



جامعة القاهرة



CS432 - Computation Theory
Assignment #2
4CS-S5

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Context Free Grammar

Q1: Find a CFG for each of the following languages over the alphabet (a, b).

Solution

$$- S \rightarrow aSa \mid bSb \mid aSb \mid bSa \mid \varepsilon$$

$$- S \rightarrow A$$

$$A \rightarrow aB \mid bB$$

$$B \rightarrow aaS \mid abS \mid baS \mid bbS$$

$$- S \rightarrow aSB \mid bSB \mid \varepsilon$$

$$B \rightarrow aa \mid bb$$

Q2: Find a CFG grammar of this form for the language of each of the following regular expressions.

Solution

$$- S \rightarrow aB$$

$$B \rightarrow abB \mid \varepsilon$$

$$- S \rightarrow Ba$$

$$B \rightarrow abB \mid \varepsilon$$

$$- S \rightarrow AcB$$

$$A \rightarrow abA \mid \varepsilon$$

$$B \rightarrow aB \mid bB \mid \varepsilon$$

Q3: Find the leftmost derivation of the word **abba** in grammar. Draw a derivation tree.

Solution

$$S \rightarrow AA$$

$$AA \rightarrow aBA \quad (A \rightarrow aB)$$

$$aBA \rightarrow abBA \quad (B \rightarrow bB)$$

$$abbA \rightarrow abbA \quad (B \rightarrow bB)$$

$$abbBA \rightarrow abbA \quad (B \rightarrow \varepsilon)$$

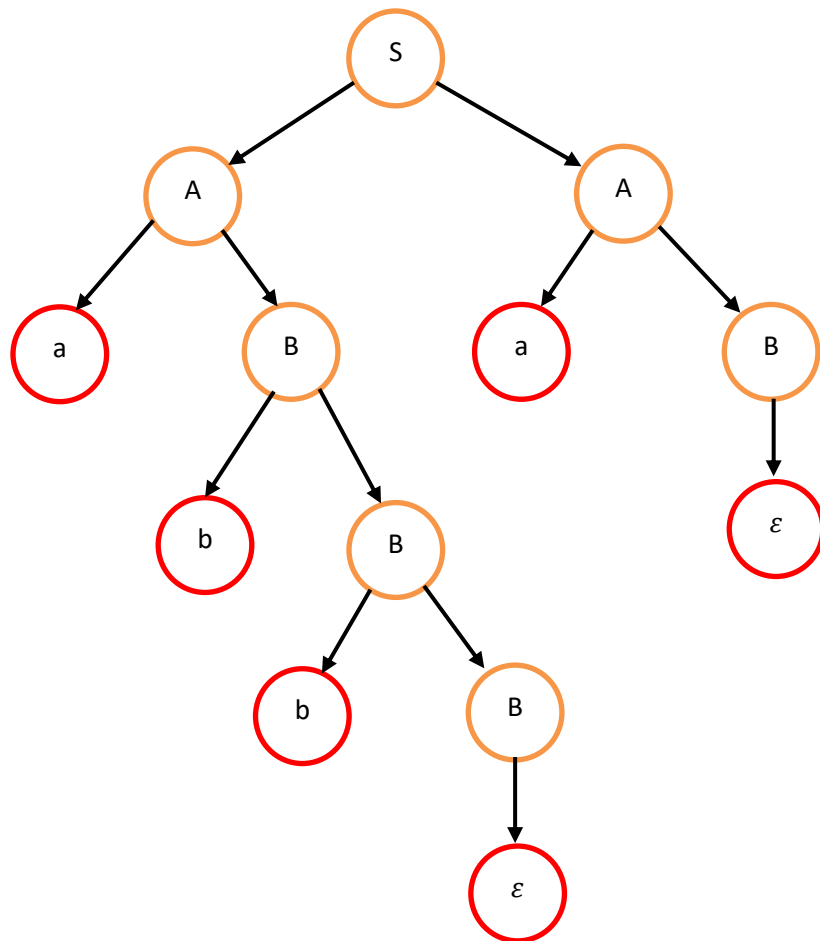
$$abbaA \rightarrow abbaB \quad (A \rightarrow aB)$$

$$abbaB \rightarrow abba \quad (B \rightarrow \varepsilon)$$

$$S \rightarrow AA$$

$$A \rightarrow aB$$

$$B \rightarrow bB \mid \Lambda$$



Q4: Find the leftmost derivation of the word **abbabaabbbabbab** in the CFG. Draw a derivation tree.

Solution

$S \rightarrow SSS \mid aXb \ (S \rightarrow SSS)$

$SSS \rightarrow aXbSS \ (S \rightarrow aXb)$

$aXbSS \rightarrow abbaSS \ (X \rightarrow bba)$

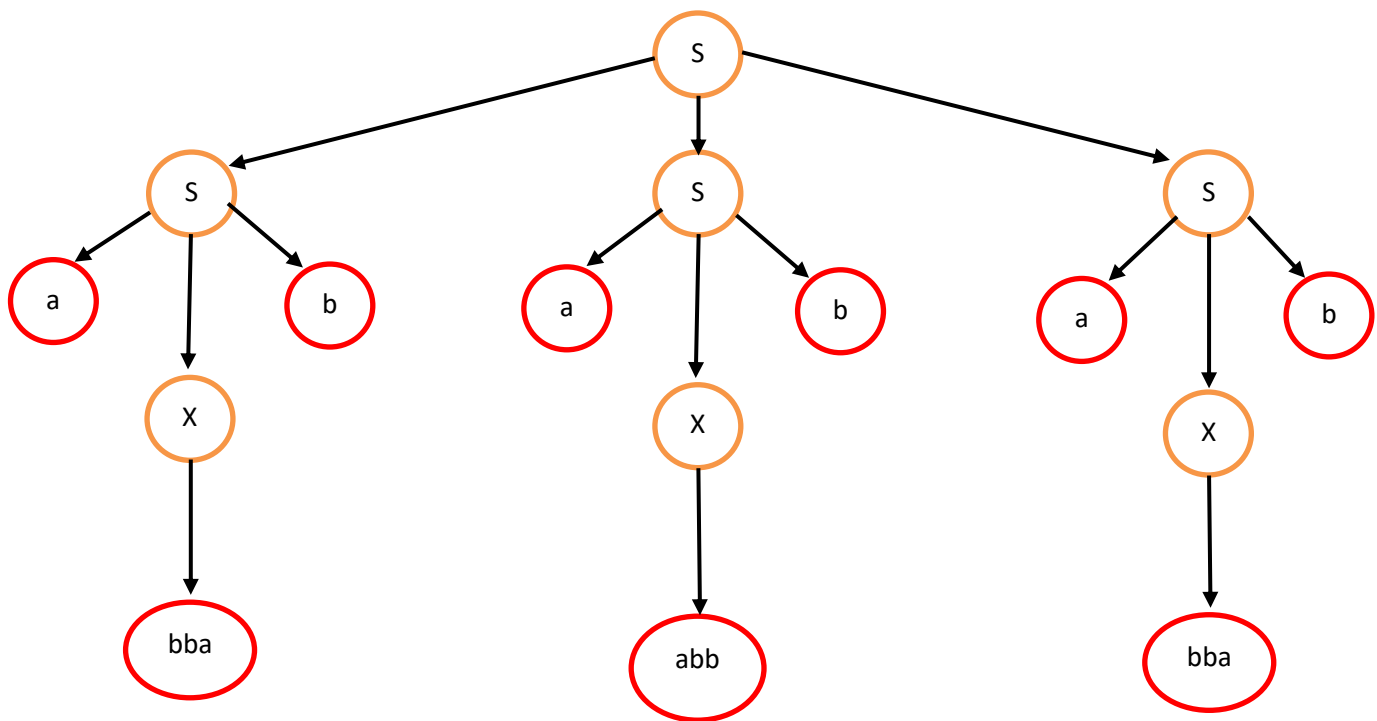
$abbabSS \rightarrow abbabaXbS \ (S \rightarrow aXb)$

$abbabaXbS \rightarrow abbabaabbbS \ (X \rightarrow abb)$

$abbabaabbbS \rightarrow abbabaabbb aXb \ (S \rightarrow aXb)$

$abbabaabbb aXb \rightarrow abbabaabbb abba \ (X \rightarrow bba)$

$S \rightarrow SSS \mid aXb$
 $X \rightarrow ba \mid bba \mid abb$



Q5: For the following CFGs, find regular expressions that define the same language and describe the language.

Solution

- (i) $S \rightarrow aS \mid bX \mid a$
 $X \rightarrow aX \mid bY \mid bZ \mid a$
 $Y \rightarrow aY \mid a$
 $Z \rightarrow aZ \mid bW$
 $W \rightarrow aW \mid a$
- (ii) $S \rightarrow bS \mid aX$
 $X \rightarrow bS \mid aY$
 $Y \rightarrow aY \mid bY \mid a \mid b$

$W \rightarrow aW \mid a$	$W \rightarrow a^*$
$Z \rightarrow aZ \mid bW$	$Z \rightarrow a(a b)^* \mid b(a b)a$
$Y \rightarrow aY \mid a$	$Y \rightarrow a^*$
$X \rightarrow aX \mid aY \mid bZ \mid a$	$X \rightarrow a(a b)^*a \mid (a b)^*a \mid b(a b)^*a \mid a$
$S \rightarrow aS \mid bX \mid a$	$S \rightarrow a(a b)^*a \mid b(a b)^*a \mid a(a b)^*a \mid b(a b)^*a \mid a$

– *Regx* : $a(a + b)^*a + b(a + b)^*a + a(a + b)^*a + b(a + b)^*a + a$

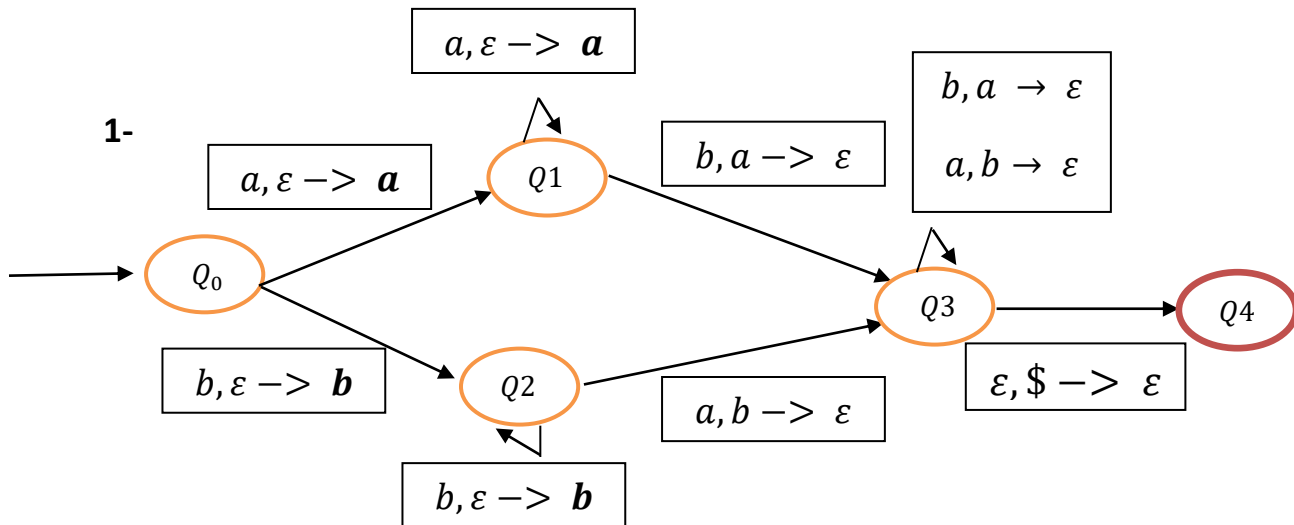
$Y \rightarrow aY \mid bY \mid a \mid b$	$Y \rightarrow (a b)^*$
$X \rightarrow bS \mid aY$	$X \rightarrow bS \mid a(a b)^*$
$S \rightarrow bS \mid aX$	$S \rightarrow b(bS a(a b)) \mid a(bS a(a b))$ $S \rightarrow a(a b)^* \mid b(a b)(a(a b) b(a b)^*)$

– *Regx* : $a(a + b)^* + b(a + b)(a(a + b)|b(a + b)^*)$

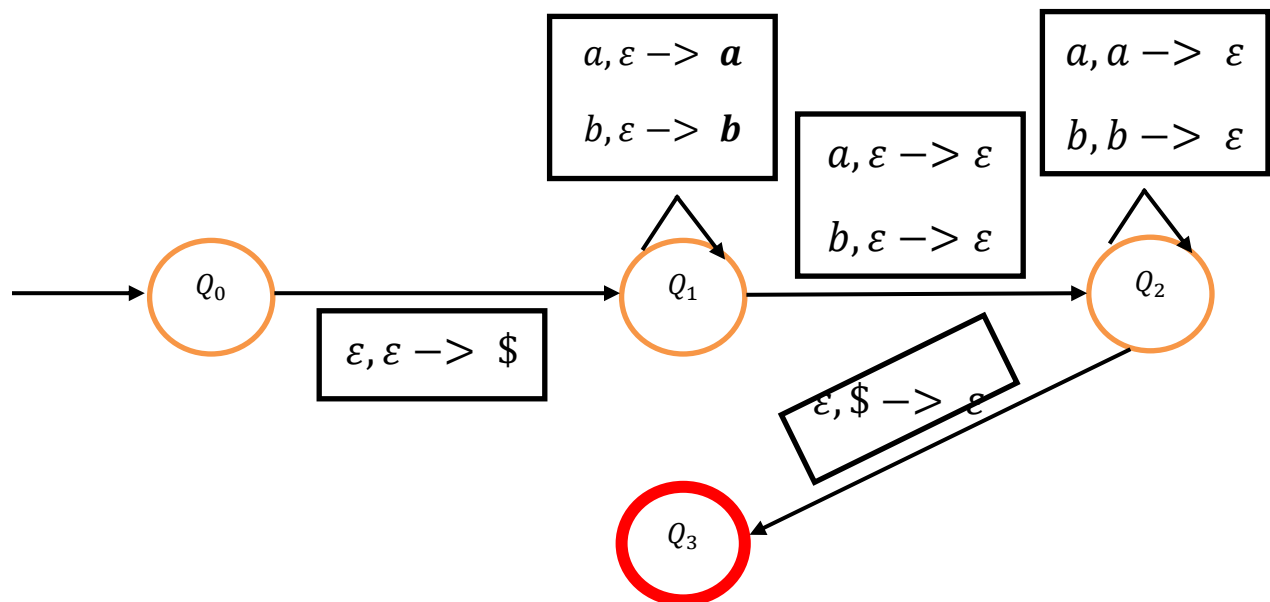
Push Down Automaton

Q6: Find a pushdown automaton for each of the following languages.

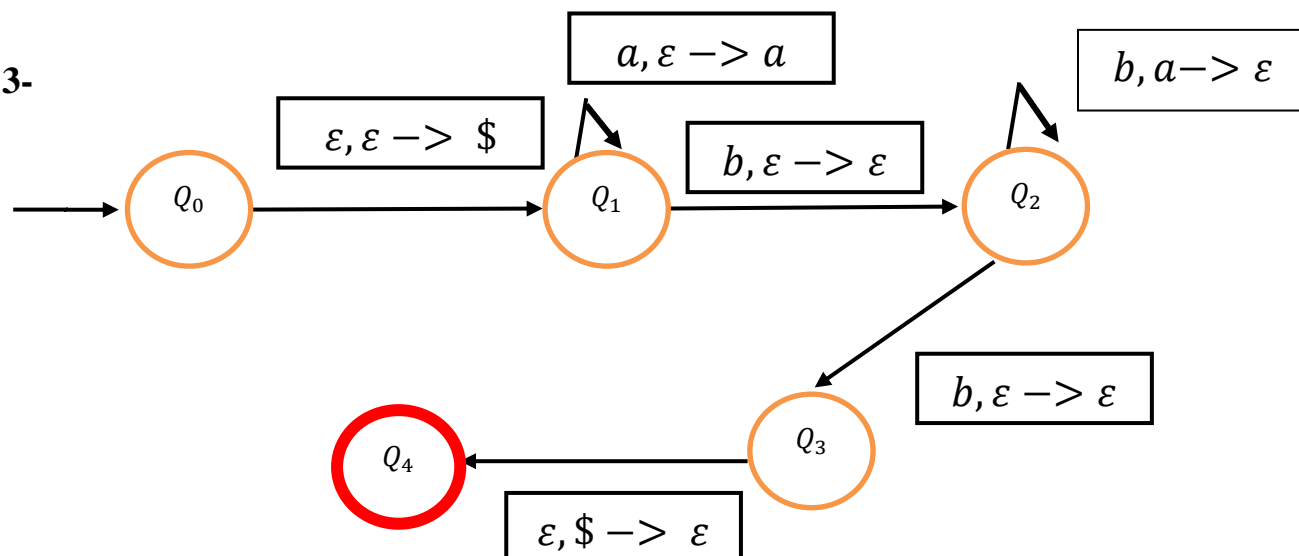
Solution



2-

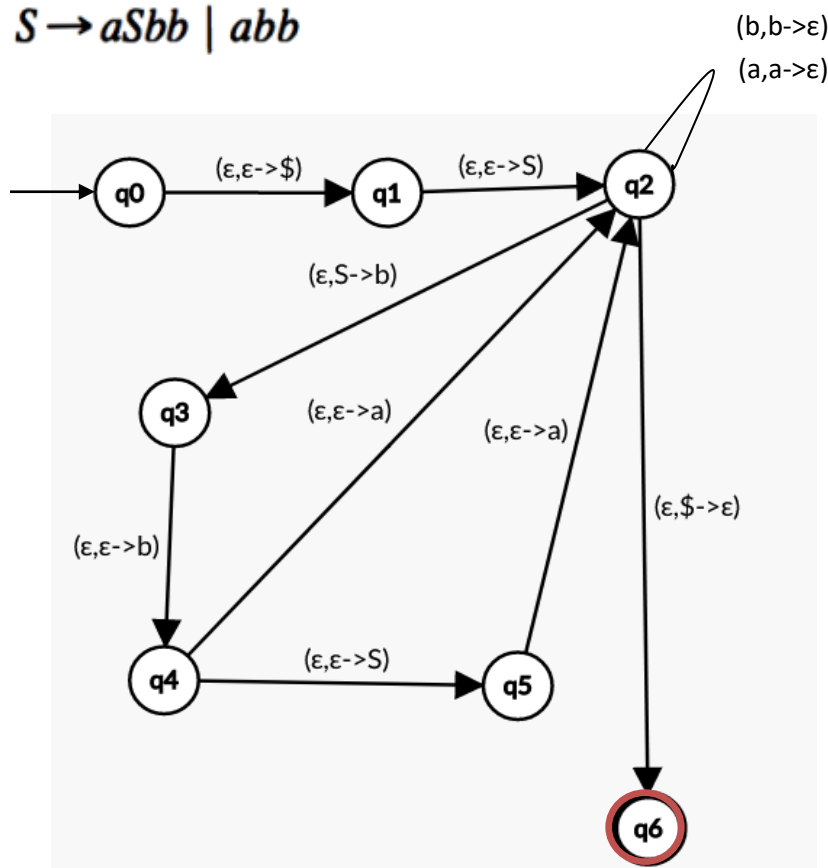


3-

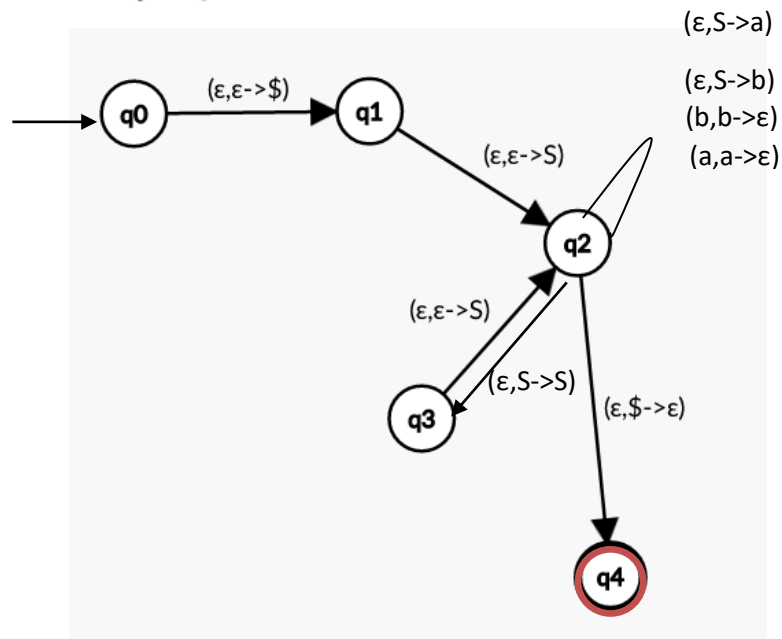


Q7: For each of the CFGs below in Problems construct a PDA that accepts the same language they generate.

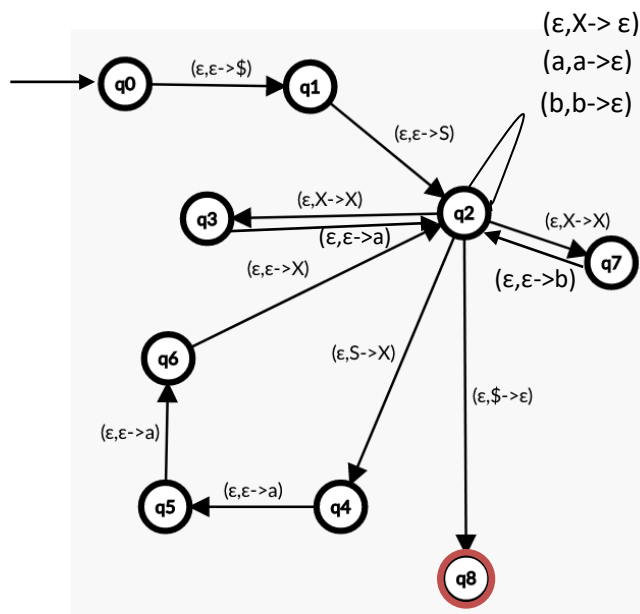
1. (i) $S \rightarrow aSbb \mid abb$



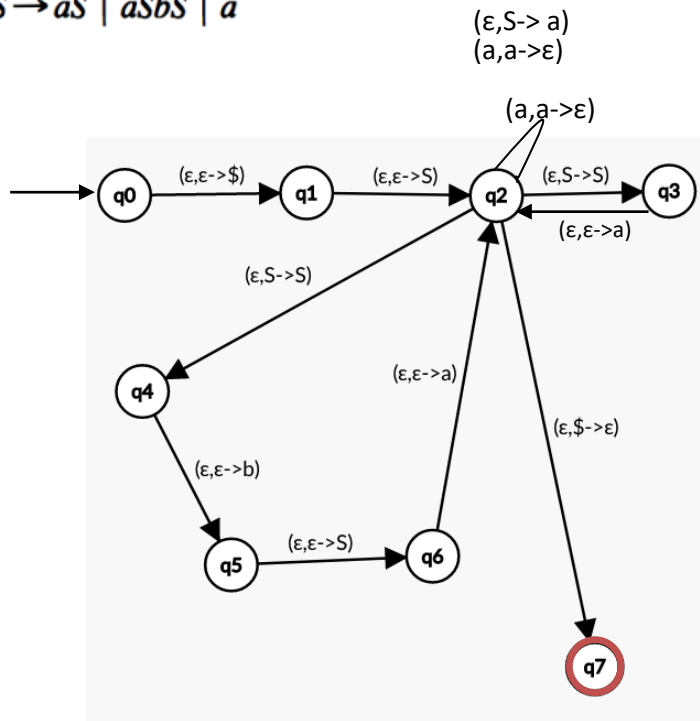
1. (ii) $S \rightarrow SS \mid a \mid b$



2. $S \rightarrow XaaX$
 $X \rightarrow aX \mid bX \mid \Lambda$



3. $S \rightarrow aS \mid aSbS \mid a$



4. $S \rightarrow XY$

$X \rightarrow aX \mid bX \mid a$

$Y \rightarrow Ya \mid Yb \mid a$

