Wireless Channels

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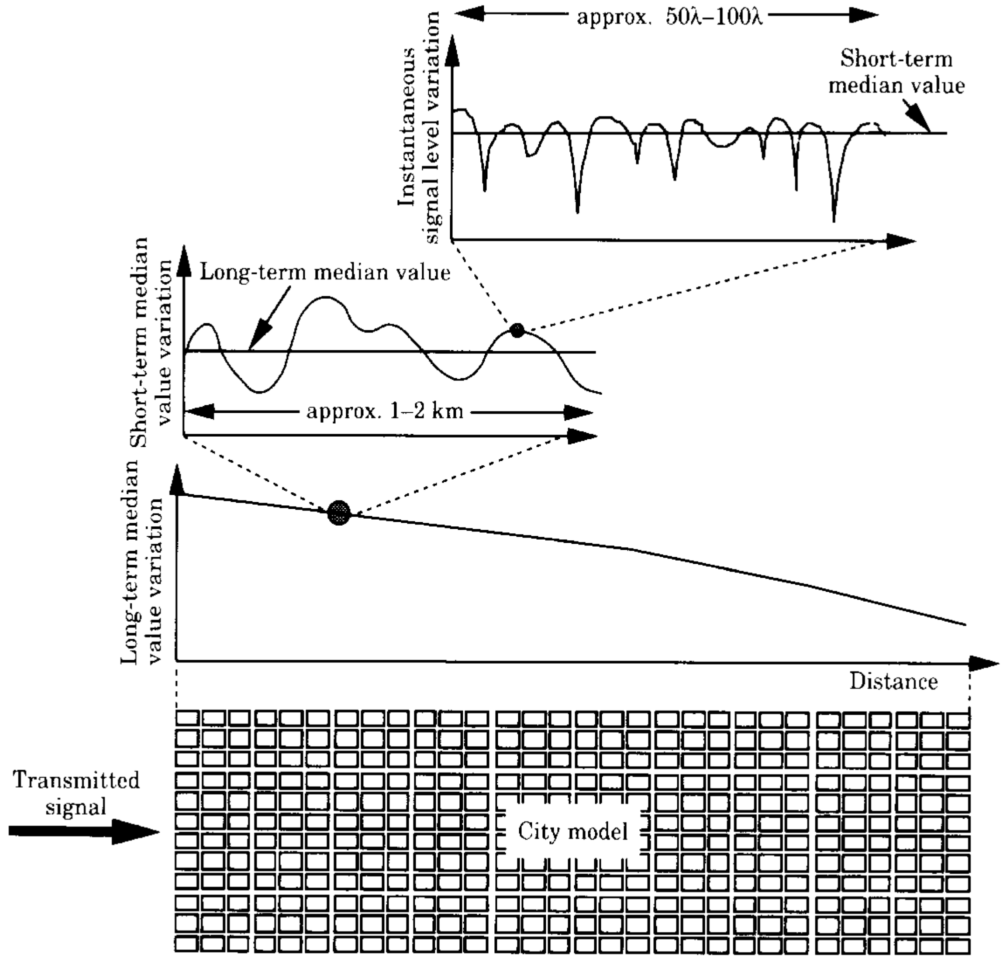
General Model for Wireless Channels

- Multipath Fading
  - constructive and destructive interference caused by multiple TX-RX paths with diff lengths arriving from diff directions
  - Signal envelope varies widely over 30 dB in the span of a few wavelengths in distance (e.g. λ = 1 ft when f_c=1GHz)
- Shadowing
  - Short-term average variation or large-scale signal variation
  - obtain by averaging over 50-100 wavelengths in distance
  - caused by local changes in terrain features or man-made obstacles (e.g. blockage)
- Path Loss Model
  - Long-term or large-scale average signal level
  - depends on the distance between TX and RX

General 3-level Model



Sampei, p. 16, Fig 2.1

General 3-level Model

- Path loss model is used for
  - system planning, cell coverage
  - link budget (what is the frequency reuse factor?)
- Shadowing is used for
  - power control design
  - 2nd order interference and TX power analysis
  - more detailed link budget and cell coverage analysis
- Multipath fading is used for
  - physical layer modem design --- coder, modulator, interleaver, etc

Ideal Path Loss Model

$$P_r = \frac{c^2 P_t G_t G_r}{16\pi^2} \frac{1}{d^2} \frac{1}{f^2}$$

dBm if P_r is in mWatt & dBW if P_r is in Watt

$$P_r\,(\text{in } dBm) = 10\log_{10} P_r$$

$$= 10\log_{10} P_t + C - 20\log_{10} f_c - 20\log_{10} d$$

$$PL_{\text{free space}}\,(\text{in } dB) = P_t\,(\text{in } dBm) - P_r\,(\text{in } dBm)$$

$$= -C + 20\log_{10} f + 20\log_{10} d$$

Path Loss Exponent = 2

$$\boxed{PL_{\text{free space}}\,(\text{in } dB) = PL(d_0) + 20\log\frac{d}{d_0}}$$

– Path Loss Exponent indicates how fast signal power drops with Tx-Rx separation
  - 2 means 6dB drop per doubling of the distance

Path Loss Exponent

- Path loss in dB depends on TX-RX distance via PL exponent, n.

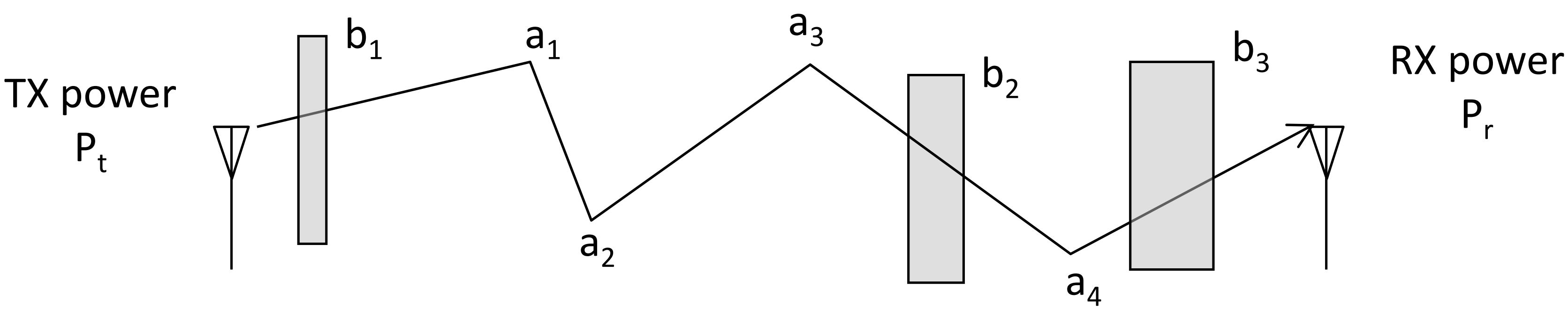
$$PL(d) \propto \left(\frac{d}{d_0}\right)^n$$

$$PL(d) = PL(d_0) + 10n\log\left[d / d_0\right]$$

Environment	Path Loss Exponent
Free Space	2
Urban area cellular	2.7 to 3.5
Shadowed urban cellular	3 to 5
In building line-of-sight	1.6 to 1.8
Obstructed in building	4 to 6
Obstructed in factories	2 to 3

Shadowing Effect

- Variations around the path loss predication due to buildings, hills, trees, etc.



  - Consider a signal undergoes multiple reflections (each with a power attenuation factor a_i) and passes through multiple obstacles (with factors b_i).

$$P_r = \prod_{i=1}^{4} a_i \prod_{i=1}^{3} b_i P_t$$

$$P_r(\text{in } dBm) = \sum_{i=1}^{4} 10\log(a_i) + \sum_{i=1}^{3} 10\log(b_i) + P_t(\text{in } dBm)$$

$$= \sum_i \alpha_i(\text{in } dB) + P_t(\text{in } dBm)$$

Shadowing Effect

- Each term introduces a random attenuation of α_i dB and they are assumed to be *statistically independent*
- As the number of these factors increases, by the central limit theorem, the sum, S, approaches a Gaussian (normal) random variable

$$P_r(\text{in } dBm) = S(\text{in } dB) + P_t(\text{in } dBm)$$

$$= m(\text{in } dB) + X(\text{in } dB) + P_t(\text{in } dBm)$$

where S~N(m,σ^2) and X~N(0,σ^2)

- the mean m is generally included in the Path loss model (that's why the path loss exponent can be larger than 2 as the number of terms generally increases with the TX-RX separation)

Shadowing Effect

– When we study only the Shadowing effect, we have

$$P_r(\text{in } dBm) = X(\text{in } dB) + P_t(\text{in } dBm)$$

where X is a zero mean Gaussian random variable with variance σ^2

– Expressing in linear scale, we have

$$P_r = 10^{(X/10)} P_t = A_s P_t$$

where A_s is the attenuation factor due to shadowing effect

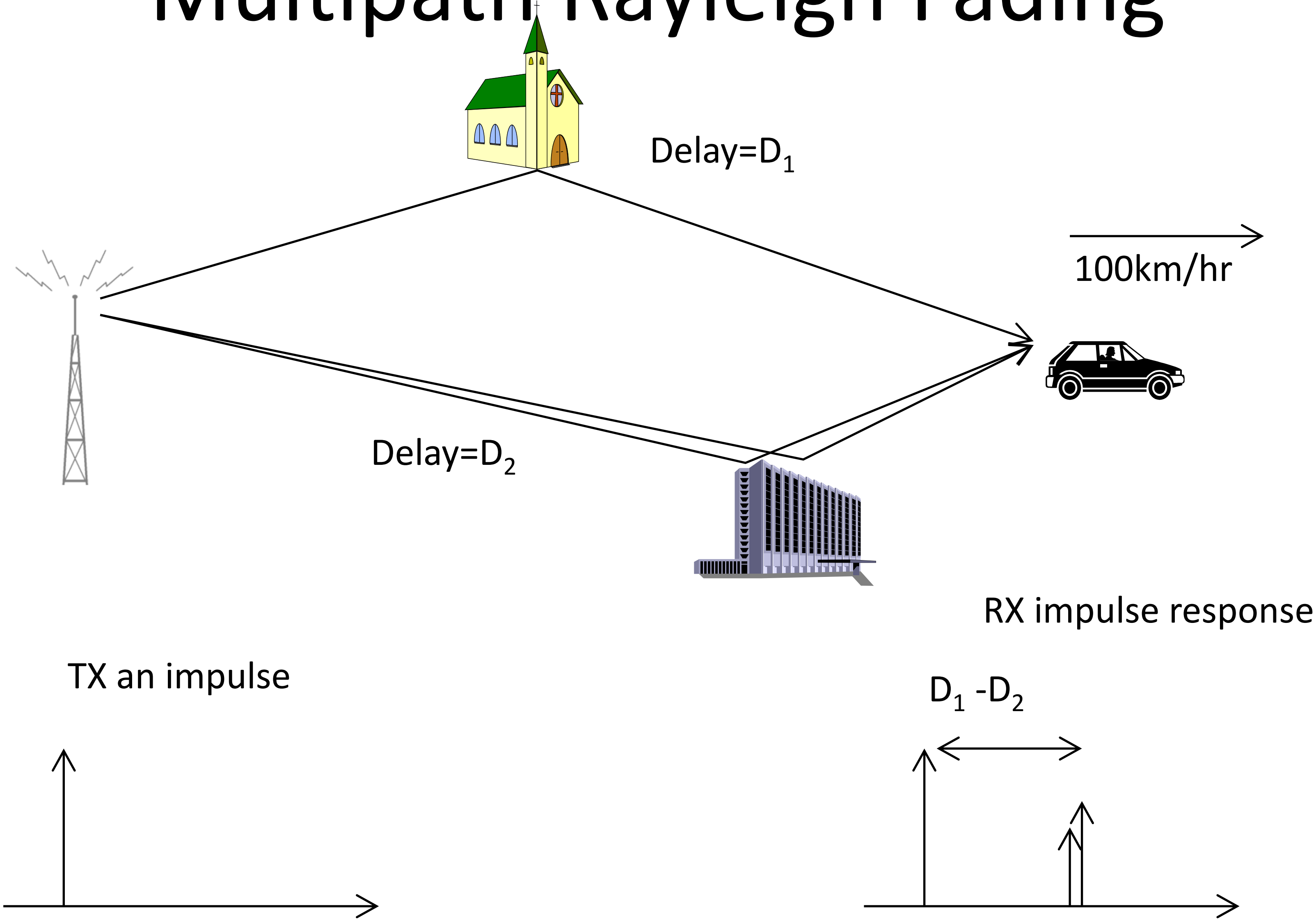
– Note that $\log(A_s)=X/10$ is normally distributed; hence, the distribution of A_s is known as the "Lognormal" distribution

– σ is called the standard deviation and has a unit of dB

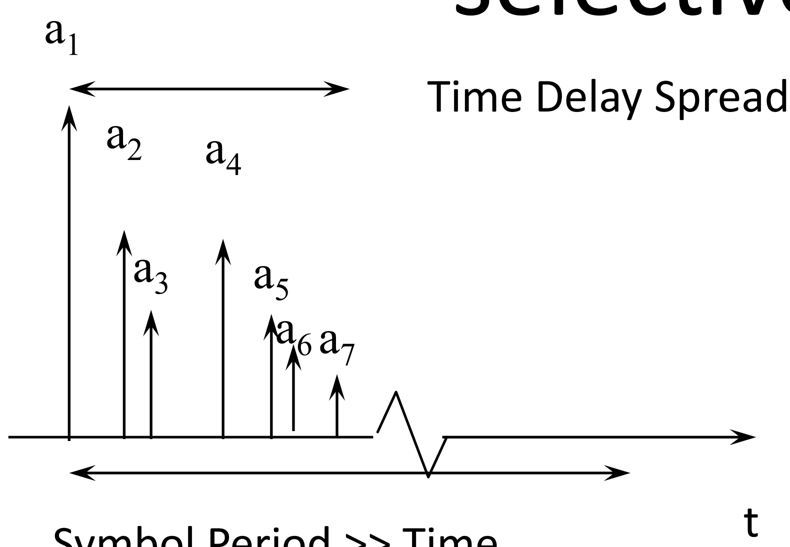
Shadowing Effect

- Variations around the median path loss line due to buildings, hills, trees, etc.
  - Individual objects introduces random attenuation of x dB, after pass through so many objects the attenuation factors multiply (or add in dB scale)
  - As the number of these x dB factors increases, the combined effects becomes Gaussian (normal) distribution (by central limit theorem) in dB scale: “Lognormal”
- $PL(dB) = PL_{avg}$ (dB) + X where X is $N(0,\sigma^2)$ where
  - PL_{avg} (dB) is obtained from the path loss model
  - σ is the standard deviation of X in dB

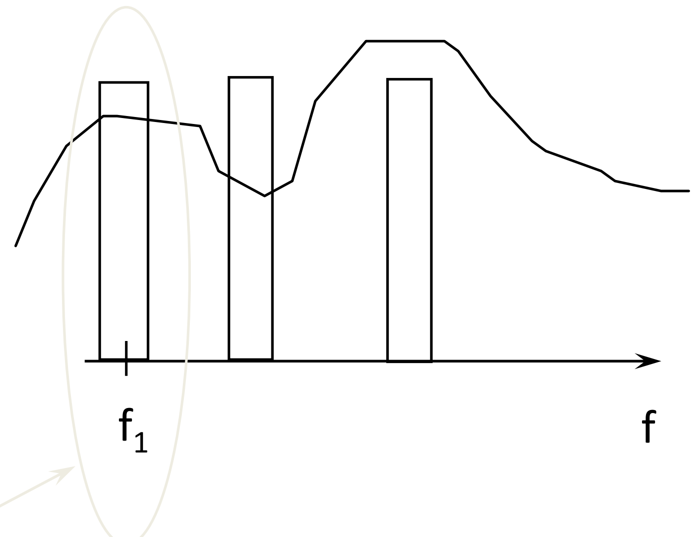
Multipath Rayleigh Fading



Narrowband TX: Frequency Non-selective Model

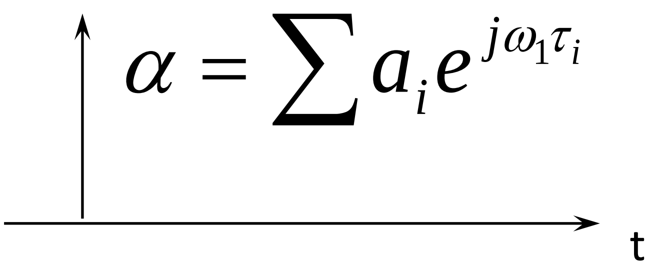


Symbol Period >> Time Delay Spread



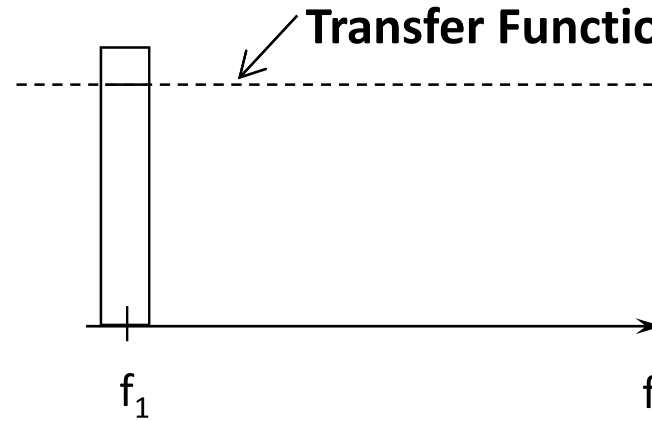
Equivalent Model:

$$\alpha = \sum a_i e^{j\omega_1 \tau_i}$$



y(t) = α x(t), t$\in$[0,T]

Equivalent Transfer Function



Rayleigh Fading (No Line of Sight)

$$\alpha = \sum \mathrm{Re}(a_i e^{j\omega_1 \tau_i}) + j\sum \mathrm{Im}(a_i e^{j\omega_1 \tau_i})$$

By Central Limit Theorem

$$= \alpha_I + j\alpha_Q$$

Independent zero mean Gaussian

$$= re^{j\theta}$$

$$f_{\alpha_I,\alpha_Q}(\alpha_I,\alpha_Q) = f_{\alpha_I}(\alpha_I) f_{\alpha_Q}(\alpha_Q) = \frac{1}{2\pi\sigma^2} e^{-\left(\alpha_I^2+\alpha_Q^2 \big/ 2\sigma^2\right)}$$

$$f_{R\Theta}(r,\theta) = f_\Theta(\theta) f_R(r) = \frac{1}{2\pi} \frac{r}{\sigma^2} e^{-\left(r^2/2\sigma^2\right)}$$

Phase is Uniform

Magnitude is Rayleigh

$$\text{where } \theta \in (-\pi,\pi], r \in [0,\infty)$$

Rayleigh Fading

$f_{\alpha_I,\alpha_Q}(\alpha_I,\alpha_Q)$: Independent Gaussian with mean σ^2

$$f_\Theta(\theta) = \frac{1}{2\pi} \quad \text{if } \theta \in [0,2\pi) \quad : \text{Uniform Phase}$$

$$f_R(r) = \frac{r}{\sigma^2}\exp\left(-\frac{r^2}{2\sigma^2}\right) \quad \text{if } r > 0 \quad : \text{Rayleigh Amplitude}$$

$$f_P(p) = \frac{1}{P_o}\exp\left(-\frac{p}{P_o}\right) \quad \text{if } p > 0 \quad : \text{Exponential Channel Power Gain}$$

where $p = r^2$ and $P_o = 2\sigma^2$ is mean channel power gain