

Homework 3

0.7/EM: application Simple

$$A=a, B=b, C=c$$

1- la vraisemblance

$$L = \prod_{i=1}^n p(A_i, B_i, C_i)$$

$$L(B, C) = \prod_{i=1}^n \sum_C p(A_i, B_i, C_i)$$

$$\log(L) = \sum_{i=1}^n \log \sum_A p(A_i, B_i, C_i) = \sum_{i=1}^n \log \sum_A p(A_i) p(B_i) p(C_i | A_i)$$

$$\log(L) = \sum_{i=1}^n \log \left(A_i \cdot \sum_A p(C_i, A_i, B_i) p(A_i) \right)$$

2- fonction auxiliaire

$$b_{n,c}(a|b) = p(A=a | B=b, C=c, \theta)$$

$$= \frac{p(A=a) p(B=b) p(C=c | A=a, B=b)}{p(A=a) \sum_b p(B=b) \sum_c p(C=c | A=a, B=b)}$$

$$= \frac{p(A=a) p(B=b) p(C=c | A=a, B=b)}{p(A=a)}$$

$$= p(B=b) p(C=c | A=a, B=b)$$

$$= p(C=c | A=a)$$

Don la fonction auxiliaire:

$$Q_i(\theta) = \sum_{i=1}^N \sum_c \delta_{B_i, c_i}(\alpha/\theta') \log \left(\frac{p(\alpha/\theta) p(B_i/\theta) p(c_i/\alpha, B_i, \theta)}{p(c_i/\alpha, B_i, \theta)} \right)$$

$$= \sum_{i=1}^N \sum_c p(c=c_i/\alpha/\theta') \log(p(\alpha/\theta) p(B_i/\theta) p(c_i/\alpha, B_i, \theta))$$

3. les équations de MAF

Step 1 $Q_i^A \propto p(A/B_i=b, C_i=c/\theta')$

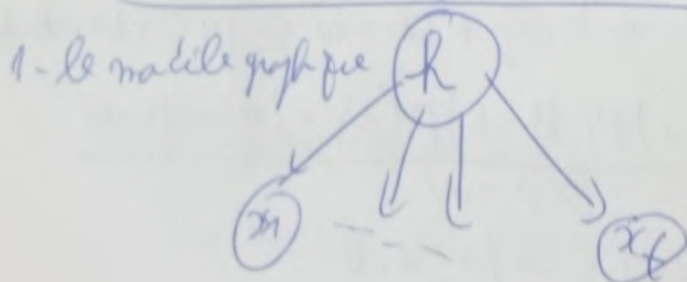
$$p(A/B=b, C=c) \propto p(A, B=b, C=c)$$

$$p(A/b, c) \propto \prod_a \prod_b \pi_{ab}^c$$

Step 2

$$Q_i \propto \arg \max_{\theta} F(g, \theta)$$

O.2/ Quelle la longueur de ce mat



② Les paramètres du modèle

$h \in H, x \in X$, Les paramètres sont $p(h)$ et $p(x_t | x_{t-1}, h)$

on pose $|h| = l, |x| = n$

d'où on a $(l + 2^n(l^n) = l + 2^n l)$ paramètres

③ La vraisemblance

$$P(X|\theta) = \prod_{i=1}^N \sum_h \prod_t p(x_t^i | x_{t-1}^i, h, \theta) p(h|\theta)$$

④ la vraisemblance

$$L = \prod_{i=1}^N \sum_h \prod_t p(x_t^i | x_{t-1}^i, h) p(h)$$

~~la vraisemblance~~

$$\log(L) = \sum_{i=1}^N \log \left(\sum_h \prod_t p(x_t^i | x_{t-1}^i, h) p(h) \right)$$

⑤ la fonction auxiliaire

$$Q_x(h|\theta) = P(h|x, \theta)$$

$$= \frac{\prod_t P(h|\theta) p(x_t | x_{t-1}, h, \theta)}{\sum_{x \in \{1, \dots, 2^n\}} \prod_t P(h|\theta) p(x_t | x_{t-1}, h, \theta)}$$

③

$$\begin{aligned}
 Q_0(\theta) &= \sum_{n=1}^N \sum_k t_{x^n}(k|\theta^n) \log p(x^n, k|\theta) \\
 &= \sum_{n=1}^N \sum_k t_{x^n}(k|\theta^n) \sum_t \log p(k|\theta) \\
 &\quad p(x_t | x_{t-1}, k, \theta)
 \end{aligned}$$

① Equation de MAF

étape 1 $Q_i^k \leftarrow p(k | x_{(1:T)}^k)$

Step $\theta \leftarrow \arg \max_{\theta} F(g, \theta)$