# Java$^{TM}$ Education & Technology Services

# Data Structures and Algorithms

# What about data structures?

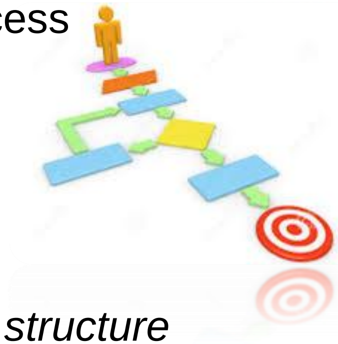
- **Structure :How data is organized ?**

  - Place data continuously
  - Place data here and there with “links”
  - Place data with “formula”

  - *Organization of a data structure may allow better algorithms to be applied*

- **Algorithms: How to access data for a result/task ?**

  A high level, language independent, description of a step-by-step process

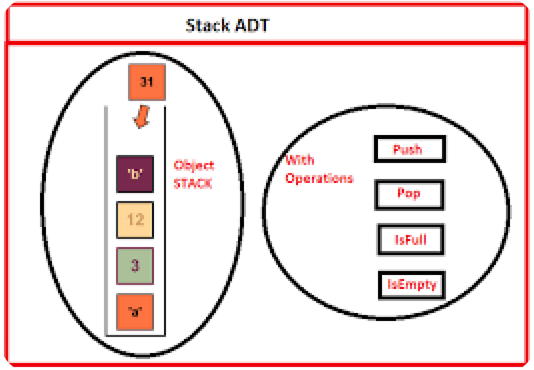


  - Scan data sequentially
  - Scan data according to the sequence of a structure
  - Scan data with “formula”

  - *Algorithms can be smarter without a special organization of a data structure*

# What about data structures?

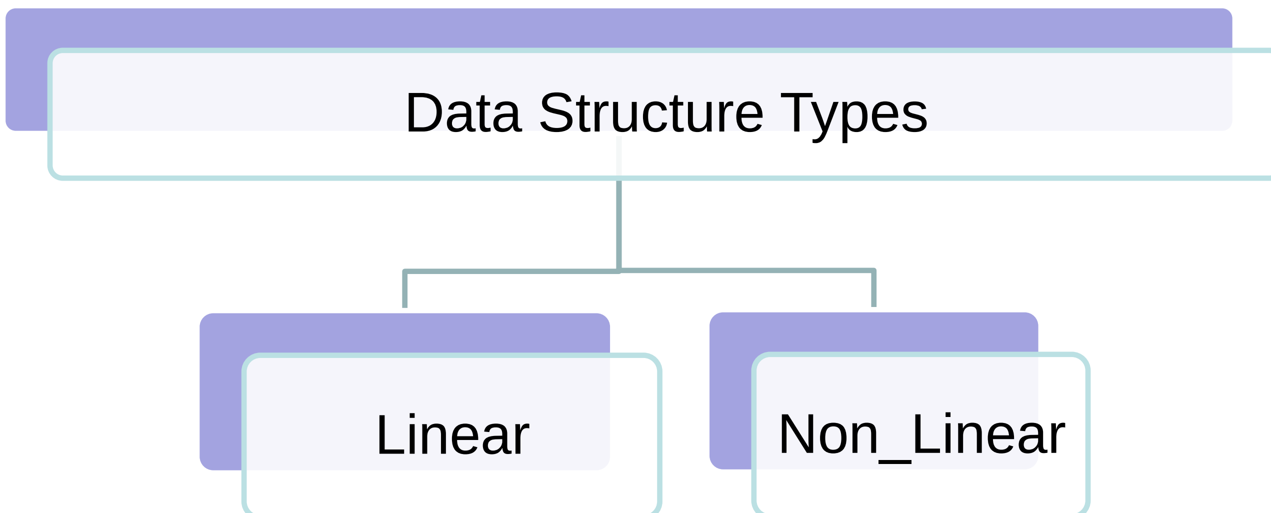
- **Abstract Data Type [ ADT]**
  - An abstract model of a data structure together with the operations (algorithms) processed on the data structure.
  - Useful building block




  - In C++ or Java, an ADT can be implemented as a class.

# What about data structures?

- **Data Structure**
  - A specific family of algorithms for implementing an ADT.




- Array
- Stack
- Queue
- Linked List

- Tree
- Graph
- Heaps

# What about data structures?

| Concepts | Mechanism |
|---|---|
| Abstract | Concrete |
| Pseudocode | Specific programming language |
| Algorithm | Program |
| Abstract Data Type (ADT) | Data structure |

# What about Algorithms ?

- **Algorithms:**
  - Is a finite set of instructions which , when followed, accomplishes a particular task.
  - There can be more than one algorithm to solve a given problem.
  - An algorithm can be implemented using different programming languages.
  - we should employ mathematical techniques that analyze algorithms independently of specific implementations, computers, or data.
  - There are two aspects of Algorithm performance:
    - Time
      - How to estimate the time required for an algorithm
      - How to reduce the time required
    - Space
      - Data structures take space
      - What kind of data structures can be used?
      - How does choice of data structure affect the runtime?

# What about Algorithms ?

- **Analysis of Algorithms:**
  - The Execution Time of Algorithms:
    - **Cost** is the amount of computer time required for a single operation in each line **[1].**
      **Times** is the amount of computer time required by each operation for all its repeats.
      **Total** is the amount of computer time required by each operation to execute.
    - It requires 1 unit of time for Arithmetic and Logical operations
    - It requires 1 unit of time for Assignment and Return value
    - It requires 1 unit of time for Read and Write operations

```
count = count + 1;
```

➔ take a certain amount of time, but it is constant say: 2 units

```
count = count + 1;
sum = sum + count;
```

Total cost = 4 units

Cost: 1 +1 Times: 1

Cost: 1+1 Time: 1

# What about Algorithms ?

- **The Execution Time of Algorithms :**
  - *Example: Simple Loop*

```
i = 1;
sum = 0;
while (i <= n) {
   i = i + 1;
   sum = sum + i;
   }
```

| Cost | Times |
|---|---|
| 1 | 1 |
| 1 | 1 |
| 1 | n+1 |
| 2 | n |
| 2 | n |

Total cost = 1 + 1+ (n+1) + 2n + 2n

– The time required for this algorithm is proportional to n

- **The Execution Time of Algorithms :**
  - *Example: Simple Loop*

| Code | Cost | Times |
|---|---|---|
| `int sum = 0, i;` | 1 | 1 |
| `for(i = 0; i < n; i++)` | 1 +1+1 | 1+(n+1)+n |
| `sum = sum + A[i];` | 2 | n |
| `return sum;` | 1 | 1 |

Total cost = 1 + (2+2n) + 2n + 1 = 4n +4

- The time required for this algorithm is proportional to n
- If the amount of time required by an algorithm is increased with the increase of input value then that time complexity is said to be Linear Time Complexity

# What about Algorithms ?

- **The Execution Time of Algorithms :**
  - *Example: Nested Loop*

| Code | Cost | Times |
|---|---|---|
| `i=1;` | 1 | 1 |
| `sum = 0;` | 1 | 1 |
| `while (i <= n) {` | 1 | n+1 |
| `j=1;` | 1 | n |
| `while (j <= n) {` | 1 | n*(n+1) |
| `sum = sum + i;` | 2 | n*(n) |
| `j = j + 1;` | 2 | n*(n) |
| `}` | | |
| `i = i +1;` | 2 | n |
| `}` | | |

Total cost = 1 + 1 + (n+1)*1 + n*1 + n*(n+1)*1+n*n*2+n*n*2+n*2

The time required for this algorithm is proportional to $n^2$

# What about Algorithms ?

- **The Execution Time of Algorithms :**
  - **General Rules for Estimation:**
    - **Loops:** The running time of a loop is at most the running time of the statements inside of that loop times the number of iterations.
    - **Nested Loops:** Running time of a nested loop containing a statement in the inner most loop is the running time of statement multiplied by the product of the sized of all loops.
    - **Consecutive Statements:** Just add the running times of those consecutive statements.
    - **If/Else:** Never more than the running time of the test plus the larger of running times of S1 and S2.

# What about Algorithms ?

- **The Execution Time of Algorithms :**
  - **Big O Notation:**
    - An algorithm's proportional time requirement is known as **growth rate.**
    - The function f(n) is called the algorithm's growth-rate function.
    - Since the capital O is used in the notation, this notation is called the Big O notation.
    - If Algorithm A requires time proportional to $n^2$, it is $O(n^2)$.
    - If Algorithm A requires time proportional to n, it is O(n).

# What about Algorithms ?

- What to Analyze:
  - An algorithm can require different times to solve different problems of the same size.
  - **Worst-Case Analysis :**The maximum amount of time that an algorithm require to solve a problem of size n.
    - This gives an upper bound for the time complexity of an algorithm.
    - Normally, we try to find worst-case behavior of an algorithm.
  - **Best-Case Analysis:** The minimum amount of time that an algorithm require to solve a problem of size n.
    - The best case behavior of an algorithm is NOT so useful.
  - **Average-Case Analysis:** The average amount of time that an algorithm require to solve a problem of size n.
    - Sometimes, it is difficult to find the average-case behavior of an algorithm.
    - We have to look at all possible data organizations of a given size n, and their distribution probabilities of these organizations.

  - Worst-case analysis is more common than average-case analysis.

# Sorting Algorithms

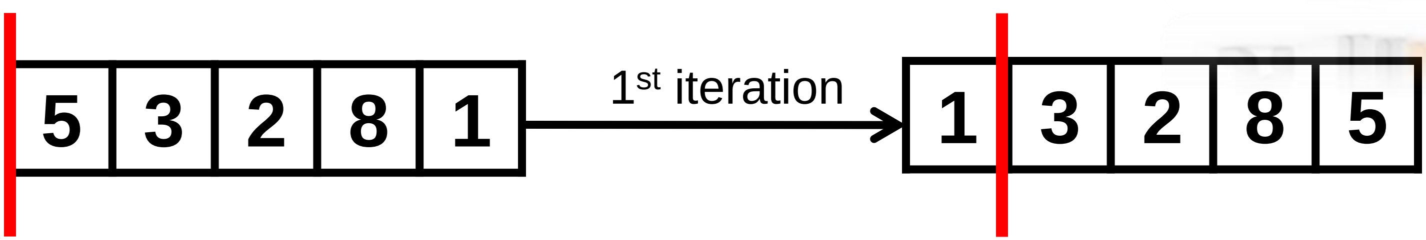
# Sorting Algorithms

- **Sorting:**
  - is the process of arranging a set of similar information into an increasing or decreasing order.
  - Sorting also has indirect uses. An initial sort of the data can significantly enhance the performance of an algorithm.
  - ***Sorting Algorithms:***
    - Selection Sort
    - Insertion Sort
    - Bubble Sort
    - Merge Sort
    - Quick Sort
  - The first three are the foundations for faster and more efficient algorithms.

| 6 | 8 | 3 | 5 | 1 | $\xrightarrow{\textit{Sorting}}$ | 1 | 3 | 5 | 6 | 8 |
|---|---|---|---|---|---|---|---|---|---|---|

# Sorting Algorithms

- Selection Sorting:
  - The list is divided into two sub lists, sorted and unsorted.
  - We find the smallest element from the unsorted sub list and swap it with the element at the beginning of the unsorted data.
  - After each selection and swapping, the wall between the two sub lists move one element ahead, increasing the number of sorted elements and decreasing the number of unsorted ones.
  - A list of n elements requires n-1 passes to completely rearrange the data.



# Sorting Algorithms

- Selection Sorting:
  - Algorithm Procedure
    1. Find the minimum value in the list.
    2. Swap it with the value in the first position.
    3. Repeat the steps above for the remainder of the list (starting at the second position and advancing each time).

**Find & Swap**

- Selection Sorting:

**Sorted** **Unsorted**

| | | | | | |
|---|---|---|---|---|---|
| 5 | 3 | 2 | 8 | 1 | 4 |

Find
Swap √

# Sorting Algorithms

- Selection Sorting:

| Sorted | | | | Unsorted | | |
|---|---|---|---|---|---|---|
| 5 | 3 | 2 | 8 | 1 | 4 | Find Swap √ |
| 1 | 3 | 2 | 8 | 5 | 4 | Find Swap √ |
| 1 | 2 | 3 | 8 | 5 | 4 | Find Swap |
| 1 | 2 | 3 | 8 | 5 | 4 | Find Swap √ |
| 1 | 2 | 3 | 4 | 5 | 8 | Find Swap |
| 1 | 2 | 3 | 4 | 5 | 8 | Sorted |

- Selection Sorting:
  - C++ code implementation

```
void  main ()
{
      int i, n;
      int  a[6]={5,3,2,8,1,4};
     selection_sort (a, 6);
    for (i = 0; i < 6; i++)  cout << a[i] << endl;
}
void swap (int &x, int &y)
{
            int temp;
            temp = x;
            x = y;
            y = temp;
}
```

- Selection Sorting:
  - C++ code implementation

```
void selection_sort (int *a, int n)   // *a: the array to sort, n: the array length
 {
    int i, j, min;                              // min : index of the min value
    for (i = 0; i < (n-1); i++)
    {
       min = i;          // assume that the first element is the min at the beginning
       for (j = (i+1); j < n; j++)
       {                                  // find the index of the min element
          if(a[j] < a[min])   min = j;
       }
        if (i != min)      swap(a[i],a[min]);     // swap if needed
    }
 }
```

- Selection Sorting:
  - Performance Analysis

    Selection sort is not difficult to analyze compared to other sorting algorithms since none of the loops depend on the data in the array.

    Selecting the lowest element requires scanning all n elements (this takes $n - 1$ comparisons) and then swapping it into the first position. Finding the next lowest element requires scanning the remaining $n - 1$ elements and so on $n(n - 1) / 2$

    1. Best Case $O(n^2)$
    2. Worst Case $O(n^2)$
    3. Average Case $O(n^2)$

    *Where, n is the number of items being sorted*

- Bubble Sorting:
  - The *Bubble Sort Algorithm* belongs to the *Exchange* Methods of sorting.
  - Bubble Sort is the *Simplest* sorting routine.
  - The general concept behind the Bubble sort is the *repealed comparisons* and, if necessary, exchanges of adjacent elements. Its name comes from the method's similarity to *bubbles in a tank of water*, where each bubble seeks its own level.

# Sorting Algorithms

- **Bubble Sorting:**
  - Algorithm Procedure
    1. Compare the first two elements of the array and swap them if they are out-of-order.
    2. Continue doing to swap the array for each pair of elements, until you reach the last entry. the array for each two adjacent elements until you reach the last entry.
    3. At this point the last entry is the largest element in the array.
    4. Continue this procedure for each next largest element until the array is fully sorted.
  - Step-by-Step Example:

| | | | | |
|---|---|---|---|---|
| 6 | 8 | 3 | 5 | 1 |

# Sorting Algorithms- Bubble

**Performing 1st Iteration:**

| | | | | |
|---|---|---|---|---|
| 6 | 8 | 3 | 5 | 1 |

Comparing 1st & 2nd numbers
No Swapping, since 8 > 6

| | | | | |
|---|---|---|---|---|
| 6 | 8 | 3 | 5 | 1 |

| | | | | |
|---|---|---|---|---|
| 6 | 8 | 3 | 5 | 1 |

Comparing 2nd & 3rd numbers
Swapping, since 3 < 8

| | | | | |
|---|---|---|---|---|
| 6 | 8 | 3 | 5 | 1 |

| | | | | |
|---|---|---|---|---|
| 6 | 3 | 8 | 5 | 1 |

Comparing 3rd & 4th numbers
Swapping, since 5 < 8

| | | | | |
|---|---|---|---|---|
| 6 | 3 | 8 | 5 | 1 |

| | | | | |
|---|---|---|---|---|
| 6 | 3 | 5 | 8 | 1 |

Comparing 4th & 5th numbers
Swapping, since 1 < 8

| | | | | |
|---|---|---|---|---|
| 6 | 3 | 5 | 8 | 1 |

| | | | | |
|---|---|---|---|---|
| 6 | 3 | 5 | 1 | 8 |

Now last position carries the right element.

Check the list if one swap at least was done run the 2nd iteration.

# Sorting Algorithms- Bubble

**Performing 2nd Iteration:**

| | | | | |
|---|---|---|---|---|
| 6 | 3 | 5 | 1 | 8 |

Comparing 1st & 2nd numbers
Swapping, since 3<6

| | | | | |
|---|---|---|---|---|
| 6 | 3 | 5 | 1 | 8 |
| 3 | 6 | 5 | 1 | 8 |

Comparing 2nd & 3rd numbers
Swapping, since 5 < 6

| | | | | |
|---|---|---|---|---|
| 3 | 6 | 5 | 1 | 8 |
| 3 | 5 | 6 | 1 | 8 |

Comparing 3rd & 4th numbers
Swapping, since 1 < 5

| | | | | |
|---|---|---|---|---|
| 3 | 5 | 6 | 1 | 8 |
| 3 | 5 | 1 | 6 | 8 |

Now last-1 position carries the right element.

Check the list if one swap at least was done run the 3rd iteration.

# Sorting Algorithms- Bubble

**Performing 3rd Iteration:**

| | | | | |
|---|---|---|---|---|
| 3 | 5 | 1 | 6 | 8 |

Comparing 1st & 2nd numbers
No Swapping, since 5>3

| | | | | |
|---|---|---|---|---|
| 3 | 5 | 1 | 6 | 8 |

| | | | | |
|---|---|---|---|---|
| 3 | 5 | 1 | 6 | 8 |

Comparing 2nd & 3rd numbers
Swapping, since 1 < 5

| | | | | |
|---|---|---|---|---|
| 3 | 5 | 1 | 6 | 8 |

| | | | | |
|---|---|---|---|---|
| 3 | 1 | 5 | 6 | 8 |

Now last-2 position carries the right element.

Check the list if one swap at least was done run the 4th iteration.

# Sorting Algorithms- Bubble

**Performing 4th Iteration:**

| | | | | |
|---|---|---|---|---|
| 3 | 1 | 5 | 6 | 8 |

Comparing 1st & 2nd numbers
Swapping, since 1<3

| | | | | |
|---|---|---|---|---|
| 3 | 1 | 5 | 6 | 8 |

| | | | | |
|---|---|---|---|---|
| 1 | 3 | 5 | 6 | 8 |

Now last-3 position carries the right element.

Check the list if one swap at least was done run the 5th iteration.

# Sorting Algorithms- Bubble

**Performing 5th Iteration:**

| | | | | |
|---|---|---|---|---|
| 1 | 3 | 5 | 6 | 8 |

Now last-4 (array 1st )position carries the right element.

There is no swap done. So, the list is sorted

- ## Bubble Sorting:
  - C++ code implementation

```
void bubble_sort (int *a, int n)   // stopping condition is no more swapping
 {
    int swapped;
    int i, j;
    for (i = 1; i < n; i++) {
        swapped = 0;    // this flag is to check if the array is already sorted
        for(j = 0; j < n - i; j++) {
          if(a[j] > a[j+1]) {
              swap(a[j],a[j+1]);
              swapped = 1;
            }
        }
        if(!swapped)  break; // if it is sorted then stop
    }
 }  // use the early main for Test
```

- Bubble Sorting:
  - Performance Analysis

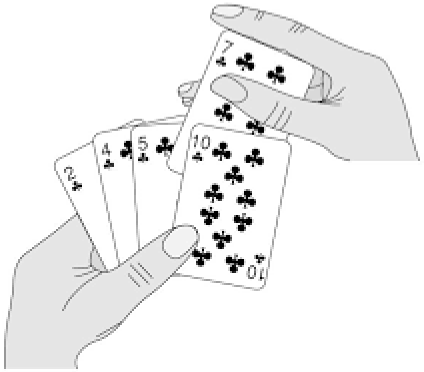
    No. of Comparisons = (n – 1) + (n – 2) + …+1

$$\tfrac{1}{2}\left(n^2 - n\right)$$

1. Worst Case O($n^2$)
2. Average Case O($n^2$)

*Where, n is the number of items being sorted*

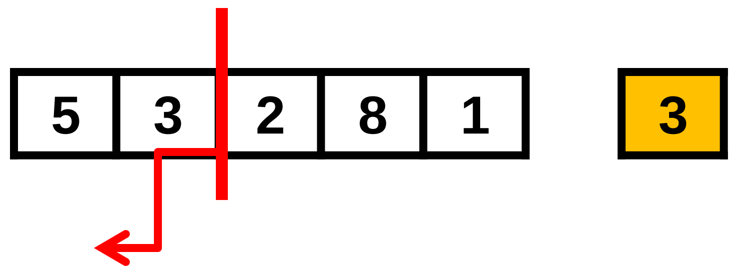
- Insertion Sorting:
  - The *Insertion Sort* Algorithm is the simplest sorting algorithm that is appropriate for *small inputs*.
    - Most common sorting technique used by card players.
  - Insertion Sort takes advantage of *presorting*.
  - It requires fewer comparisons than bubble sort.
  - Insert the number in *its location* and shift all of the next elements.



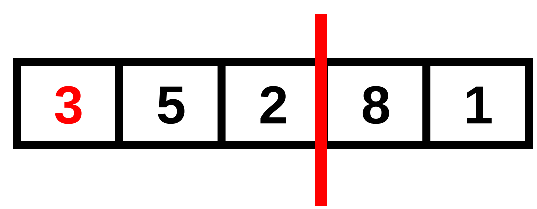
- Insertion Sorting:
  - Algorithm Procedure
    1. Insertion Sort is somewhat similar to the Bubble Sort in that we compare adjacent elements and swap them if they are out-of order.
    2. Unlike the Bubble Sort however, we do not require that we find the next largest or smallest element.
    3. Instead, we take the next element and insert it into the sorted list that we maintain at the beginning of the array.

# Sorting Algorithms

- Insertion Sorting:
  - **<u>1st Iteration</u>**



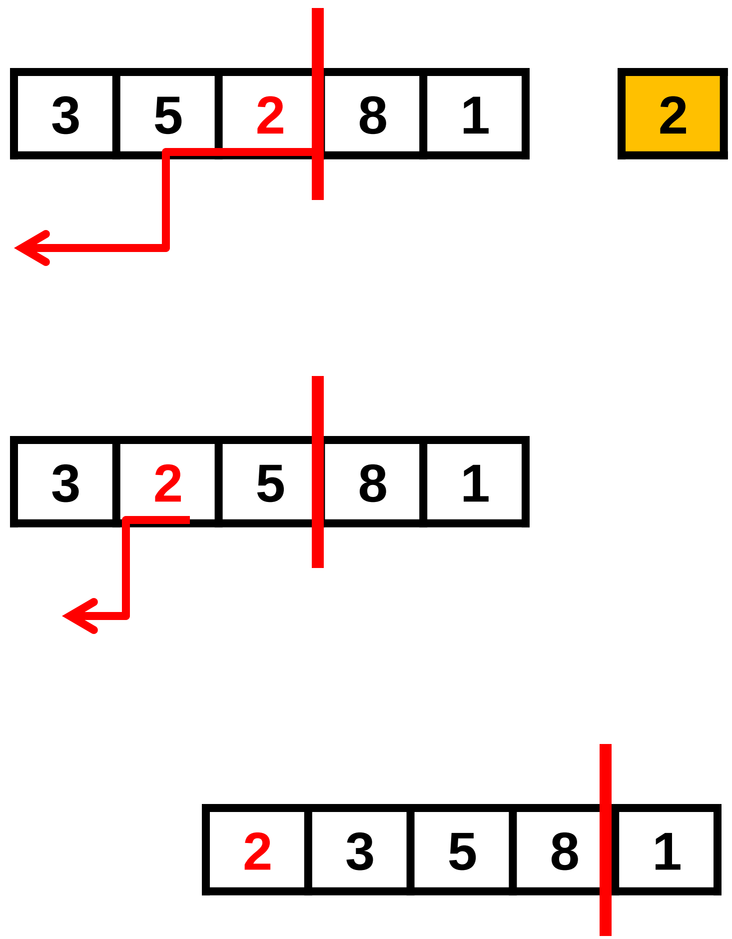

1. Compare with 5
2. Swap
3. Go to next iteration



# Sorting Algorithms

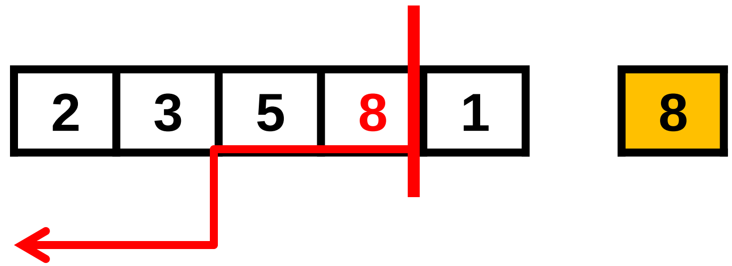
- Insertion Sorting:
  - **2nd Iteration**

1. Compare with 5
2. Swap
3. Compare with 3
4. Swap
5. Go to next iteration

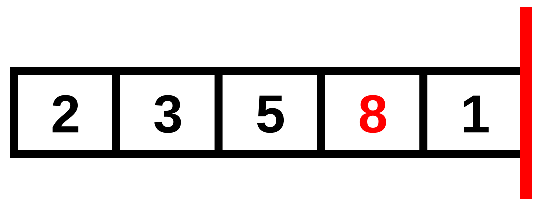


# Sorting Algorithms

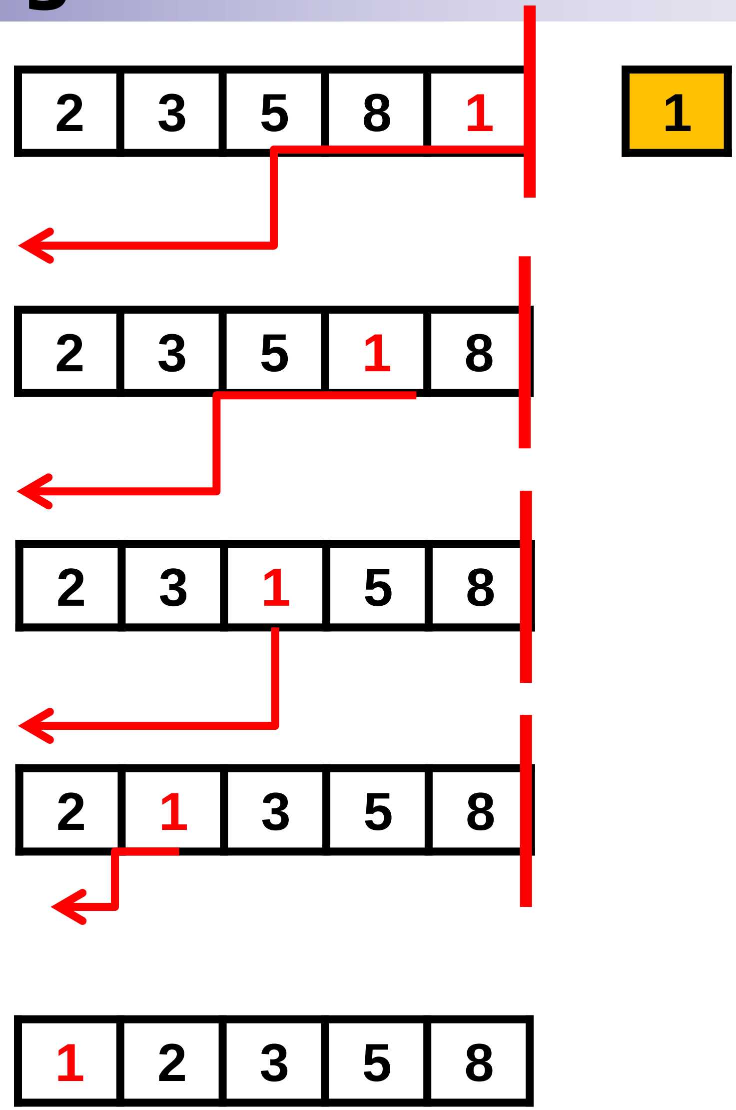
- Insertion Sorting:
  - **3rd Iteration**




1. Compare with 5
2. No Swap
3. Go to next iteration



- Insertion Sorting:
  - **4th Iteration**




1. Compare with 8
2. Swap
3. Compare with 5
4. Swap
5. Compare with 3
6. Swap
7. Compare with 2
8. Swap

- Insertion Sorting:
  - C++ code implementation

```
void insertion_sort (int *a, int n)
{
    int i, j, value;
    for(i = 1; i < n; i++) {
        value = a[i];
         j= i – 1;
        while ( (j >= 0 ) && (a[j] > value)) {
              a[j + 1] = a[j];
              j = j - 1;
        }
       a[j + 1] = value;
    }
 }  // use the early main for rest
```

# Sorting Algorithms

- **Insertion Sorting:**
  - Performance Analysis
    - Running time depends on not only the size of the array but also the contents of the array.
    - ***Best-case:*** **O(n)**
      - Array is already sorted in ascending order.
      - Inner loop will not be executed.
      - The number of key comparisons: (n-1) **O(n)**
    - ***Worst-case:*** **$O(n^2)$**
      - Array is in reverse order:
    - ***Average-case***: **$O(n^2)$**
      - We have to look at all possible initial data organizations.
    - So, Insertion Sort is **$O(n^2)$**

# Lab Exercise

# Lab Exercise

- **1st Assignment :**

1. Implement Selection Sort on array of integers.
2. Implement Bubble sort on array of integers.
3. Implement Insertion Sort on array of integers.

// Build a class for sorting algorithms.

1. Bonus: Implement any of the sort algorithms on array of Employees.
2. Search: Merge Sort & Quick Sort

# Searching Algorithms

- Searching Algorithms:
  - Searching is to find the *location* of *specific value* in a list of data elements.
  - Searching Methods can be divided into two categories:
    - Searching methods for *unsorted* as well as sorted lists.
      - e.g., Sequential Search.
    - Searching methods for *sorted* lists.
      - e.g., Binary Search.
    - Direct access by key value (hashing)

## • Sequential Search Algorithm :

- is also know as *Linear Search*.
- can be used to search both *sorted* and *unordered* lists.
- operates by checking every element of a list one at a time in sequence until a match is found.
- **Algorithm Procedure:**
  1. For each item in the list, check if the item we are looking for matches the item in the list as follows:
     1. If it matches, return the location where the item is found(i.e., the item index) and end the search.
     2. Otherwise, continue searching until a match is found or the end of the list is reached.
  2. If the end of the list is reached without finding a match, then the Item does not exist in the list.

# Searching Algorithms

- Sequential Search Algorithm :

**Initial state :** find **9**

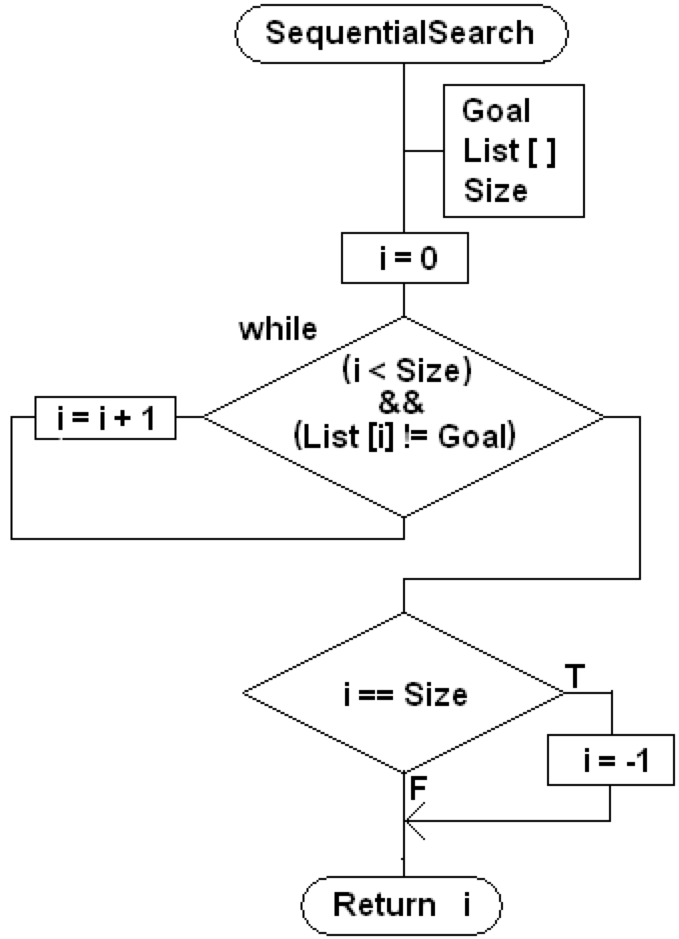
| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| 6 | 8 | 3 | 5 | 1 | 4 | 9 | 2 | 7 |

Unsorted list

---

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| Is 1st element 6 == 9 ? False | **6** | 8 | 3 | 5 | 1 | 4 | 9 | 2 | 7 |
| Is 2nd element 8 == 9 ? False | 6 | **8** | 3 | 5 | 1 | 4 | 9 | 2 | 7 |
| Is 3rd element 3 == 9 ? False | 6 | 8 | **3** | 5 | 1 | 4 | 9 | 2 | 7 |
| Is 4th element 5 == 9 ? False | 6 | 8 | 3 | **5** | 1 | 4 | 9 | 2 | 7 |
| Is 5th element 1 == 9 ? False | 6 | 8 | 3 | 5 | **1** | 4 | 9 | 2 | 7 |
| Is 6th element 4 == 9 ? False | 6 | 8 | 3 | 5 | 1 | **4** | 9 | 2 | 7 |
| Is 7th element 9 == 9 ? True [stop here] | 6 | 8 | 3 | 5 | 1 | 4 | **9** | 2 | 7 |

- Sequential Search Algorithm :



- Sequential Search Algorithm :
  - C++ code implementation

```
void main ()
{
   int i,n;
   int i, n;
   int  a[9]={6,8,3,5,1,4,9,2,7};
   cout << ”Found at : ”<< sequential_search (a, 9,9);
 }
```

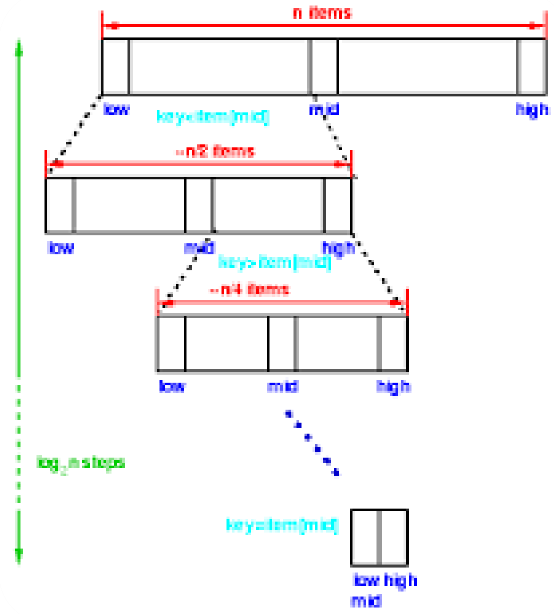
- Sequential Search Algorithm :
  - C++ code implementation

```
int sequential_search (int *a, int n, int num)
{
    int i = 0, found = 0;
    while ((!found) && (i < n)) {
       if ( num == a [i])
            found = 1;
       else
            i++;
    }
    if  ( found)
          return i;
    else
          return -1;

}
```

- Sequential Search Algorithm :
  - Performance Analysis
    - Best Case = 1 (Only, one element will be tested).
    - Worst Case = n (All n elements will be tested).
    - Time Complexity: **O(n).**

- Binary Search Algorithm :
  - *Binary Search* is fast searching algorithm, but it can be used only to search *sorted* lists.
  - The *Binary Search* method uses the *"divide-and-conquer"* approach.



- Binary Search Algorithm :
  - **Algorithm Procedure:**
    1. we need three indexes:
       1. **high**: index of the last element in the array.
       2. **low**: index of the first element in the array.
       3. **mid**: index of the middle element ( high + low )/2
    2. First test the middle element:
       1. If the element equals the key , stop and return mid ( index of middle).
       2. If the element is lager than the key, then test the middle element of the first half;
       3. Otherwise, test the middle element of the second half.

    3.Repeat this process until either a match is found, or there are no more elements to test.

- Binary Search Algorithm :

**Initial state :** find **4**

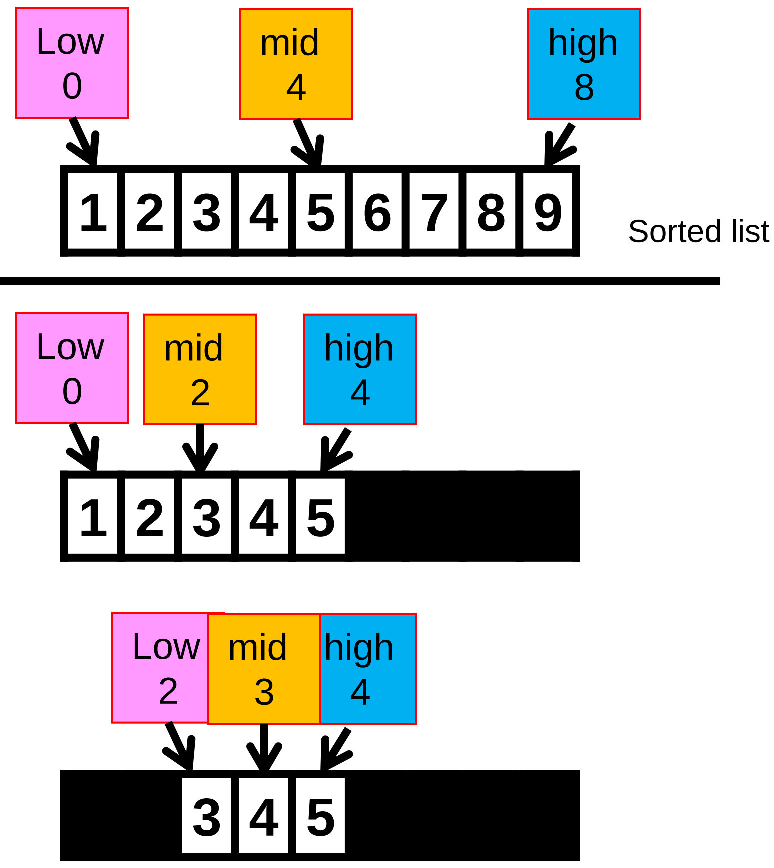
mid value 5 > 4

Consider only right half

mid value 3 < 4

Consider only left half

mid value 4 == 4 found stop



- Binary Search Algorithm :
  - C++ code implementation

```
void main ()
{
   int i,n;
   int i, n;
   int  a[9]={1,2,3,4,5,6,7,8,9};
   cout << ”Found at : ”<< binary_search (a, 9,4);
 }
```

- Binary Search Algorithm :
  - C++ code implementation

```
void binary_search (int *a, int n, int num)
{
int found = 0;
int high = n ;
int low = 0;
int mid;
while ((!found) && (low < = high )) {
      mid = (hi + lo) / 2;
      if (a [mid] > num)
            high = mid – 1; // go right
     else if (a [mid] < num)
            low = mid + 1; // go left
          else
            found = 1;
}
if  ( found) return mid;
else return -1;
}
```

# Lab Exercise

- ## 2nd Assignment :

1. Implement sequential search on array of integers.
2. Implement binary search on array of strings.

// Build a class for sorting algorithms.

1. Bonus: Implement any of the search algorithms on array of Employees.
2. Search: Hashing