^2 - 4x + 10$

C1:

0	0	0	0	1	1	1

Phenotype: $x = 7$

Fitness: $7^2 - 4 * 7 + 10 = 31$

C2:

0	0	1	1	0	0	0

Phenotype: $x = 24$

Fitness: $24^2 - 4 * 24 + 10 = 490$

C3:

0	1	1	0	0	1	1

Phenotype: $x = 51$

Fitness: $51^2 - 4 * 51 + 10 = 2407$

C4:

1	0	0	0	0	0	0

Phenotype: $x = 64$

Fitness: $64^2 - 4 * 64 + 10 = 3850$

C5:

1	0	0	0	1	1	1

Phenotype: $x = 71$

Fitness: $71^2 - 4 * 71 + 10 = 4767$

C6:

0	0	1	1	0	1	1

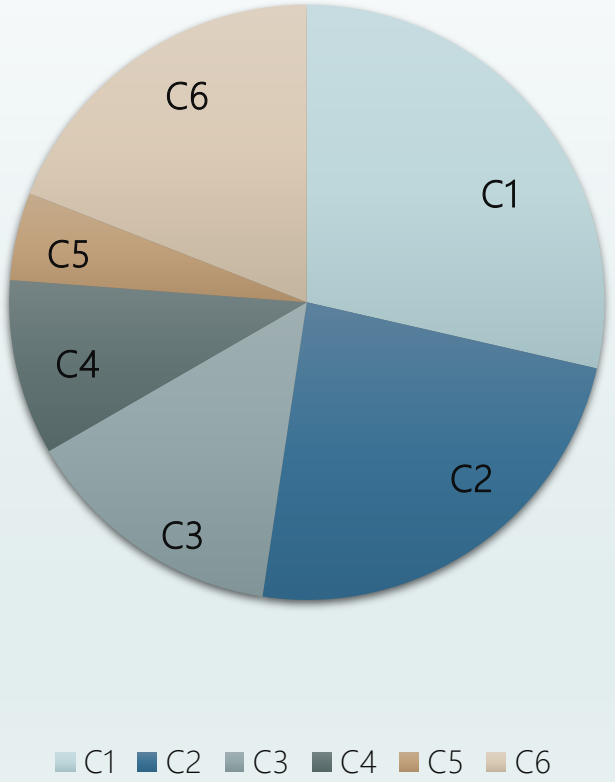
Phenotype: $x = 27$

Fitness: $27^2 - 4 * 27 + 10 = 631$

SOLUTION (3. APPLY GA)

3. Selection *(Rank Selection):*

	Fitness	Rank	Normalized	Cumulative
C1	31	6	$^{6}/_{21}$	$^{6}/_{21} = 0.285$
C2	490	5	$^{5}/_{21}$	$^{11}/_{21} = 0.523$
C3	2407	3	$^{3}/_{21}$	$^{14}/_{21} = 0.666$
C4	3850	2	$^{2}/_{21}$	$^{16}/_{21} = 0.761$
C5	4767	1	$^{1}/_{21}$	$^{17}/_{21} = 0.809$
C6	631	4	$^{4}/_{21}$	1

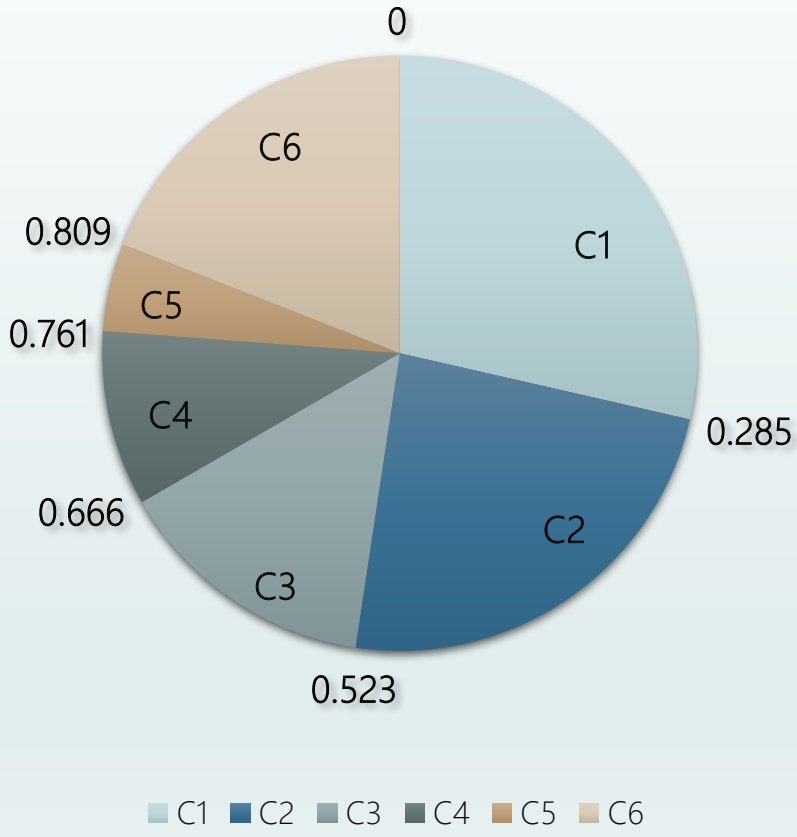


SOLUTION (3. APPLY GA)

3. Selection *(Rank Selection):*

*Generate **N** Random Numbers (**r**), where* **r ∈ [0,1]**

r	Selected Chromosome
0.1	C1
0.5	C2
0.8	C5
0.23	C1



SOLUTION (3. APPLY GA)

4. Crossover *(2-point crossover):* $P_c = 0.7$

Apply crossover on C1 & C2

- *r = 0.2* → *apply crossover*
- *Generate 2 random numbers* ∈ [1, 6]

$x_1 = 1, x_2 = 4$

C1:	0	0	0	0	1	1	1
C2:	0	0	1	1	0	0	0
O1:	0	0	1	1	1	1	1
O2:	0	0	0	0	0	0	0

Apply crossover on C5 & C1

- *r = 0.8* → *no crossover*

C5:	1	0	0	0	1	1	1
C1:	0	0	0	0	1	1	1
O3:	0	0	0	0	1	1	1
O4:	1	0	0	0	1	1	1

SOLUTION (3. APPLY GA)

5. Mutation: $P_m = 0.02$

Generate random number $r_{i,j} \in [0,1]$, *where* i *is number of chromosome,* j*=index of bit*

Applying on O1

$r_{1,0} = 0.1$ $r_{1,1} = 0.01$ $r_{1,2} = 0.7$ $r_{1,3} = 0.65$ $r_{1,4} = 0.008$ $r_{1,5} = 0.247$ $r_{1,6} = 0.7676$

O1:

0	0	1	1	1	1	1

→

0	1	1	1	0	1	1

Doing the same on O2, O3, O4

O2:

0	0	0	0	0	0	0

→

0	0	0	1	0	0	0

O3:

0	0	0	0	1	1	1

→

0	0	0	0	1	0	1

O4:

1	0	0	0	1	1	1

→

1	0	0	0	1	1	1

SOLUTION (3. APPLY GA)

6. Replacement *(Elitist Strategy):*

First, evaluate the parents and offsprings using fitness function

Chromosome	C1	C2	C3	C4	C5	C6	O1	O2	O3	O4
Fitness	31	490	2407	3850	4767	631	3255	42	15	4767

*Second, sort and choose first **pS** chromosomes*

O3	C1	O2	C2	C6	C3	O1	C4	C5	O4

New generation:
O3, C1, O2, C2, C6, C3

RANK VS ROULETTE WHEEL SELECTION

Soft Computing

Selection Method

Rank

Minimization Problem

→ Give the chromosome with smallest fitness, largest rank (pop_size)

→ Apply Rank Selection

	Fitness	Rank	Norm	Cum.
C1	2	4	4/10	4/10
C2	20	1	1/10	5/10
C3	10	2	2/10	7/10
C4	5	3	3/10	1

Maximization Problem

→ Give the chromosome with highest fitness, largest rank (pop_size)

→ Apply Rank Selection

	Fitness	Rank	Norm	Cum.
C1	2	1	1/10	1/10
C2	20	4	4/10	5/10
C3	10	3	3/10	8/10
C4	5	2	2/10	1

Roulette Wheel

Minimization Problem

→ Take the inverse of Fitness values

→ Apply Roulette wheel

	Fitness	Norm	Cum.
C1	1/2	10/17	10/17
C2	1/20	1/17	11/17
C3	1/10	2/17	13/17
C4	1/5	4/17	1

Maximization Problem

→ Take Fitness values as it is

→ Apply Roulette wheel

	Fitness	Norm	Cum.
C1	2	2/37	2/37
C2	20	20/37	22/37
C3	10	10/37	32/37
C4	5	5/37	1

C1: [... | ... | ... | ...] C2: [... | ... | ... | ...] C3: [... | ... | ... | ...] C4: [... | ... | ... | ...]

Fitness: 2 | 20 | 10 | 5

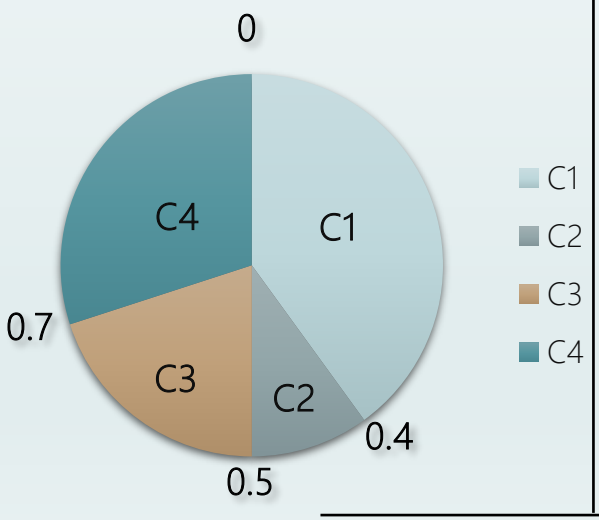
Selection Method

Rank

Minimization Problem

→ Give the chromosome with smallest fitness, largest rank (pop_size)

→ Apply Rank Selection



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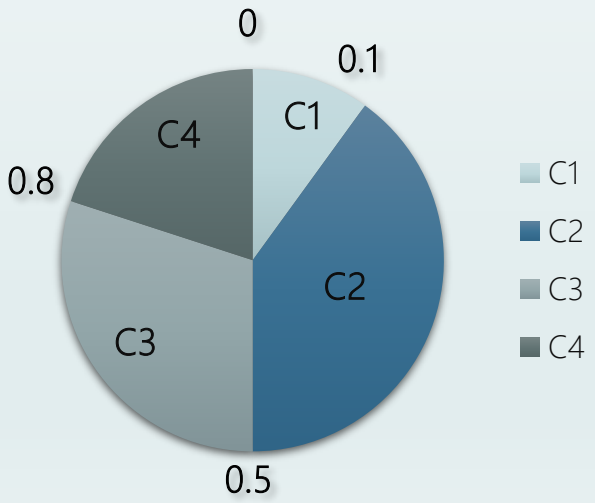

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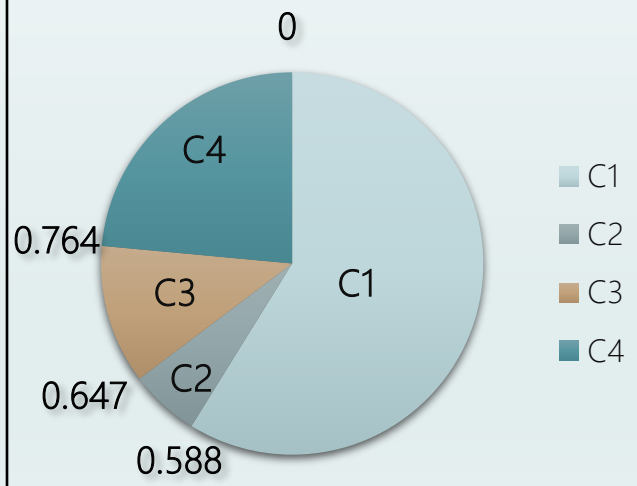

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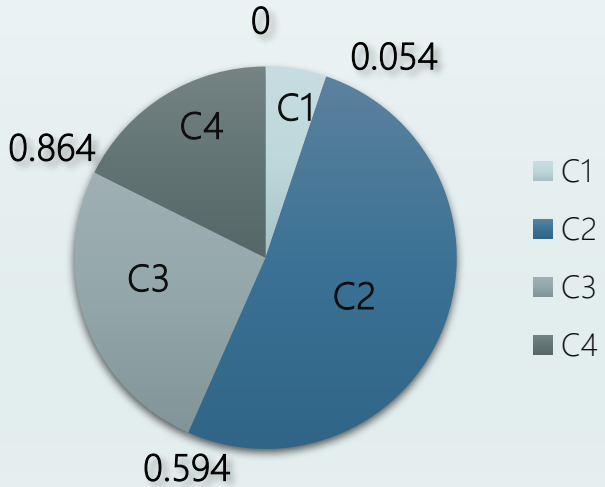
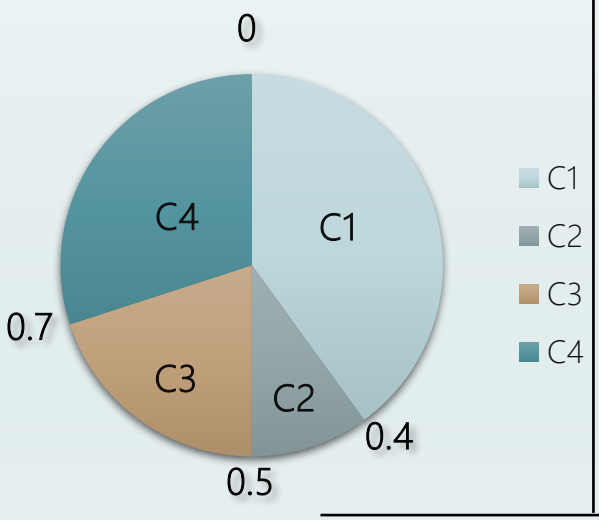


	C1	C2	C3	C4
Fitness:	2	20	10	5

Rank

Minimization Problem

	Fitness	Rank	Norm	Cum.
C1	2	4	4/10	4/10
C2	20	1	1/10	5/10
C3	10	2	2/10	7/10
C4	5	3	3/10	1



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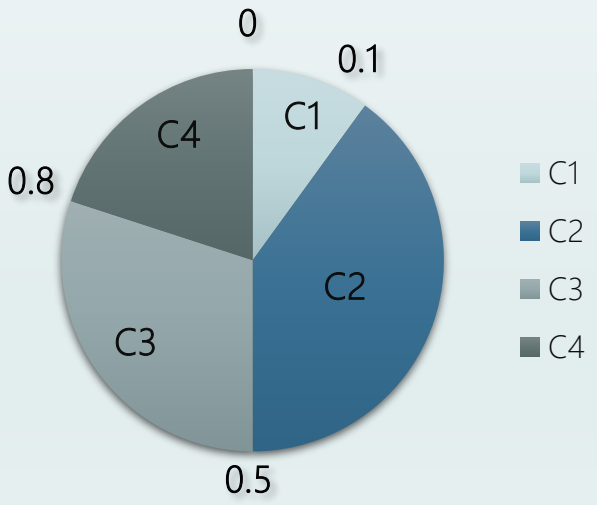

Roulette Wheel

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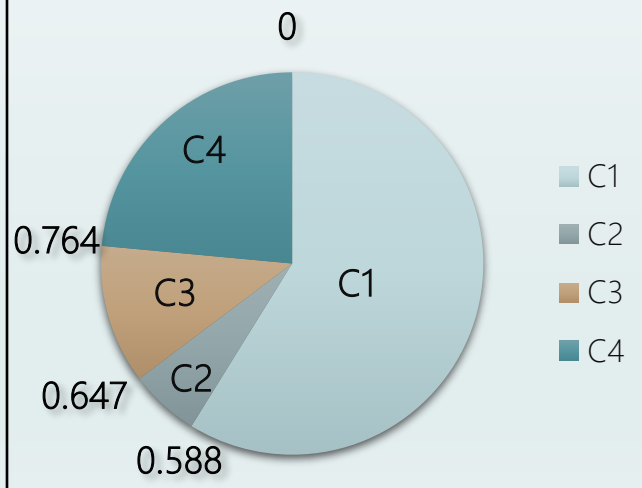
C3	1/10	2/17	13/17
C4	1/5	4/17	1



Maximization Problem

	Fitness	Norm	Cum.
C1	2	2/37	2/37
C2	20	20/37	22/37
C3	10	10/37	32/37
C4	5	5/37	1