CS432 - Computation Theory

Assignment #1

4CS-S5

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Regular Expressions

- Find a regular expression to describe each of the following five languages

Solution

1. $\wedge\ + a\ (b\ .\ b)^*$
2. $\wedge\ + (a^* +\ b^* +\ c^*)$
3. $\wedge\ +(\ c^*\ .\ a + b\ .\ c^*\ )$
4. $(a\ .\ a)^* + b\ .\ (b\ .\ b)^*$
5. $a^*\ .\ b\ .\ c^*$

- Find a regular expression over the alphabet (0, 1) to describe the set of all binary numerals without leading zeros (except 0 itself). So, the language is the set (0, 1, 10, 11, 100, 101, 110, 111, ...).

Solution

$$0 + 1\ .\ (\ 0 + 1)^*$$

- Find a regular expression for each of the following languages over the alphabet (a,b).

Solution

a. $(aa + bb + ab + ba)^*$
b. $((a + b)\ .\ (a + b)\ .\ (a + b))^*$
c. $(a + ba + bba)^*(b\ + bb + \wedge\ )$
d . $(a\ .\ (aa + bb)^* b) +\ (b\ .\ (aa + bb)^* a)$

- Describe in English phrases the languages associated with the following regular expression:

Solution

a. Strings must include an **odd number of b’s**.

b. String’s length must be $\{\boldsymbol{3x + b : x \geq 0 \; and \; b} \in [\mathbf{0}, \mathbf{1}]\}$.

c. String in which the letters b , a is never doubled , This means that **no word contains the substring aa** .

- Construct a regular expression defining each of the following languages over the alphabet {a b}:

Solution

$$(b^{*}.\mathrm{a}b^{*}.\mathrm{a}b^{*}.\mathrm{a}b^{*})^{*}$$

- Describe (in English phrases) the languages associated with the following regular expressions:

Solution

a) Strings must include at **least a** and will never end with any number of b’s **excluding 4 b’s**.

b) Strings may be **empty** or , **start with a and end with a or bb.**

c) Strings may be **empty** or **start with a and have odd number of a’s and b’s**.

d) Strings may be **empty** or have **an odd number of a’s**.

e) Strings may be **empty** or **consist of a’s only or b’s only** or consist of a’s and b’s **but occurrences of contagious a’s and contagious b’s are not even** .

f) Strings may be **empty** or **have even length and end with a**

- Show that the following pairs of regular expressions define the same language over the alphabet {a,b}:

Solution

a) These regular expressions are equals and match the same strings that consist of ababab….aba , match with first regular expression as : (ab)(ab)(ab)….(ab)a , match with second regular expression as: a(ba)(ba)(b….a)(ba)

b) These regular expressions are equals and match the same strings that consist of any number of a's and b's in any order as : aaabbaa match first regular expression as : (aaa)(b)(b)(aa) and match second regular expression as : (a)(a)(a)(b)(b)(a)(a)

c) These regular expressions are equals and match the same strings that consist of any number of a's and b's in any order as : aaabbaa match first regular expression as : (aaa)(bb)(aa) and match second regular expression as : (a)(a)(a)(b)(b)(a)(a)

d) These regular expressions are equals and match the same strings that consist of any number of a's and followed by number of b's divisible by 3 then followed by any number of a's as : aaaabbbbbbaa match with first regular expression as : ((aaaa)bbb)(bbb)(aa) and match second regular expression as : (aaaa)(bbb)(bbb(aa))

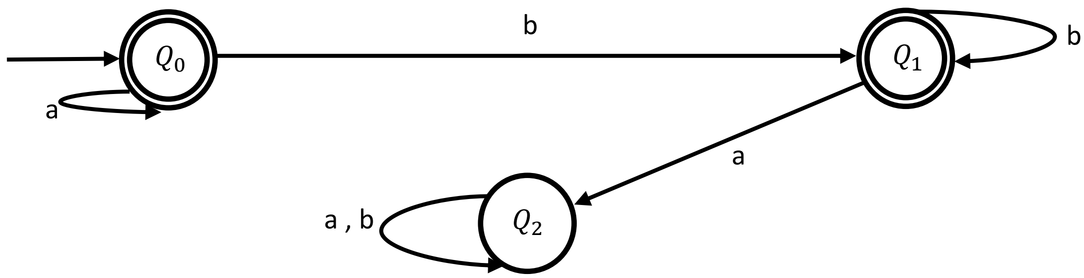
Finite Automata

DFA

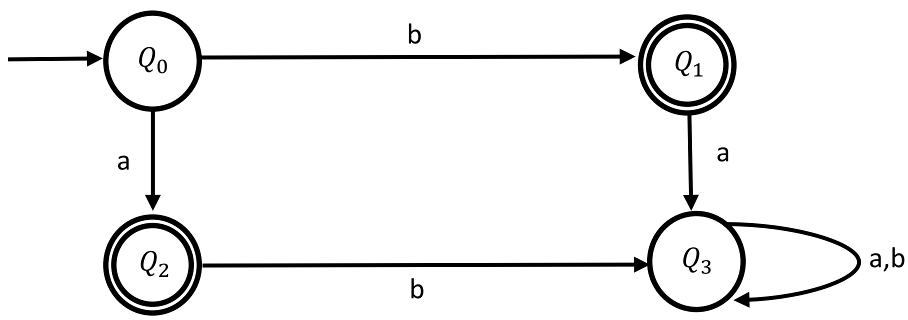
Transform each of the following regular expressions into a DFA.

Solution

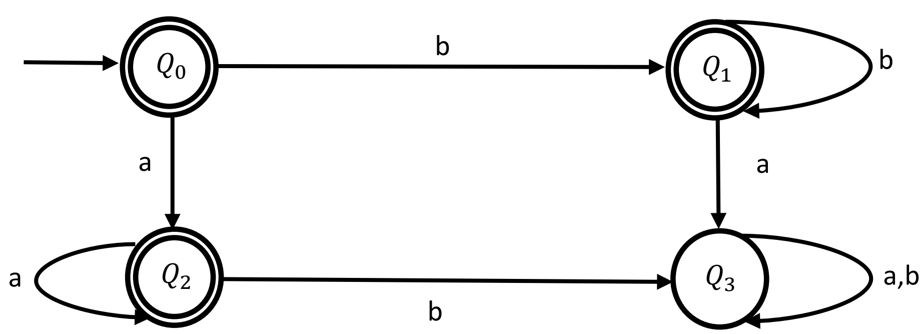
- $a * b *$



- $(a + b)$



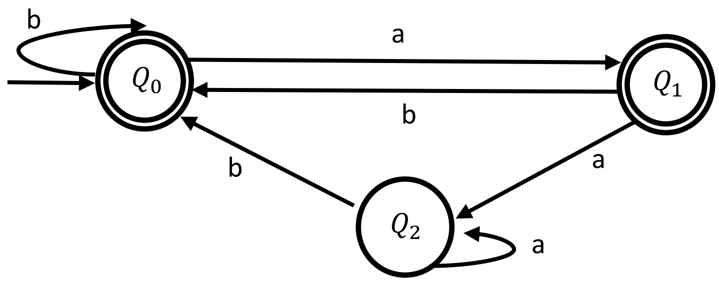
- $(a * + b *)$



Design a DFA that accepts all strings over {a, b}

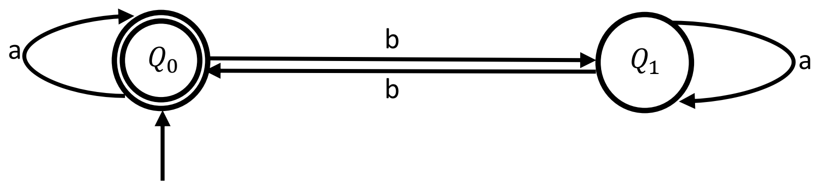
- All strings that do not end with aa.

  Regular expression : $\ni + a + b + (a + b) * (ab + bb + ba)$



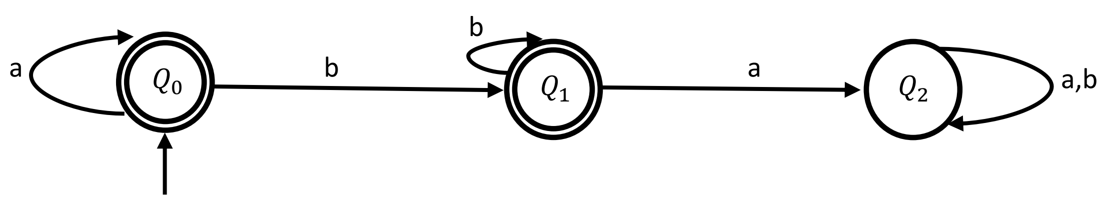
- All strings that contain an even number of b’s

  Regular expression : $a * (b\, a * b\, a *) *$



- All strings which do not contain the substring ba

  Regular expression : $a * b *$

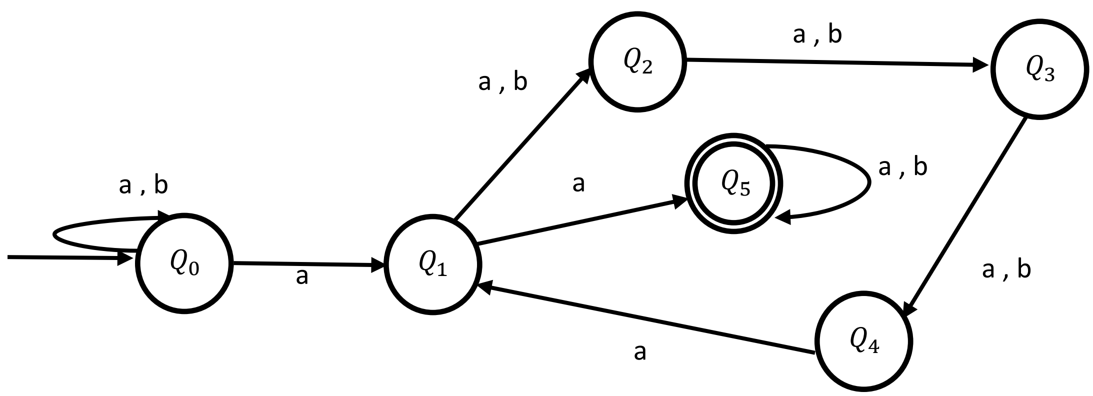


NFA

Draw NFA for each of the following languages over the alphabet {a,b}

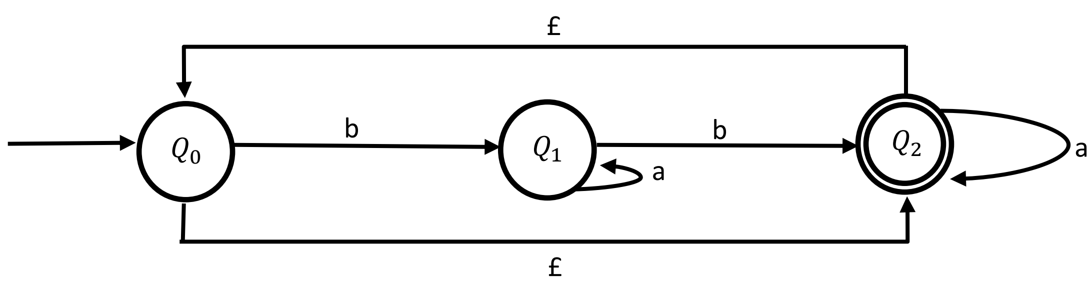
- All strings that contain two a's separated by a substring whose length is a multiple of 3.

  Regular expression : $(a+b)*a\big((a+b)(a+b)(a+b)\big)*a(a+b)*$



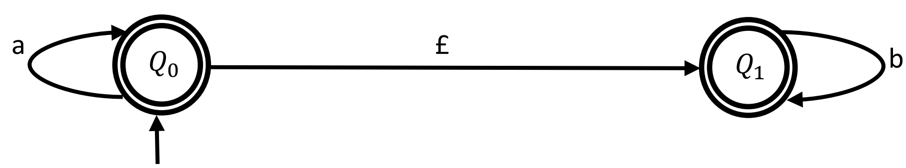
- All strings that contain an even number of b's.

  Regular expression : $a*(b\,a*\,b\,a*)*$



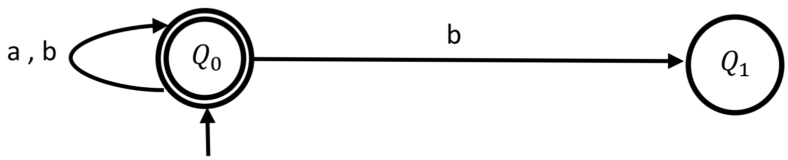
- All strings which do not contain the substring ba.

  Regular expression : $a*\,b*$



NFA to DFA

Q1

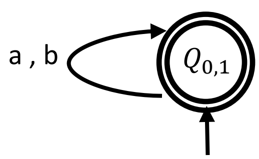


DFA

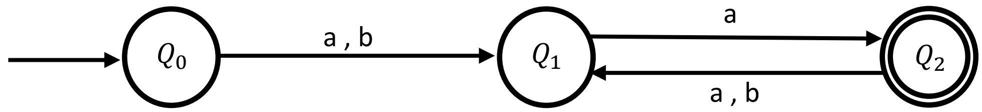
	a	b
Q_0	Q_0	Q_0, Q_1
Q_0, Q_1	Q_0	Q_0, Q_1
Q_1	—	—

NFA

	a	b
Q_0	Q_0	Q_0, Q_1
Q_1	—	—



Q2

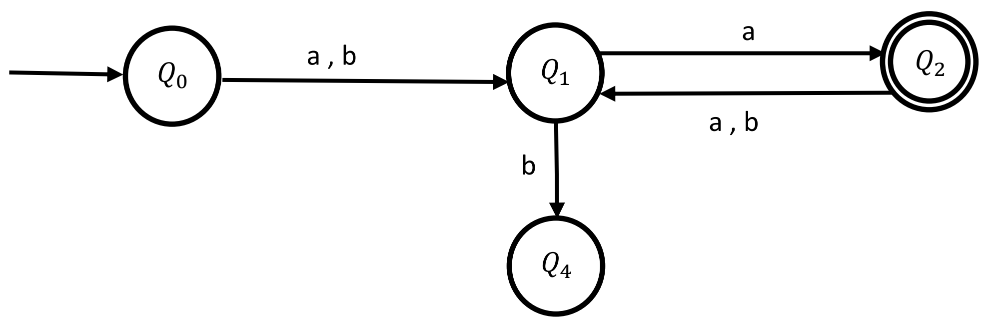


NFA

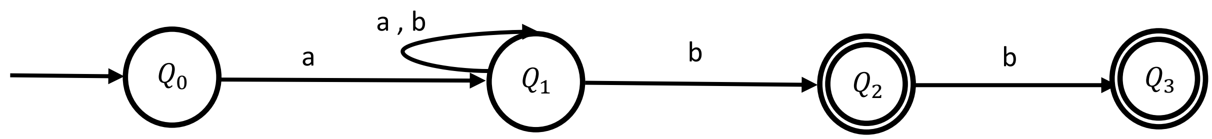
	a	b
Q_0	Q_1	Q_1
Q_1	Q_2	—
Q_2	Q_1	Q_1

DFA

	a	b
Q_0	Q_1	Q_1
Q_1	Q_2	Q_4
Q_2	Q_1	Q_1



Q3

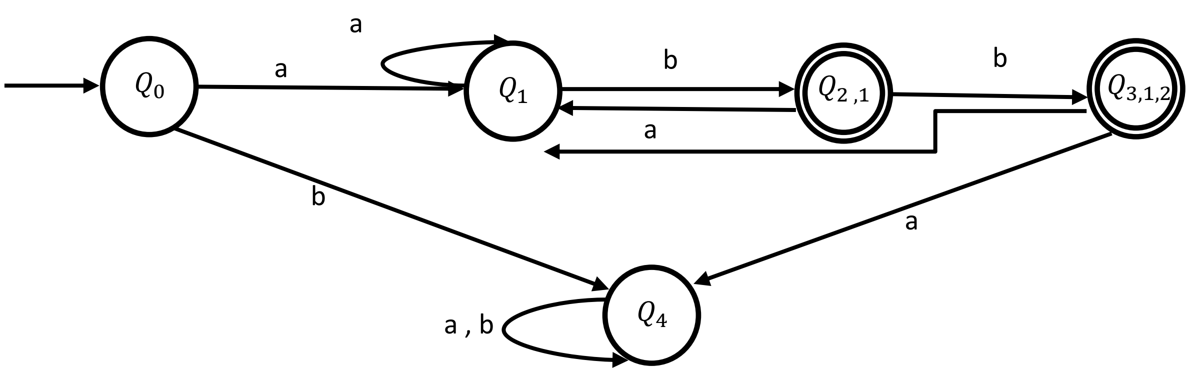


NFA

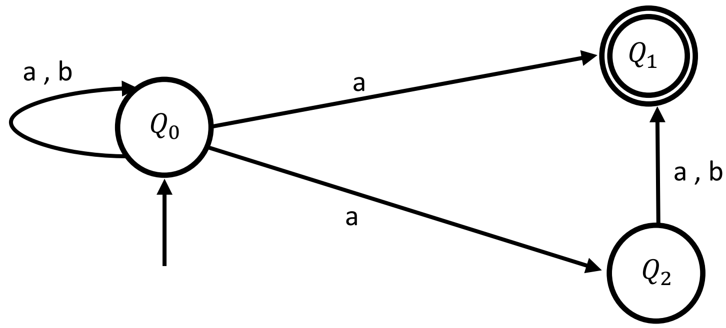
	a	b
Q_0	Q_1	$-$
Q_1	Q_1	Q_1Q_2
Q_2	$-$	Q_3
Q_3	$-$	$-$

DFA

	a	b
Q_0	Q_1	Q_4
Q_1	Q_1	Q_1Q_2
Q_1Q_2	Q_1	$Q_1Q_2Q_3$
$Q_1Q_2Q_3$	Q_1	$Q_1Q_2Q_3$



Q4



NFA

	a	b
Q_0	$Q_0Q_1Q_2$	Q_0
Q_1	$-$	$-$
Q_2	Q_1	Q_1

DFA

	a	b
Q_0	$Q_0Q_1Q_2$	Q_0
$Q_0Q_1Q_2$	$Q_0Q_1Q_2$	Q_0Q_1
Q_0Q_1	$Q_0Q_1Q_2$	Q_0