Applied Soft Computing 109 (2021) 107529

Contents lists available at ScienceDirect

Applied Soft Computing

journal homepage: www.elsevier.com/locate/asoc

Design and modeling of adaptive IIR filtering systems using a weighted sum - variable length particle swarm optimization

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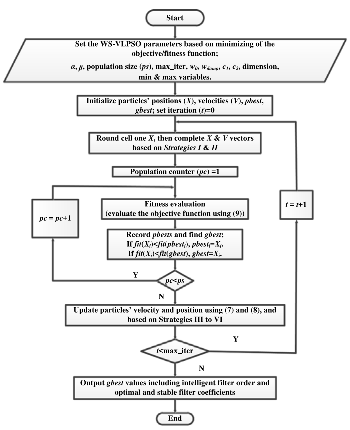
GRAPHICAL ABSTRACT



ARTICLE INFO

Article history:
Received 31 March 2019
Received in revised form 2 March 2021
Accepted 18 May 2021
Available online 27 May 2021

Keywords:
Variable length particle swarm optimization
Weighted sum fitness function
Adaptive IIR modeling
Variable length optimization problems
Optimum modeling indicator

ABSTRACT

The use of optimization algorithms for designing Infinite Impulse Response (IIR) filters has been considered in many studies. The concern in this area is the multimodal error surface of such filters and their fitting with filter coefficients. The order of the modeled system has a direct effect on the number of coefficients, complexities of the error surface, and the filter's stability. This paper proposes an efficient approach based on a variable length particle swarm optimization algorithm with a weighted sum fitness function (WS-VLPSO). The proposed WS-VLPSO is utilized as an effective adaptive algorithm for designing optimal IIR filters. The approach is based on the inclusion of the order as a discrete variable in the particle vector, which is done with the goal of intelligent minimizing of order and thereby reducing the design complexity of IIR filters. To ensure the optimality of the systems, the objective function is considered as a weighted sum. Also, a new criterion called Optimum Modeling Indicator (OMI) is introduced, a measure to determine the percentage reduction of order and the success rate of the proposed approach. The proposed algorithm is also applied for solving the sensor coverage problem as another real-world variable length engineering optimization application. Evaluation of simulation results, with Monte-Carlo simulation, indicates the acceptable improvement of identified structures and the significant performance of the proposed approaches. Note that the source codes of the paper will be publicly available at https://github.com/ali-ece.

1. Introduction

The use of digital filters in the field of digital signal processing (DSP) and other related fields, such as image and video processing and digital communications, has been widespread and of

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https://doi.org/10.1016/j.asoc.2021.107529
1568-4946/

great interest. On the other hand, the limitations and problems of designing such filters have led researchers to move towards utilizing cost-effective solutions with minimal time and computational cost instead of using traditional design techniques. Typically, digital filtering is divided by the length of the impulse response into two categories of digital filters with finite impulse response (FIR) and infinite impulse response (IIR). Since the use of IIR filtering systems has a smaller group delay, a lower computational cost (less order), and a much higher and better accuracy in meeting the performance characteristics than the equivalent FIR system, the use of them is more common [1–3]. In contrast, their error surface is usually nonlinear and multimodal (the solution space is multimodal), and it has a direct relationship to the filter coefficients [2,4,5]. However, gradient-based design as conventional methods is easily trapped in local optimal minimums in the solution space, and the effective optimization of the objective/cost/fitness function has encountered a problem [5–8]. Hence, to overcome these limitations, researchers have taken advantage of heuristic and evolutionary optimization methods as powerful, effective, and practical solutions in this regard [2–9]. These methods are intelligent optimization algorithms inspired by physical, biological, and natural processes that have attracted much attention in recent years. Some popular and common examples include genetic algorithms (GA) [10], particle swarm optimization (PSO) [11], and gravitational search algorithm (GSA) [12].

Optimal design and modeling of the IIR systems have been among the most common topics in recent research [6,13–17]. Many studies have proposed different methods and structures based on intelligent optimization techniques for adaptive IIR system identification. Two points are especially important in previous research: their novelty, which reveals the importance and needs to pay attention to the system identification area based on the IIR models and the critical challenges facing it. Another is the lack of attention to filter order as an available, effective, and determinant factor to reduce the complexity of designing such filtering systems (the design error surface). Because according to the theory of designing IIR digital filters, it is expected that by involving the filter order along with the parameters of the filter coefficients in the optimal design process, the modeling of the restricted mode (only with a reduced order of the unit) becomes dynamic, and at the same time optimal.

In the current research, for the first time, to overcome the challenges of designing digital IIR filters, such as their multidimensional error surface, complexity, and stability, a weighted sum version of the particle swarm optimization algorithm with variable length particle vector (WS-VLPSO) is presented. It is used as a robust adaptive algorithm in the system identification configuration for optimal modeling of such filters. The proposed approach is based on the simultaneity and contraction of optimal filter order modeling along with filter coefficients in the design. So that, the procedure is implemented with the inclusion of the filter order as a discrete (rounded as a positive integer) variable in the particle vector to minimize and optimize it, and thus reducing the complexity of the design and implementation of the filter. The proposed VLPSO structure is such that each search agent (particle) in the algorithm can select the discrete and integer values for the filter order, in addition to estimating continuous values for the filter coefficients. Therefore, the vector dimensions of each search agent will vary, and these conditions must be intelligently and appropriately considered by the user in the structure of the algorithm. In this regard, versions of GA and PSO algorithm with variable length vector proposed in applications, and each research, different strategies have been considered in the structure of each algorithm [18–21].

In this work, to ensure the optimality of the modeled IIR systems, the final objective function has been considered in the form of a weighted sum, and stability is also guaranteed for it. Also, a new criterion called optimum modeling indicator (OMI) is introduced, a measure to determine the percentage reduction of filter order and the success rate of the proposed approach in optimal modeling of the obtained IIR models. The OMI criterion will show how much the collection of adaptive systems derived from a limited number of trials of the modeling process has improved, in terms of the number of estimated orders from the set of possible orders in the problem space. As another real-world variable length engineering optimization application, we also apply the proposed VLPSO algorithm to solve and optimize the sensor coverage problem. To analyze the reliability and robustness features of the proposed solutions, the Monte-Carlo (MC) analysis is presented. The MC simulation method employs random number generators for the solution of practical problems where a closed-form or analytical solution is not possible. This feature makes it an excellent choice for the study of the system identification problems in adaptive IIR filtering.

To summarize, the main contributions of this paper can be listed as:

- A new approach based on synchronizing the modeling of dynamic order adaptive IIR filtering systems and estimating the optimal coefficients of its corresponding filter;

- A novel and dynamic version of variable length PSO algorithm as an effective adaptive algorithm in the IIR systems modeling process, as well as for solving coverage problem in wireless sensor networks;

- An intelligent weighted sum objective function, to satisfy all qualitative indicators such as establishing an acceptable trade-off for achieving optimal responses, stability, and minimum filter order;

- A new qualitative criterion called OMI to verify the intelligent minimization of filter order.

Five benchmark challenging IIR systems with different orders (at least 2 to 6) for a comprehensive assessment of the proposed approach have been simulated. To demonstrate the performance of the proposed method, a comprehensive quantitative and qualitative comparison, and accurate analysis of the results and indexes of the problem have been reported simultaneously. They include the estimated coefficients of all cases, filter orders, run-times, fitness convergence curves, stability conditions, bode diagrams, step and impulse responses, statistical indicators.

The remainder of the paper is that in the second section, the mathematical formulation of the adaptive IIR filtering system design under the system identification configuration is presented. The proposed approach and considerations regarding the implementation of the WS-VLPSO algorithm are described in detail in Section 3. The fourth section contains the results and analysis of simulations. The considerations of the sensor coverage problem, along with its outputs are also reported in this section. Finally, in the last section, the conclusion is given, and some information has been addressed about future works.

2. Design and modeling of IIR digital filters under system identification configuration

System identification techniques form a versatile tool for many problems in science and engineering [22]. In the proposed IIR modeling configuration, an adaptive algorithm attempts to set up the adaptive model parameters (here filter order and coefficients of the adaptive IIR system) in a repetitive manner (see Fig. 1).

A recursive IIR system is described by the z transform of the impulse response of the system, which can be displayed in the form of the following transfer function:

$$H(z) = \frac{Y(z)}{X(z)} = \frac{b_0 + b_1 z^{-1} + \cdots + b_P z^{-P}}{1 + a_1 z^{-1} + \cdots + a_Q z^{-Q}} \tag{1}$$

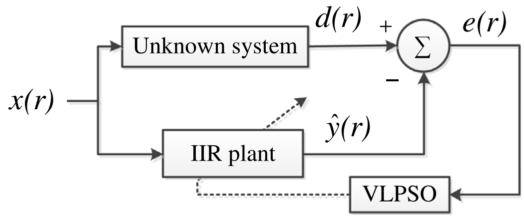

Fig. 1. IIR filtering system modeling using the VLPSO adaptive optimization algorithm.

In (1), $Y(z)$ and $X(z)$ are the system output and system input in the z domain, respectively. Also, P and Q are the number of the numerator and denominator coefficients, respectively, and Q ($\geq P$) represents the order of the filter. b_i and a_j are filter coefficients. The differential equation of the above relation is written in the following form:

$$y\,(r) = \sum_{j=1}^{Q} a_j y(r-j) + \sum_{i=0}^{P} b_i x(r-i) \tag{2}$$

where $x(r)$ and $y(r)$ represent the r th input and output of the system, respectively. The vector of filter coefficients (unknown parameters) is also considered by (3):

$$u = [b_0, b_1, \ldots, b_P, a_1, a_2, \ldots, a_Q]^T \tag{3}$$

The system modeling for filter designing is assumed as a minimization/optimization problem, and its value is regarded as a mean-square-error (MSE), objective function, J:

$$MSE = \underset{u\in U}{J(u)} = \frac{1}{K}\sum_{r=1}^{K}(d(r)-\hat{y}(r))^2 \tag{4}$$

where $e(r) = \hat{y}(r) - d(r)$ is the error signal, K is the number of samples that are used to calculate the function, U is coefficients space. Also, $\hat{y}(r)$ and $d(r)$ are the response of the IIR estimated system, and the unknown plant response in the r th step, respectively.

3. The proposed approach and considerations for implementing the weighted sum variable length particle swarm optimization (WS-VLPSO)

In this section, the proposed approach, along with its considerations, is described.

3.1. Description of the problem

The obvious point in the review of previous researches is that they estimate only filter coefficients and model an optimal filter with minimum reduced order of unit for system identification. Also, the lack of stability analysis of the designed filter overshadows the correctness and efficiency of the modeled IIR filtering system.

3.1.1. Filter order

The maximum delay, in samples, used in creating each output sample is called the *order* of the filter. The filter order is also equal to the number of poles in the frequency response. The filter order determines the number of filter delay lines, i.e., the number of input and output samples that should be saved so that the next output sample can be computed. In the optimal IIR filter design, by choosing a low order filter, the computation time can be reduced. The high-order IIR filters may have problems with instability, arithmetic overflow, and limit cycles, and require careful design to avoid such pitfalls.

3.1.2. Filter coefficients

A digital filter is designed using electronic circuits, which perform addition, multiplication, and delay operations [14]. So, we are faced with three arithmetic units; adder, multipliers, and delay to design a digital IIR filter. Numerical coefficients (b_i, a_j) are multiplier coefficients of a digital filter. We change the transfer function of the IIR filter via changing the coefficients.

3.1.3. Filter stability

There is a potential instability that is typical of IIR filters. Since we know that a recursive filter is stable if and only if all its poles have a magnitude less than 1 [23], a distinct method for checking stability is to find the roots of the denominator polynomial $X(z)$ in the filter transfer function (Eq. (1)). If the moduli of all roots are less than 1, the filter is stable. A filter transfer function is stable if and only if all its poles strictly are inside the unit circle ($|z|$ =1) in the complex z plane. A filter can also be said to be stable if its impulse response decays to 0 as r goes to infinity. We usually require that the impulse response decay to zero over time; otherwise, we say the filter is unstable.

In this study, the filter order Q is included as an integer variable in the particle vector of the PSO algorithm (rounded as a positive integer) to estimate the optimal value for filter order (an optimal value and, at the same time minimal). This causes the length of the coefficients vector to be changed based on the proposed value for the filter order (namely, a variable length particle vector), which should be correctly and intelligently considered into the structure of the PSO algorithm.

3.2. Particle swarm optimization (PSO)

The PSO algorithm was first proposed by Kennedy and Eberhart in 1995 based on the social behavior of organisms such as birds and fish [11]. In PSO, at each step of the movement of a category or group, the position of each member or agent (one particle) with two values of "*the best*" is determined. The best values are determined based on the value function (optimization criterion). The first value is named "*the local best*" or "*the local fitness*" (*pbest*) and is the best place a particle had during the repetition of the algorithm up to the moment t. The second value is named "*the global fitness*" (*gbest*), and it is the best place that particles have had up to moment t. Each particle in PSO is represented by a position vector and a velocity vector as follows:

$$X_i = (x_{i1}, x_{i2}, \ldots, x_{iD}) \tag{5}$$

$$V_i = (x_{i1}, x_{i2}, \ldots, x_{iD}) \tag{6}$$

The position and velocity of the particle i in d *th* and iteration $t+1$ are updated by using (7) and (8), respectively:

$$\begin{aligned} v_{id}\,(t+1) &= w(t)v_{id}(t) + c_1 r_{1i}(p_{id}^{best} - x_{id}(t)) \\ &\quad + c_2 r_{2i}(g_d^{best} - x_{id}(t)) \end{aligned} \tag{7}$$

$$x_{id}\,(t+1) = x_{id}(t) + v_{id}\,(t+1) \tag{8}$$

In the above equations, w (inertial weight) is the impact factor of the previous velocity, c_1 and c_2 are cognitive, and social coefficients, respectively, and r_1 and r_2 are uniformly distributed random numbers between 0 and 1 [11]. The maximum velocity is also restricted to $0.1\times d_x$, of which d_x is the different values of maximum and minimum vectors of the variables (d_u = X_{max}-X_{min}).

3.3. Particle swarm optimization with variable length particle vector (VLPSO)

In variable length optimization problems, the number of variables is not necessarily fixed. Since traditional optimization methods were designed for fixed-length design structures, they can be employed by assuming a fixed number of variables. Still, a suboptimal length will lead to a suboptimal solution. On the other hand, depending on the value of the determinant variable the design vector length, the problem-solving space will vary in each value. In other words, the distinct search space makes the algorithm execution process more special for space proper exploration. Besides, control values to accommodate these changes must also be taken into account. However, the standard and improved versions of the PSO algorithm in this problem are practically ineffective.

A better approach is to use an algorithm whose solution vectors can vary in length. Therefore the standard version of PSO needs to be modified to adapt and apply variable length vectors. A key to the success of a heuristic/evolutionary algorithm is its ability to accommodate its design vector length corresponding to the number of determining variables (here, filter order, or node). Essential considerations on variable length particle swarm optimization are as follows. The following strategies explain how to configure the proposed new version of the PSO algorithm with variable vector length, and the difference between the proposed VLPSO and intelligent consideration of each strategy compared to similar versions.

Strategy I: *Match the length of particle vectors based on deleting or adding cells with random values*
To match the particle vector dimensions during the search process, the removal of additional cells to match the larger vector to the smaller and adding random values (in terms of the non-violation of the minimum and maximum variables) for matching the short vector to larger is used. The random values generated consist of "uniformly distributed random numbers" that are selected from the allowable range of problem variables (between -2 and $+2$).
Because, according to the argument and experimental results, adding random values instead of zeroing, moving, or copying the corresponding cells for matching, will cause the process to be more innovative and more effective, and the lack of overcoming the larger vector and the effective improvement of the results.

Strategy II: *Match the particle vector length based on the number for the filter order*
Accordingly, for each d in the t th iteration of the algorithm, the size of particle vector length is updated based on the estimated value of the order variable ($L = ((2 \times O) + 1) + 1$}. Where L is the particles vector length, $((2 \times O) + 1)$ the formula for calculating the number of filter coefficients $(Q + P + 1)$, O is the proposed filter order of the algorithm, and the second number 1 is corresponding to the number of cells of the filter order variable. It is worth noting that, depending on the main transfer function of the benchmark plant, if its order Q is equal to P $(Q = P)$, then the formula for calculating the number of filter coefficients $(Q + P + 1)$ is considered $(2 \times O)$. The structure of the variable length particle vector of the proposed VLPSO method for the IIR modeling problem is shown in Fig. 2.

Strategy III: *Match the particle vector length for the difference in the second expression of (7)*
Accordingly, and based on the argument that the values of the *pbest* vector represent the conditions of local superiority in the previous step and its fitness value is corresponding to its positions, the matching is done in terms of the vector dimensions of the *pbest* particles.

Strategy IV: *Match the particle vector length for the difference in the third expression of (7)*
Similar to the reasoning expressed for the third strategy, the matching is done in terms of *gbest* dimensions.

Strategy V: *Match the particle vector length for the sum of the second and third expressions of (7)*
Accordingly, and with the argument that the pruning of each response vector of the second and third terms will cause the optimizing process of the algorithm to be lost and will not affect the velocity of the next step of particles, the matching will be done based on the vector with the larger-dimension.

Strategy VI: *Match the particle vector length for the sum of the first expression with the other two terms in (7) and the sum of the two expressions of (8).*
Similar to the fifth strategy, the matching of the first expressions in (7) and (8) is also done based on the larger response vector length of the second and third terms of (7).

Due to the intelligence of modeling order and the difference in values of coefficients of the identified filters with a different order, the minimum and maximum selectable range for particle vector variables have particular importance. Also, the maximum accuracy should be considered for choosing any control parameter (such as population, repetitions, etc.) based on the maximum modeling order.

3.4. Optimal, minimal order, and stable adaptive modeling of IIR systems using the WS-VLPSO adaptive algorithm

In this subsection, the final step to implement the proposed method is described. To make the algorithm more intelligent for less-order modeling and to reduce the implementation complexities and error surface of the designed filter, as well as to ensure the stability of the filter (the absence of poles outside the unit circle or on the boundary unit circle with boundary stability), the final objective function of the VLPSO algorithm is considered in the form of (9) (optimal, minimal order, and stable adaptive modeling). With this implementation, it is possible to evaluate all estimated values of the proposed algorithm (values leading to a stable or unstable filter and different order) using the proposed objective function. Therefore, the algorithm cleverly evaluates all candidate models first, and after identifying the order characteristics and the range of stable coefficients, zeroing the second part of the objective function. Then, the rest of the search process is followed for the optimum point with the same order. On the other hand, this kind of implementation will be the only way out to identify unstable unknown systems.

In this case, the stability constraint will act as a damping factor, and as soon as the stability condition of the proposed filter is obtained, the second part of the cost function becomes zero. Then the algorithm proceeds to search for coefficient estimation (optimum) and overcome the challenge of multimodal error surface in the desired order space. In each iteration, it is possible to analyze all optimal and stable solutions for each estimated order.

$$fitness = (\alpha \times MSE) + ((1 - \alpha) \times (\frac{O}{Q - 1} \cdot \frac{\Omega}{\Omega_T})) \tag{9}$$

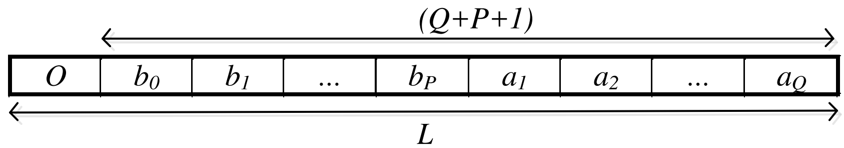

Fig. 2. Variable length particle vector for the IIR modeling problem.

where α is the effect factor of adaptive IIR system coefficients optimal design, and $(1 - \alpha)$ the effect factor of optimal and minimal filter order modeling. The parameter O is the optimal and minimum estimated integer for the filtering system order, $Q - 1$ is the maximum modelable order (Q is the filter order of main/unknown plant), Ω is the numbers of filter's poles outside and on the boundary of unit circle, and Ω_T is the numbers of all filter's poles. The desired value of it is zero, and the optimization process continues until the constraint is satisfied. As the fitness function is deduced, the identification process is performed intelligently. If optimal filter order modeling results in the stable filter design, the algorithm searches for optimization of the identified filter. This procedure will lead to the simultaneity of minimal modeling and optimal and stable design.

It is worth noting that different structures of the fitness function were considered, and many corresponding results were obtained. The overall estimation has shown that the algorithm tends to easily satisfy the stability condition only by decreasing the filter order, which results in a decrease in the amount of fitness function. Also, according to experimental results, it was observed that changes in the aggregation function coefficients did not significantly improve the results. Therefore, it was found that the multi-objective version of the algorithm is not effective in the current situation. The complete flowchart of the proposed WS-VLPSO is shown in Fig. 3.

3.5. Optimum modeling indicator (OMI)

Here, we present a new and important criterion called optimum modeling indicator (OMI) (as Eq. (10)). The proposed OMI clarifies the usefulness and effectiveness of the optimal filter order modeling based on reducing the order of IIR filters for improving the complexity and overcoming the challenge of designing and implementing adaptive IIR systems. Accordingly, the OMI is expressed in percentages, so that low percentages indicate an undesirable drop in order, and the high percentage indicates that optimal and minimal filter order modeling is acceptable. The motivations for this indicator is to be able to calculate the percentage of filter order reduction/improvement (as a critical and desired variable in the main process that led to present a new version of the VLPSO algorithm) compared to the default mode, and the common in other studies (estimate only one order less than the main filter).

The success rate for the number of independent runs of the adaptive algorithm is expandable as (10). In this case, for each e, the estimated order value is deducted from the fixed value of the actual filter, and is divided by the denominator (the total number of orders in the response space: obtained by multiplying the real filter order with the number of independent runs minus the minimum number of selected order of 1, which will be equal to the number of independent runs). In this way, the success rate of reducing the filter order and achieving an intelligent and optimal order, and consequently, reducing the number of filter coefficients will be determined.

$$OMI(\%) = \sum_{e=1}^{E} \frac{Q - O(e)}{E.Q - I} \times 100\% \tag{10}$$

where e is the number of the current run, and E is the total number of independent runs. Also, Q is the filter order of the main plant, $O(e)$ is the current estimated filter order in the e th run, $E.Q$ is the total sum of the available orders in the problem modeling space (total E times from the main plant order Q), and the I parameter represents the total number of runs with a minimum possible order 1, based on the filter design theory, which in this research is equal to the value of E. The OMI takes a value between zero and one hundred. An OMI = 0 means that all identified models are only with a unit reduced order (one order less than the main plant), and effective filter order minimization has not been performed. In contrast, OMI = 100 means that all models are minimized to one order (the minimal possible).

4. Results and analysis

In this section, the results of implementations are presented for five complex, common, and challenging benchmark IIR plants with different orders (min 1 and max 6). In this paper, the Monte Carlo simulation [24] is used to analyze the reliability and robustness features of the proposed solutions (the proposed approaches and not necessarily the proposed algorithm). It is worth noting that the MC not a competitor and requires different and heavy considerations to achieve the desired and definite result.

Results are including the best solutions (for five better runs) selected based on global optimum (convergence properties, fitness, filter order and coefficients, OMI criterion, execution time, stability, bode diagram, impulse response, and step response) along with computation time over total 30 independent runs for WS-VLPSO and five independent runs with 100,000 iterations for both MC method with weighted sum fitness function (WS-MC), (as Eq. (9)), and MC with simple objective function (Eq. (4)). Note that the time analysis (computational time and time complexity) is an important performance indicator in terms of the complexity of the proposed algorithm and the system under modeling, especially in real-time systems modeling. Based on simulations, the best results of the MC method are obtained by using the proposed intelligent objective function of Eq. (9). Therefore, comparative results between the proposed WS-VLSPO with the WS-MC method have been reported. All implementations are performed in MATLAB 2015b environment.

The values of the control parameters of the WS-VLPSO algorithm are listed in Table 1. As described before, value α (and 1-α) was chosen based on the desired objective (preferring the optimal global solution with a stable low-order filter). The inertia weight w is reduced linearly with the friction factor w_{damp} based on the repetition steps of the algorithm. The parameters of c_1 and c_2 are adjusted for maintenance of the exploration and exploitation mechanisms of the proposed algorithm, similar to settings presented in similar works. Also, the variables boundary and numbers of iteration along with the population size of the WS-VLPSO algorithm defined based on Section 3.4 of Section 3. How to select the control parameters of the proposed approach is defined to reduce the computational and time complexities. On the other hand, the robust and powerful display of the proposed algorithm with the minimum computational/timing volume compared to the extensive and weighty considerations of other research has been considered.

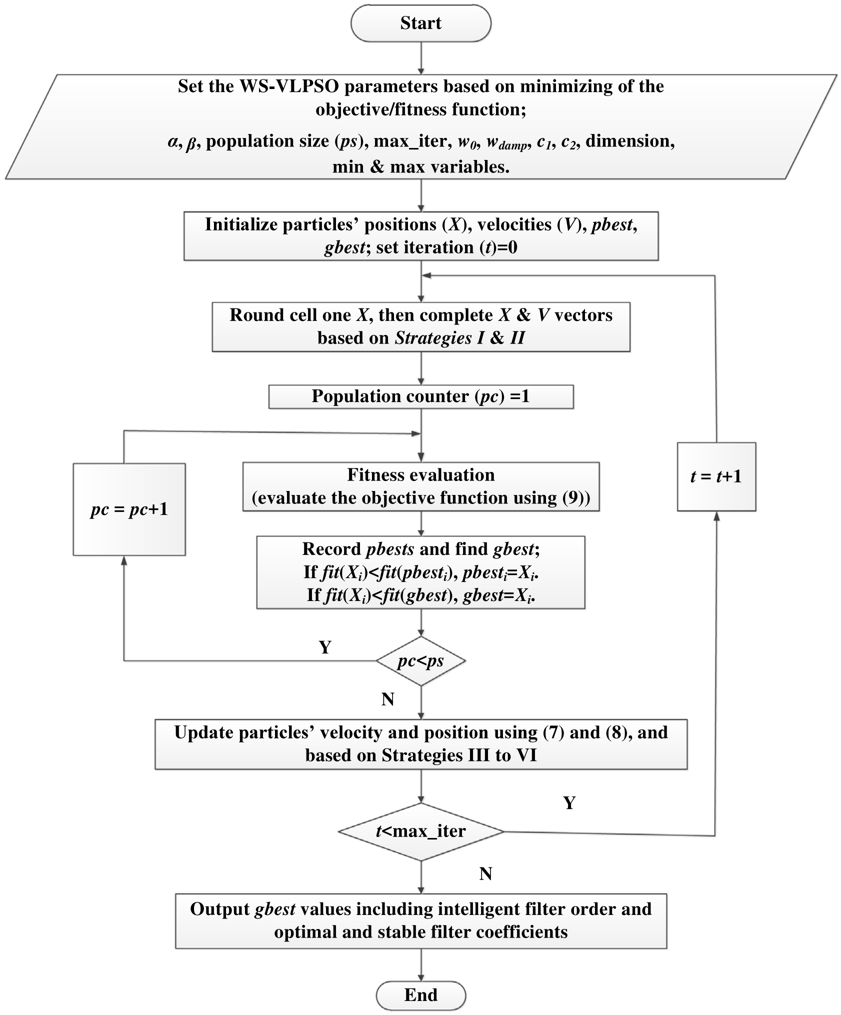

Fig. 3. Flowchart of the WS-VLPSO algorithm.

It is worth noting that the iteration values considered for WS-VLPSO and the rival method MC are the minimum possible value to achieve the desired answer, and certainly selecting more values will give a more desirable response. On the other hand, the solution mechanism of the two methods is different and this is also involved in the selection of their control parameters. Because one of the aspects of the advantage of using such soft computing optimization techniques against numerical and mathematical solution methods is the low computational volume and achieving the desired response compared to the large computational volume and time of rival methods. Also, the focus is on highlighting the advantage of the proposed algorithm to meet the initial design objectives with minimum computational values and the time that led to the selection of such values.

Following the research goal, the simulation results are reported in terms of fitness convergence curves, estimated coefficients, the fitness value of the weighted sum objective function, estimated order (O) for the modeled IIR system, and the computation time (all three in terms of best/min, worst/max, mean/average, and standard deviations (SD)) along with the OMI criterion, and numbers of coefficients. Also, to analyze the accuracy modeling process and ensuring the matching of obtained IIR filtering systems with their equivalent benchmark IIR plant transfer function, comprehensive evaluations including Pole-Zero plots, Bode diagrams, Step responses, and Impulse responses are verified. Finally, comparative Bar graphs (for fitnesses, filter orders, timing, and OMI) along with the comparative table with other works are also provided. The runs with italic values in the result tables are corresponding to the selected solutions for representing minimum and mean fitness curves for WS-MC, and, bode diagrams, pole-zero plots, impulse and step responses of the identified adaptive IIR models for both WS-VLPSO and WS-MC methods, respectively.

The input signal is a white noise string with a length of $K = 200$ (the data length). It is worth mentioning that, based on experimental results, the algorithm satisfies the optimality, stability, and other desired conditions for any number of independent runs.

The general form of the adaptive IIR system transfer function is considered as (11) for all examples.

$$H_s\left(z^{-1}\right) = \frac{b'_0 + b'_1 z^{-1} + b'_2 z^{-2} + \cdots + b'_{(P-1)} z^{-(P-1)} + b'_P z^{-P}}{1 + a'_1 z^{-1} + a'_2 z^{-2} + \cdots + a'_{(Q-1)} z^{-(Q-1)} + a'_Q z^{-Q}} \tag{11}$$

The specifications and definitions for the examples are as follows:

• ***Example 1: 2nd-order benchmark IIR plant***

The first example is a second-order benchmark plant [3,6,7,17] with the following transfer function:

$$H_p\left(z^{-1}\right) = \frac{0.05 - 0.4 z^{-1}}{1 - 1.1314 z^{-1} + 0.25 z^{-2}} \tag{12}$$

Table 1
Values of control parameters of the WS-VLPSO algorithm.

Parameter										
Iteration	Population	α	c_1	c_2	w_0	w_{damp}	$X_{min}(1)$, O_{min}	$X_{max}(1)$, O_{max}	X_{min}	X_{max}
200	50	0.6	2	2	0.99	0.99	1	Q - 1	−2	+2

In this case, the goal is the optimal adaptive system identification and stable designing of IIR digital filters with the minimum and maximum order of one ($O = 1$).

• ***Example 2: 3rd-order benchmark IIR plant***

In this case, a third-order IIR plant [3,6,7] shown in (13) is modeled using IIR systems with the order boundary $1 \leq O \leq 2$ under adaptive system identification configuration.

$$H_p\left(z^{-1}\right) = \frac{-0.3 + 0.4z^{-1} - 0.5z^{-2}}{1 - 1.2z^{-1} + 0.5z^{-2} - 0.1z^{-3}} \tag{13}$$

• ***Example 3: 4th-order benchmark IIR plant***

The transfer function of the IIR plant is given by, [17]:

$$H_p\left(z^{-1}\right) = \frac{1 - 0.9z^{-1} + 0.81z^{-2} - 0.729z^{-3}}{1 + 0.04z^{-1} + 0.2775z^{-2} - 0.2101z^{-3} + 0.14z^{-4}} \tag{14}$$

In this case, IIR digital filters are designed with a minimum order of one and a maximum of three ($1 \leq O \leq 3$) using an adaptive IIR modeling configuration.

• ***Example 4: 5th-order benchmark IIR plant***

The plant transfer function for this benchmark IIR system is the following (15) is given in Box I, [6,7].

In this case, the goal is optimal adaptive IIR modeling along with stable designing of IIR digital filters with an order boundary as $1 \leq O \leq 4$.

• ***Example 5: 6th-order benchmark IIR plant***

The fifth example is a sixth-order benchmark plant [2,17,25, 26] with the following transfer function:

$$H_p\left(z^{-1}\right) = \frac{1 - 0.4z^{-2} - 0.65z^{-4} + 0.26z^{-6}}{1 - 0.77z^{-2} - 0.8498z^{-4} + 0.6486z^{-6}} \tag{16}$$

In this case, the goal is the optimal modeling of adaptive IIR systems with the order boundary $1 \leq O \leq 5$.

In each case, the transfer function of the candidate IIR model is considered in the form of (11), and it is repeatedly adjusted by adaptive modeling process using the proposed WS-VLPSO to finally get the most optimal filter (with optimal order and coefficients, the closest functional similarity to the main IIR plant, and stable). The fitness convergence curves, mean/minimum fitness curves, estimated coefficients, and the values of performance parameters based on five better runs using WS-VLPSO and the best run from all five independent runs using MC methods with 100,000 iterations for all examples are shown in Fig. 4, 5, and Tables 2 and 3, respectively, for all examples. The X-axis is logarithmically displayed for better resolution of the convergence mechanism. Also, stability conditions (pole-zero plots) of the main plants and the obtained models along with bode diagrams, impulse and step responses are shown in Figs. 6, 7, and 8, simultaneously.

For example 1, it is observed that WS-VLPSO obtains the best fitness without any abrupt oscillations. As for the multimodal error surface and the challenge of finding the optimal solution, in this case, it has a good convergence speed of about 20 initial iterations for all five runs. The 100% success of the optimal and stable adaptive modeling can also be seen from the results of both methods. So that, OMI 100%, the average time 11.1 s, the average fitness 0.032436, and the minimum matching error between responses of the identified model and the actual IIR plant, show the desired and acceptable performance of the proposed structure and the WS-VLPSO algorithm, especially against MC methods. For example 2, the results indicate WS-VLPSO can reach the minimum values (its best fitnesses). Also, the performance measures reveal that the proposed approach guarantees the designing of optimal, simple, and smart IIR filters (the mean fitness 0.0316264, OMI 70%, the mean modeling order 1.6 along with the good variety of the designed filters with different ordering and stable, the average time 9.94 s, and behavioral similarity in comparative graphs).

For example 3, it can be seen that the IIR modeling technique has a global search capability, and it can identify the IIR models more accurate and very close to the main IIR plant performance but with the lower order and stable, resulting in less complexity and the easier implementation. The statistical and intuitive evaluations based on good convergences, stability, good bode diagrams, similar step and impulse responses, OMI 46.67% with the mean modeling order 2.6, average fitness 0.0271972, and average time 10.3 s demonstrate the high performance and efficiency of the WS-VLPSO. Overall assessment of the first three examples, in contrast to the Monte-Carlo simulation outputs, clearly demonstrates the reliability and robustness of the proposed solutions by using the proposed WS-VLPSO algorithm. For example 4, with high order 5, WS-VLPSO has a unique initial maneuver in extracting an optimal solution about 50 initial iterations. The quality of the results for this example (e.g., estimated orders with OMI 65%, low timing, good stability, and very well matching) indicates the dynamism, success, and efficiency of the current research, especially for the order variety of final models obtained. For example 5, the WS-VLPSO obtained the desired fitness with a nearly appropriate convergence and has performed well concerning estimating coefficients of modeled filters, fitness values, estimated order values, OMI of 68%, numbers of coefficients, and execution times. In the overall assessment of the two high-level challenging plants, the tangible advantage of using an intelligent structure (including objective function and optimizer algorithm) over the Monte-Carlo direct solving method is established.

To obtain the best overall performance of the proposed approach, we present the comparative bar graphs in Fig. 9 for fitness values, estimated minimum and optimum orders, and execution times based on five better runs of Table 3, as well as the graphic representation of the OMI criterion based on the five runs and the total of 30 independent runs using WS-VLPSO for all IIR filtering examples. Besides, a comprehensive comparison of the simulation results obtained with the WS-VLPSO over 30 independent runs and WS-MC over five independent runs for all IIR filtering examples is also provided in Table 4, in the form of statistical analyzes. From the results, it is concluded that the fitness level is proportional to the filter order. So that, as the estimated order a candidate model approaches to the main plant order, the problem space becomes simpler, and the fitness function level decreases. In contrast, with decreasing order, the error surface of the objective function (design cost function) becomes more complicated, and the fitness level increases and the convergence probability of the algorithm to the sub-optimal points increase.

The overall estimation of Fig. 9 and Table 4 shows a fitness average of less than 0.05, an average modeling order of less than 4 (effective reduction of the filter order), the average execution time of less than 13 s (that indicates the acceptable promptitude and the time savings and efficiency of the proposed offline design tool), and a success rate of over 50% by using the WS-VLPSO

$$H_p\left(z^{-1}\right)=\frac{0.1084+0.5419z^{-1}+1.0837z^{-2}+1.0837z^{-3}+0.5419z^{-4}+0.1084z^{-5}}{1+0.9853z^{-1}+0.9738z^{-2}+0.3864z^{-3}+0.1112z^{-4}+0.0113z^{-5}} \tag{15}$$

Box I.

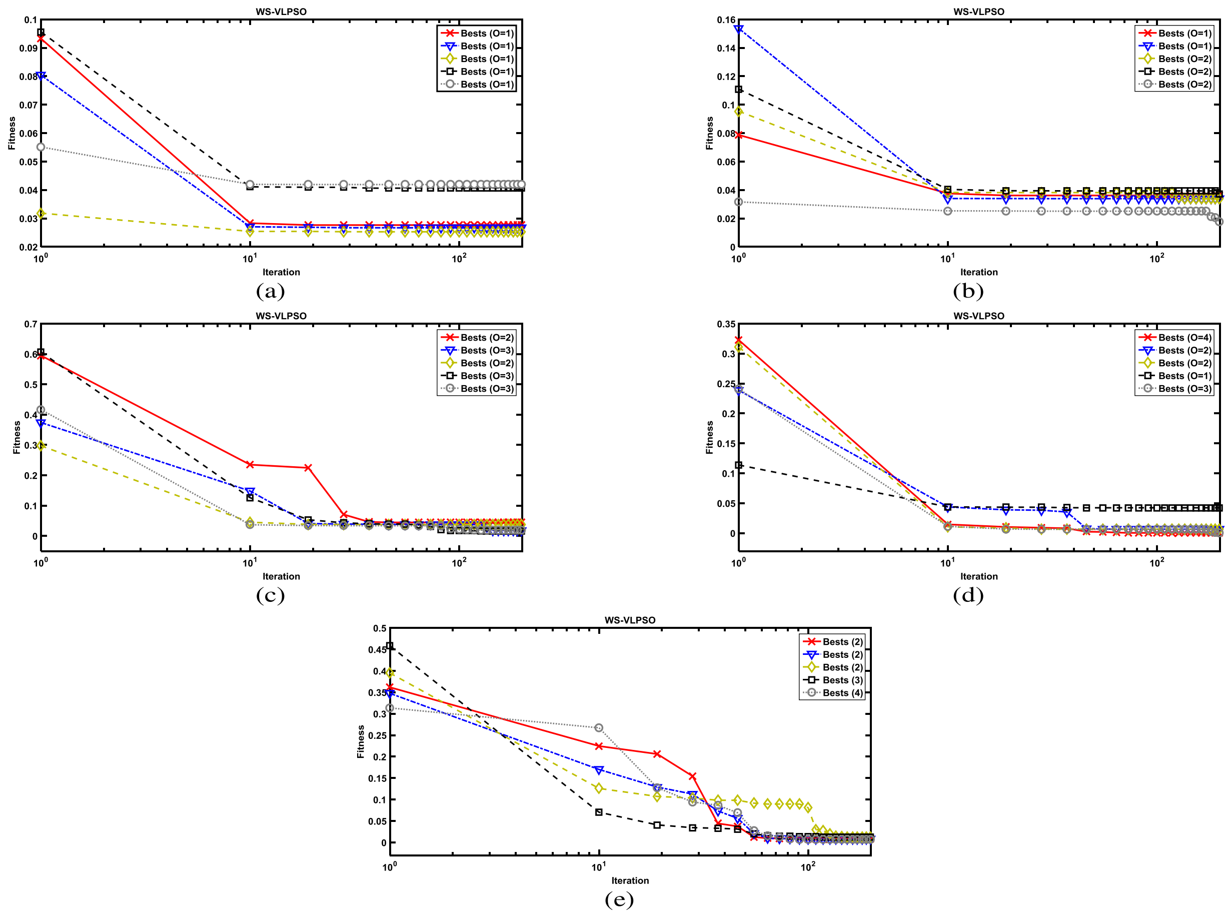

Fig. 4. Fitness curves of the WS-VLPSO algorithm for all IIR filtering examples. (a) Example 1, (b) Example 2, (c) Example 3, (d) Example 4, (e) Example 5 (X scale: logarithmic).

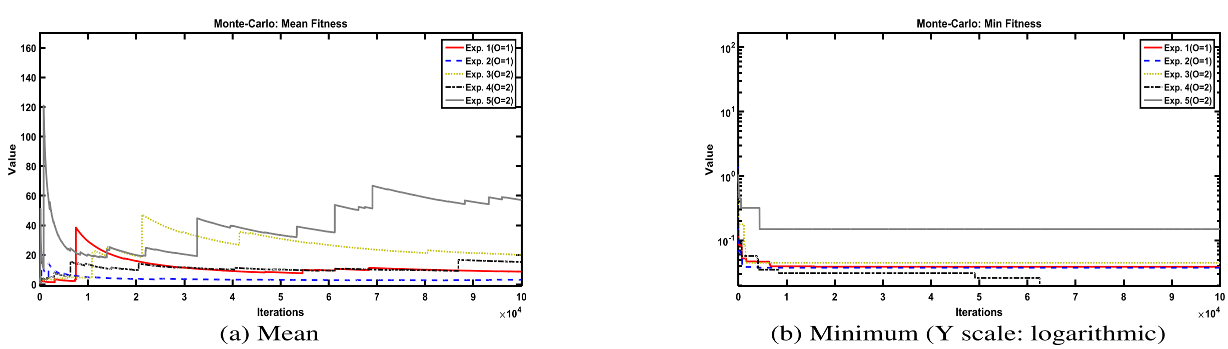

Fig. 5. Mean/Minimum fitness curves for the best run from all five independent runs using WS-MC method for all IIR filtering examples.

algorithm in the modeling of a complete range of complex IIR models with different degrees. The results of the Monte-Carlo simulation in Table 4 also confirm the accuracy, performance, efficiency, and reliability of the proposed approach in this paper.

In the end, a comprehensive comparing of the simulation results between the proposed WS-VLPSO approach along with the selected WS-MC method and other techniques is reported in Table 5, and the better values are presented in bold, and dash (–) means unreported. Thus, the proposed approach is compared with GA, PSO, GSA, CSO, SOA, LWOA, HS [27], IPO [28], FPA [29], BA [30], MFO [31], WOA [32], OBA [33], and their modified and combined versions including: QPSO [34], MuQPSO [34], PSO_QI [35], MIPO, OHCRO, HPSO-GSA, comprehensive learning

Table 2
Estimated coefficients of the modeled filters based on five better runs using WS-VLPSO and WS-MC methods for all IIR filtering examples.

Examples	Coefficients	Runs									
		I		II		III		IV		V	
		WS-VLPSO	WS-MC	WS-VLPSO	WS-MC	WS-VLPSO	WS-MC	WS-VLPSO	WS-MC	WS-VLPSO	WS-MC
Exp. 1	b_0	−0.3211	*−0.3016*	−0.3068	−0.3010	*−0.3107*	−0.3193	−0.3153	−0.2963	−0.3194	−0.3263
	a_1	−0.9009	*−0.9040*	−0.9044	−0.9118	*−0.9062*	−0.8987	−0.9055	−0.9079	−0.9025	−0.9017
Exp. 2	b_0	−0.3214	*−0.3056*	*−0.3168*	−0.4185	−0.4013	−0.3256	−0.3569	−0.3270	−0.1196	−0.1157
	b_1	–	–	–	0.0012	−0.0734	–	−0.1442	–	−0.0624	−0.0279
	a_1	−0.8643	*−0.8744*	*−0.8621*	−0.1991	−0.2120	−0.8591	−0.1547	−0.8557	−1.5113	−1.4260
	a_2	–	–	–	−0.6178	−0.5779	–	−0.6547	–	0.6081	0.4981
Exp. 3	b_0	1.1153	*0.7197*	1.0895	1.1667	*0.8743*	0.7124	0.9072	0.9800	1.0254	1.0792
	b_1	−1.0802	−0.4184	0.1898	0.2851	−0.1987	−0.0445	−0.0326	−0.1343	−0.1727	−0.0371
	b_2	–	–	0.7740	–	–	–	0.5883	–	0.7884	–
	a_1	−0.0229	*0.9638*	0.9870	1.1625	*1.0108*	1.1358	0.9697	1.0008	0.6969	0.9710
	a_2	−0.7440	*0.5276*	0.8070	0.5873	*0.5545*	0.6091	0.7583	0.4882	0.5170	0.4775
	a_3	–	–	0.1391	–	–	–	0.1086	–	−0.0098	–
Exp. 4	b_0	0.1254	0.1815	0.1013	*0.1051*	0.1151	0.2393	0.1183	−0.2055	*0.0911*	0.2871
	b_1	0.3772	0.4109	0.3989	*0.2811*	0.3950	0.3793	0.4269	0.4793	*0.4621*	0.3016
	b_2	0.4822	0.5429	0.5122	*0.5523*	0.5125	0.4799	–	0.3172	*0.7044*	0.3546
	b_3	0.0690	–	–	–	–	–	–	–	*0.4777*	–
	b_4	−0.1351	–	–	–	–	–	–	–	–	–
	a_1	−0.2897	−0.0417	−0.2372	*−0.4394*	−0.2387	0.0797	−0.5122	−0.2493	*0.3083*	−0.3434
	a_2	0.4693	0.3802	0.3583	*0.3363*	0.3492	0.1664	–	0.4086	*0.4937*	0.5348
	a_3	−0.3023	–	–	–	–	–	–	–	*−0.0207*	–
	a_4	0.0725	–	–	–	–	–	–	–	–	–
Exp. 5	b_0	0.9985	0.8312	*1.0203*	*1.0267*	1.1579	0.8990	1.0423	0.8142	1.0053	1.1933
	b_1	0.0042	−0.0917	*−0.0036*	*−0.4107*	0.0129	0.5863	−0.3408	0.1858	0.5658	−0.1418
	b_2	−0.5285	0.2194	*−0.5430*	*−0.0796*	−0.6825	–	−0.4851	0.0081	0.2029	−0.3909
	b_3	–	–	–	–	–	–	0.1606	–	−0.3070	–
	b_4	–	–	–	–	–	–	–	–	−0.3661	–
	a_1	−0.0011	−0.0862	*−0.0015*	*−0.0827*	0.0025	0.9451	−0.2648	0.0211	0.5330	0.0557
	a_2	−0.9340	−0.7800	*−0.9330*	*−0.8419*	−0.93	–	−0.9189	−0.6923	−0.2148	−0.7293
	a_3	–	–	–	–	–	–	0.2358	–	−0.4984	–
	a_4	–	–	–	–	–	–	–	–	−0.6665	–

Table 3
Comparative results of adaptive IIR filtering systems modeling between WS-VLPSO, WS-MC, and MC for all examples.

Examples	Runs	Performance parameters											
		Fitness			O			Time (s)			Stability (Yes or No)		
		WS-VLPSO	WS-MC	MC	WS-VLPSO	WS-MC	MC	WS-VLPSO	WS-MC	MC	WS-VLPSO	WS-MC	MC
Exp. 1	I	0.02765	*0.0390*	0.0689	1	*1*	1	12.37	*90.47*	141.73	Y	*Y*	Y
	II	0.026697	0.0407	0.0694	1	1	1	11.86	117.85	97.45	Y	Y	Y
	III	*0.025287*	0.0397	0.0677	*1*	1	1	*11.57*	236.55	173.21	*Y*	Y	Y
	IV	0.040638	0.0352	0.0735	1	1	1	9.74	96.37	115.36	Y	Y	Y
	V	0.041908	0.0414	0.0650	1	1	1	9.78	110.39	146.45	Y	Y	Y
Exp. 2	I	0.036232	*0.03774*	0.0684	1	*1*	2	9.71	*105.83*	158.61	Y	*Y*	Y
	II	*0.033876*	0.0400	0.0748	*1*	2	1	*9.84*	147.77	100.97	*Y*	Y	Y
	III	0.033781	0.03767	0.0642	2	1	2	10.39	113.92	126.67	Y	Y	Y
	IV	0.036634	0.0359	0.0611	2	1	2	9.99	258.15	100.27	Y	Y	Y
	V	0.017609	0.0309	0.0540	2	2	2	9.77	156.04	165.29	Y	Y	Y
Exp. 3	I	0.044488	*0.0450*	0.0876	2	*2*	3	10.20	*100.49*	179.58	Y	*Y*	N
	II	0.018091	0.0550	0.0732	3	2	2	10.42	259.75	101.62	Y	Y	Y
	III	*0.037341*	0.0594	0.1075	*2*	2	2	*10.15*	143.10	101.0	*Y*	Y	Y
	IV	0.01693	0.0492	0.0813	3	2	2	10.38	146.69	100.42	Y	Y	Y
	V	0.019136	0.0469	0.0921	3	2	2	10.41	131.62	137.94	Y	Y	Y
Exp. 4	I	0.00039316	0.0137	0.0302	4	2	3	10.96	112.25	111.69	Y	Y	N
	II	0.0066315	*0.0197*	0.0323	2	*2*	3	10.30	*125.07*	105.15	Y	*Y*	N
	III	0.0065778	0.0223	0.0279	2	2	4	10.27	120.12	192.88	Y	Y	N
	IV	0.042349	0.0307	0.0210	1	2	3	10.26	112.67	146.24	Y	Y	N
	V	*0.00073908*	0.0271	0.0218	*3*	2	2	*10.25*	127.0	108.72	*Y*	Y	Y
Exp. 5	I	0.0081967	0.1188	0.1638	2	2	4	10.30	175.99	193.34	Y	Y	N
	II	*0.0078999*	*0.1500*	0.0930	*2*	*2*	2	*11.04*	*150.13*	173.81	*Y*	*Y*	Y
	III	0.012473	0.1226	0.1433	2	1	4	10.38	103.88	171.0	Y	Y	N
	IV	0.012126	0.1263	0.2279	3	2	4	8.30	102.18	239.62	Y	Y	N
	V	0.0074506	0.1521	0.2487	4	2	5	8.38	141.93	150.96	Y	Y	N

particle swarm optimizer (CLPSO) [36], PSO with adaptive inertia weight (PSO-w) [37], standard PSO 2007 (SPSO-2007) [3], multi-strategy immune cooperative evolutionary particle swarm algorithm (ICPSO-MS) [38].

The purpose of this table is to evaluate the proposed approaches of the manuscript based on the possibility of smartening the process of designing digital filters under the system identification structure. To this end, the standard PSO algorithm is

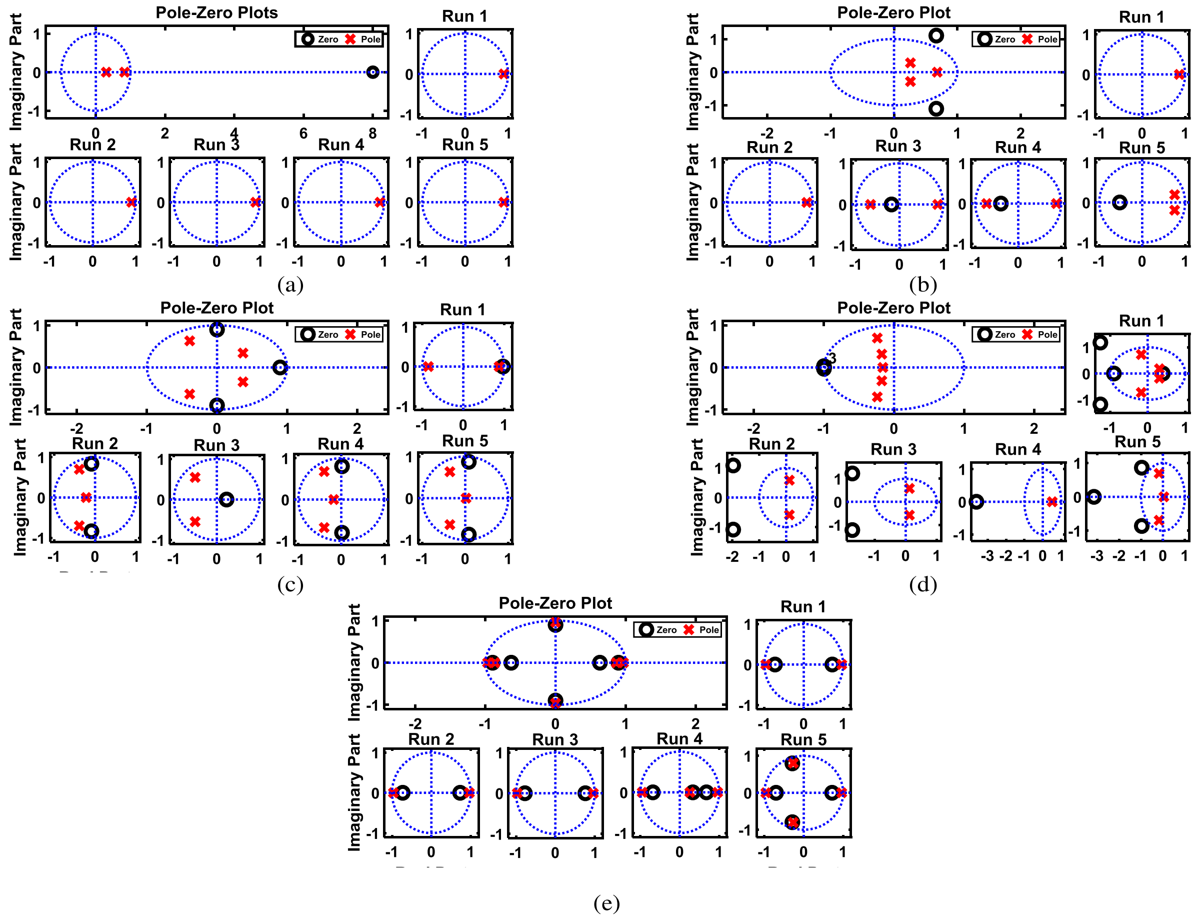

Fig. 6. Pole-zero plots of the benchmark IIR plant and modeled systems obtained using WS-VLPSO for all examples. (a) Example 1, (b) Example 2, (c) Example 3, (d) Example 4, (e) Example 5.

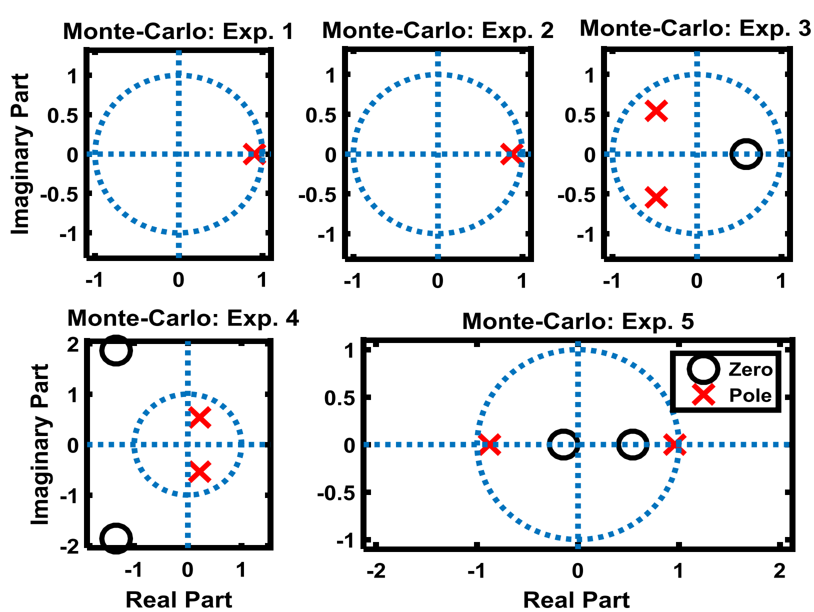

Fig. 7. Pole-zero plots of the best system modeled for all IIR filtering examples based on the best run from all five independent runs using the WS-MC method.

upgraded to a new and effective variable length version. It is then used to estimate the optimal order of benchmark IIR filters and their optimal coefficients. Besides, embedding an expert objective function of the weighted sum is another distinction of the present article from previous work. Minimal control settings to reduce the computational and time complexities of the proposed approach, along with the presentation of a new index, are including the differences with other research that is tried to be shown in Table 5. As it is clear, the overall evaluation of the results, despite these considerations, indicates the superiority and competitive

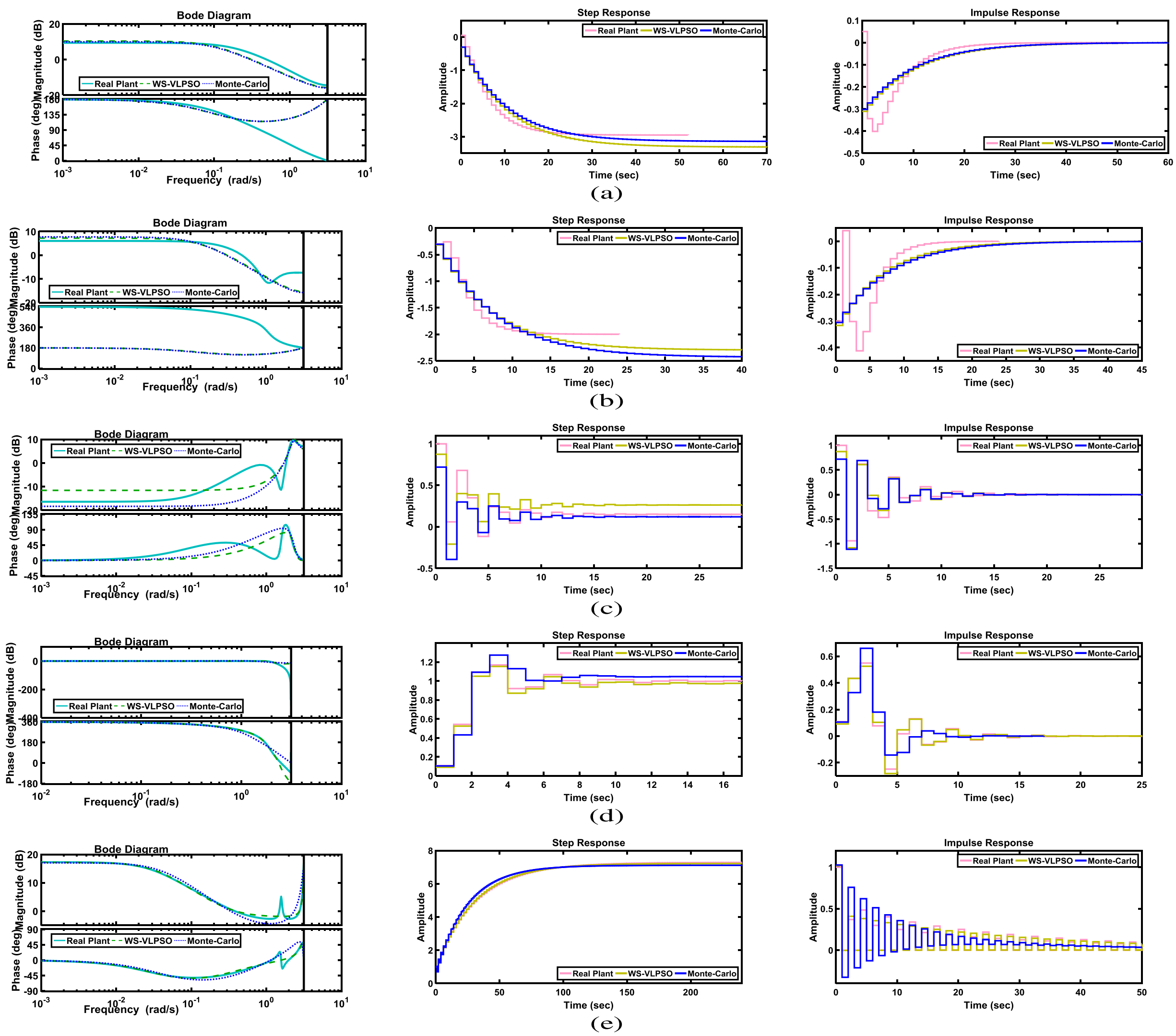

Fig. 8. Comparative graphs (bode diagrams; step & impulse responses) using WS-VLPSO and WS-MC methods for all IIR filtering examples. (a) Example 1, (b) Example 2, (c) Example 3, (d) Example 4, (e) Example 5.

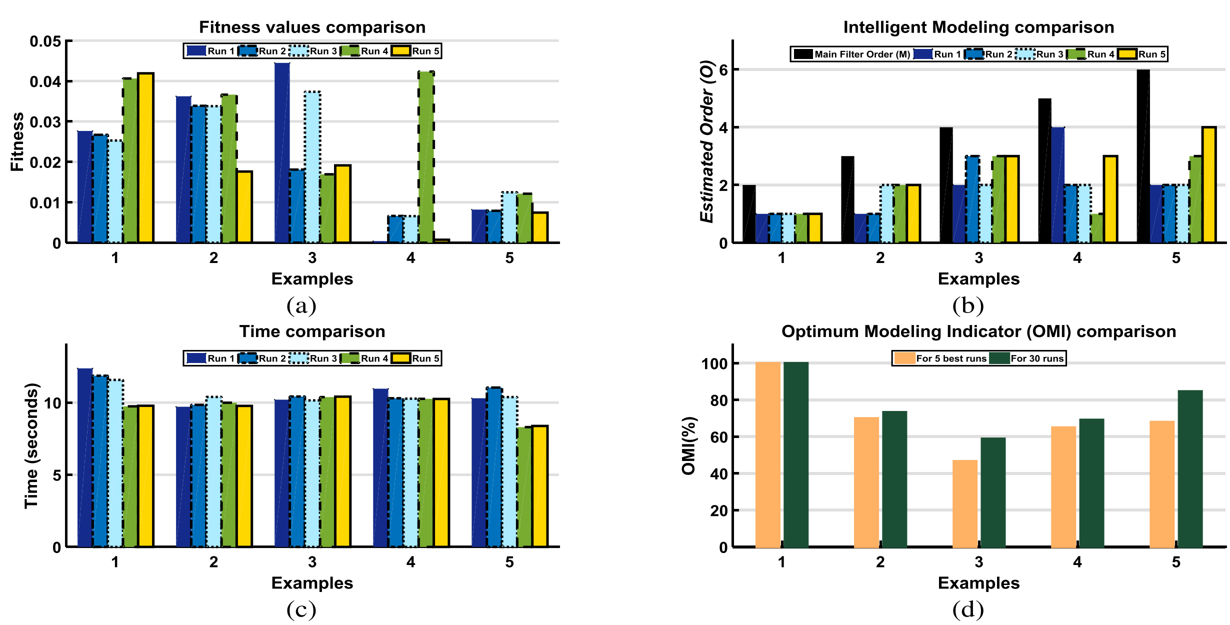

Fig. 9. Comparative bar graphs of WS-VLPSO results for all IIR filtering examples based on Table 3; (a) fitnesses, (b) estimated filter orders, (c) timing, (d) OMI.

Table 4
Comparative results between the WS-VLPSO (over 30 runs) and WS-MC (over five runs) for all IIR filtering examples.

Indexes		Examples									
		Exp. 1 (Q = 2)		*Exp. 2* (Q = 3)		*Exp. 3* (Q = 4)		*Exp. 4* (Q = 5)		*Exp. 5* (Q = 6)	
		WS-VLPSO	WS-MC	WS-VLPSO	WS-MC	WS-VLPSO	WS-MC	WS-VLPSO	WS-MC	WS-VLPSO	WS-MC
Fitness	Best	0.025287	0.035155	0.017609	0.03085	0.01011	0.045001	0.00031	0.01370	0.00623	0.11878
	Worst	0.047922	0.041376	0.045238	0.039984	0.044488	0.059405	0.050191	0.030681	0.22714	0.15208
	Mean	0.039761	0.039176	0.034923	0.036432	0.024268	0.051102	0.010434	0.022689	0.046336	0.13394
	SD	0.0053213	0.0024226	0.0058607	0.0034386	0.0062137	0.0059606	0.013399	0.0065758	0.057912	0.01585
O	Min	1	1	1	1	2	2	1	2	1	1
	Max	1	1	2	2	3	2	4	2	4	2
	Mode	1	1	2	1	2	2	3	2	2	2
	Mean	1	1	1.5333	1.4	2.2667	2	2.2	2	1.8	1.8
	SD	0	0	0.50742	0.54772	0.44978	0	0.76112	0	0.66436	0.44721
Time (s)	Min	8.34	90.47	9.42	105.83	9.4	100.49	9.5	112.25	8.3	102.18
	Max	12.37	236.6	10.39	258.15	11.06	259.75	10.96	127	11.04	175.99
	Mean	9.4203	130.32	9.7557	156.34	9.9627	156.33	9.827	119.42	9.7367	134.82
	SD	0.89804	60.372	0.20607	60.808	0.3319	60.606	0.32604	6.8345	0.48027	31.633
OMI (%)		100	100	73.33	80	58.89	66.67	69.17	75	84.67	84

performance of the proposed method. Presenting the details of other research in Table 5 will allow users to make a complete, accurate and correct assessment so that all the positive and negative aspects of the proposed approaches are found and clarified.

Finally, we can deduce that the research claim based on the simultaneity and contraction of the minimal, optimal, and stable modeling approach of IIR filtering systems has been achieved. And, the global optimization, the significant efficiency, and the success of the proposed approach based on the use of a variable length PSO algorithm with a weighted sum fitness function are proven.

4.1. A real-world variable length engineering problem

In this subsection, a real engineering problem is applied, and the performance success of the proposed algorithm, along with MC methods, is presented. The term coverage can be used to encompass several different problems. The common characteristic of these problems is their attempt to position some nodes such that they can cover a specified domain as efficiently as possible. Common examples include sensor coverage or wireless transmitter networks, such as radio or cellular networks. Many works have been performed on the sensor coverage problem in the literature [39–41].

Both coverage problems (constrained or unconstrained formulations) use relatively simple fitness models where each node covers a circular area (a 2 × 2 square domain). Here, each discrete variable represents a single node defined by an x-position, y-position, and radius. Cost is measured as the sum of all node radii in the solution. Coverage is approximated using a square point lattice covering the domain, with a spacing of 0.01 between points. The calculated value will be in the range of [0; 1], where one would indicate a fully covered domain. Large sensors are more expensive but also more cost-efficient. Therefore, the number of nodes should be reduced, while the network coverage should be maximized. It will result in a change in the length of design vectors corresponding to the number of nodes proposed in the first cell of the estimated vector $\{D = ((3 \times N) + 1)\}$. Here, D is the vector length, $(3 \times N)$ the positions of nodes, N is the number of proposed nodes, and the second number 1 is corresponding to the cell number of nodes (see Fig. 10). The unconstrained coverage uses a weighted sum of cost and uncovered area as the objective function. The weights used here result in an intense pressure toward solutions that cover most of the domain (as [40]). The optimization statement is given in (17):

$$\begin{cases} fitness\,(U) = \sum_{n=1}^{N} r_n + 50 \times (1 - coverage\,(U)) \\ U\,(N, x, y, r)\,; \qquad (x, y, r) \in \mathbb{R}^{N \times v}, N \in \mathbb{Z}^{+} \\ (x - position) \quad 0 \le x_n \le 2 \\ (y - position) \quad 0 \le y_n \le 2 \qquad n = 1, 2, \ldots, N \\ (radius) \qquad 0.10 \le r_n \le 0.25 \end{cases} \tag{17}$$

Results are reported over a total of 20 independent trials for WS-VLPSO and five trials for WS-MC (with 500,000 iterations) with weighted sum fitness function as (17) including fitness convergence curves and values, the number of network sensors, covering, cost, and time, as well as their statistical analysis. The variables ranges are as (17) and $10 \le N \le 50$. For WS-VLPSO, w is reduced linearly between [0.7 1], c_1 and c_2 are equal to 2, iteration = 3000, and population equal to 100.

Output comparative results in Table 6, and the sensor coverage sample solutions with the minimum and maximum fitness as well as two selected solutions for both WS-VLPSO and MC methods are shown in Fig. 11. The statistical analysis of the numerical values reported in Table 6, as well as the graphical representation of the nodes embedded by the proposed method along with Monte Carlo in Fig. 11, indicates the continuity of the superiority of the proposed WS-VLPSO approach in another type of real-world problems. This superiority is best illustrated by the simultaneous comparison of the higher convergence index, while minimizing the number of embedded nodes. The average run-time of about 32.34 min also indicates its success, along with time and computational efficiency. Among the 25 indicators evaluated in Table 6, the Monte Carlo has only had a relative superiority in only two indicators, which demonstrates a 92% success of the WS-VLSPSO method over it.

5. Conclusions & future work

Variable length optimization problems are a type of complex engineering problem in which the number of design variables can vary among solutions. In this research, the possibility of optimal and stable modeling of IIR digital filtering systems, as the first candidate of real variable length optimization applications, using a weighted sum version of the PSO algorithm with variable length particle vector under adaptive system identification configuration has been investigated. So that based on the results obtained by Monte-Carlo simulation and other researches, in addition to the knowing selection of the best order and optimal filter coefficients, the proposed adaptive WS-VLPSO algorithm also ensures

Table 5
Comparisons between WS-VLPSO with WS-MC and similarly reported researches for all IIR filtering examples.

Examples	Reference, Year	Method	Q	Pop.	Iters.	K	Performance parameters: Time (s)	Fitness: Value	Fitness: O	Mean estimated order	OMI (%)	Stability
Exp. 1	[3], 2010	GA	2	100	1000	100	–	0.08929	1	1	100	–
	[3], 2010	CLPSO	2	100	1000	100	–	0.08943	1	1	100	–
	[3], 2010	PSO-w	2	100	1000	100	–	0.08975	1	1	100	–
	[3], 2010	SOA	2	100	1000	100	–	0.08277	1	1	100	–
	[34] 2010	QPSO	2	10	100	500	**2.334**	0.225	1	1	100	100%
	[34] 2010	MuQPSO	2	10	100	500	**2.308**	0.21	1	1	100	100%
	[33] 2013	OBA	2	25	300	–	**2.4785**	**0.00626**	1	1	100	–
	[6], 2013	PSO	2	30	300	50	–	0.16233	1	1	100	–
	[7], 2017	IPO	2	30	100	50	–	0.0537	1	1	100	–
	[7], 2017	MIPO	2	30	100	50	–	0.04342	1	1	100	–
	[17], 2017	OHCRO	2	50	300	200	–	0.1320	1	1	100	–
	Present	**WS-MC**	2	–	100,000	200	130.32	0.035155	1	1	100	100%
		WS-VLPSO	2	50	200	200	**9.4203**	**0.02529**	1	**1**	**100**	**100%**
Exp. 2	[2], 2010	GA	3	100	1000	100	–	0.00716	2	2	50	–
	[2], 2010	CLPSO	3	100	1000	100	–	0.00551	2	2	50	–
	[2], 2010	PSO-w	3	100	1000	100	–	0.00581	2	2	50	–
	[2], 2010	SPSO	3	100	1000	100	–	0.00566	2	2	50	–
	[2], 2010	SOA	3	100	1000	100	–	**0.0052**	2	2	50	–
	[34] 2010	QPSO	3	30	500	500	51.174	0.0145	2	2	50	100%
	[34] 2010	MuQPSO	3	30	500	500	44.071	0.014	2	2	50	100%
	[6], 2013	PSO	3	30	300	50	–	0.11972	2	2	50	–
	[7], 2017	IPO	3	30	200	50	–	0.03829	2	2	50	–
	[7], 2017	MIPO	3	30	200	50	–	0.0327	2	2	50	–
	Present	**WS-MC**	3	–	100,000	200	156.34	0.03085	2	1.4	80	100%
		WS-VLPSO	3	50	200	200	**9.7557**	**0.0176**	2	**1.5333**	**73.33**	**100%**
Exp. 3	[17], 2017	GA	4	50	300	200	–	0.0219	3	3	33.333	–
	[13], 2018	GSA	4	40	300	200	–	0.0107	3	3	33.333	–
	[13], 2018	IPO	4	40	300	200	–	0.0102	3	3	33.333	–
	[17], 2017	PSO	4	50	300	200	–	0.0277	3	3	33.333	–
	[17], 2017	ICPSO-MS	4	50	300	200	–	0.0682	3	3	33.333	–
	[17], 2017	OHCRO	4	50	300	200	–	**0.0019**	3	3	33.333	–
	[16], 2017	HPSO-GSA	4	40	500	200	**10.713**	0.0428	3	3	33.333	–
	Present	**WS-MC**	4	–	100,000	200	156.33	0.045001	2	2	66.67	100%
		WS-VLPSO	4	50	200	200	**9.9627**	**0.0101**	3	**2.2667**	**58.89**	**100%**
Exp. 4	[4], 2011	GA	5	50	500	25000	–	0.0846	4	4	25	–
	[4], 2011	CSO	5	50	500	25000	–	0.00007	4	4	25	–
	[27], 2014	HS	5	25	300	–	**5.283**	0.000006	4	4	25	–
	[5], 2015	GSA	5	60	1000	200	11.462	0.0003	4	4	25	–
	[5], 2015	HPSO-GSA	5	60	1000	200	11.969	0.00001	4	4	25	–
	[13], 2018	IPO	5	40	300	200	–	0.0001	4	4	25	–
	[6], 2013	PSO	5	30	300	50	–	0.000005	4	4	25	–
	[7], 2017	MIPO	5	30	200	50	–	**0.000001**	4	4	25	–
	Present	**WS-MC**	5	–	100,000	200	119.42	0.01370	2	2	75	100%
		WS-VLPSO	5	50	200	200	**9.827**	**0.00031**	**3**	**2.2**	**69.17**	**100%**
Exp. 5	[35] 2010	DEPSO	6	25	500	50	**2.329**	0.002	5	5	20	–
	[25], 2009	PSO	6	20	500	200	7.082	0.0056	4	4	40	100%
	[35] 2010	PSO_QI	6	25	500	50	**4.008**	0.002	5	5	20	–
	[27], 2014	HS	6	25	300	–	8.573	0.0003	4	4	40	–
	[5], 2015	GSA	6	60	1000	200	11.071	0.0408	5	5	20	–
	[5], 2015	HPSO-GSA	6	60	1000	200	12.729	0.0001	5	5	20	–
	[17], 2017	GA	6	50	300	200	–	0.0026	5	5	20	–
	[17], 2017	OHCRO	6	50	300	200	–	**0.000035**	5	5	20	–
	[26], 2017	FPA	6	50	1000	100	–	0.027896	4	4	40	–
	[26], 2017	BA	6	50	1000	100	–	0.268377	4	4	40	–
	[26], 2017	MFO	6	50	1000	100	–	0.032100	4	4	40	–
	[26], 2017	WOA	6	50	1000	100	–	0.030306	4	4	40	–
	[26], 2017	LWOA	6	50	1000	100	–	0.021095	4	4	40	–
	Present	**WS-MC**	6	–	100,000	200	134.82	0.11878	2	1.8	84	100%
		WS-VLPSO	6	50	200	200	**9.7367**	**0.0062**	**2**	**1.8**	**84.67**	**100%**

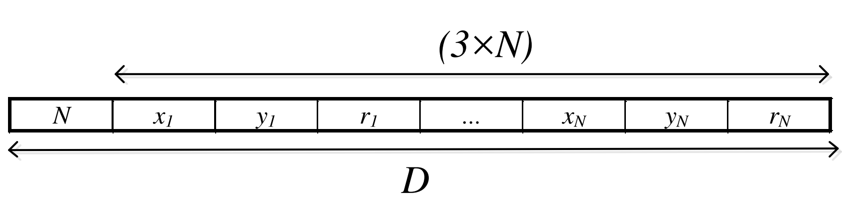

Fig. 10. Structure of variable length particle vector for sensor coverage.

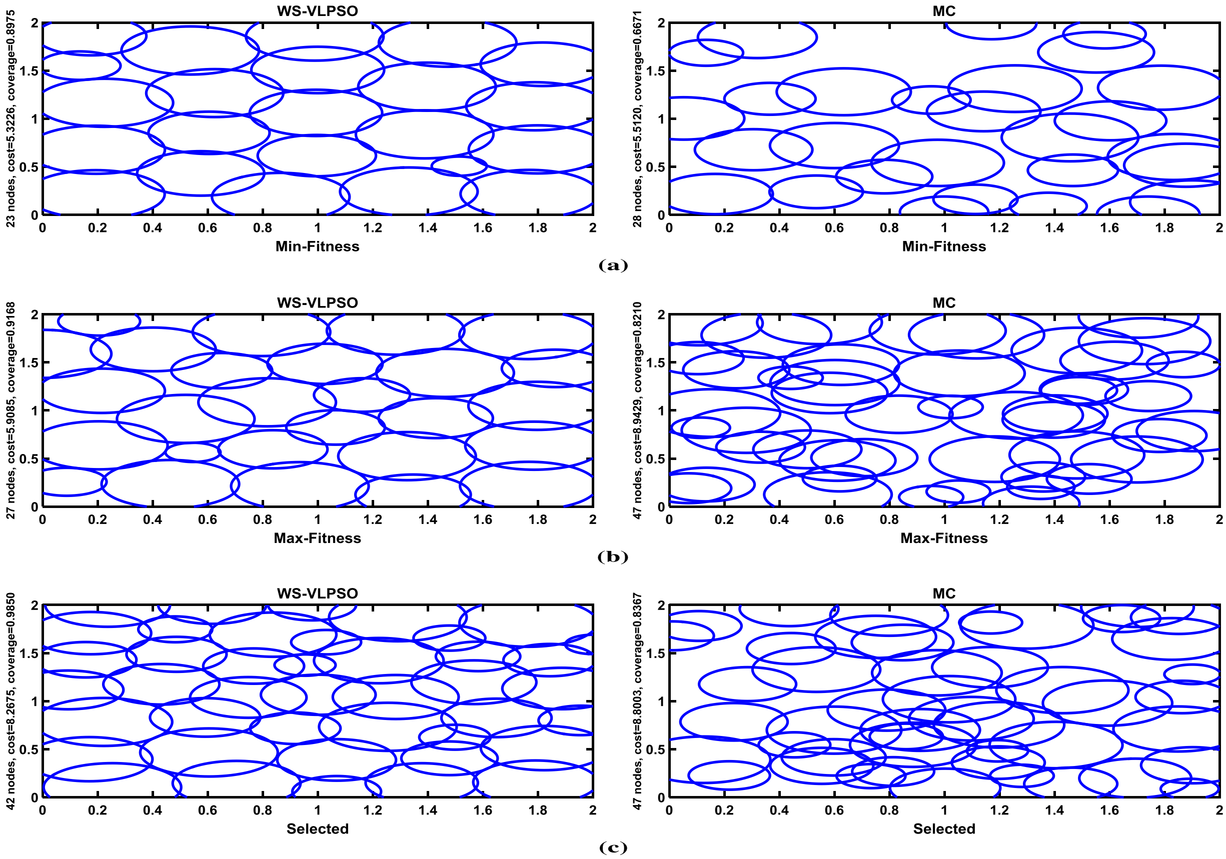

Fig. 11. Sensor coverage sample solutions using WS-VLPSO and WS-MC for: (a) minimum fitness, (b) maximum fitness, (c) selected.

Table 6
Comparative results between the WS-VLPSO and WS-MC for Sensor covering problem.

Indexes		WS-VLPSO	WS-MC
Fitness	Min	*6.8596*	*10.5054*
	Max	*10.0669*	*17.8919*
	Mean	9.16282	16.0898
	Median	9.1944	17.4044
	SD	0.7390	3.1409
Cost	Min	5.3226	5.512
	Max	9.5457	**9.4819**
	Mean	6.9188	8.3623
	Median	6.5976	8.9429
	SD	1.1994	1.6135
Coverage	Min	0.8975	0.6671
	Max	0.9938	0.8367
	Mean	0.9515	0.7988
	Median	0.9555	0.8334
	SD	0.0295	0.0739
Node	Min	23	28
	Max	50	**49**
	Mean	32.8	44
	Mode	30.50	47
	SD	7.9313	9
Time (m)	Min	21.6815	40.9105
	Max	48.0871	59.846
	Mean	32.3356	48.1088
	Median	30.9743	45.1534
	SD	7.3089	7.3636

the stability of modeled filtering systems. Therefore, it can be acknowledged that this performance proved the accuracy and efficiency of the proposed approach in the development of adaptive IIR digital filtering systems modeling. Also, its performance in solving the sensor coverage problem was verified in the form of another real-world variable length optimization problem, and the output of the implementations indicated the continued success of the new and dynamic version of the proposed VLPSO algorithm.

In future research, the proposed method needs to be explored for more complex systems modeling, such as adaptive nonlinear filtering systems and complex fractional systems. In addition to the PSO algorithm, future work can be concentrated on other optimization methods or more complex systems modeling, such as nonlinear systems and real-time modeling.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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