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HOMEWORK 5

1/ Elimination des variables

X_V : l'ensemble de toutes les variables

X_O : l'ensemble des variables observées

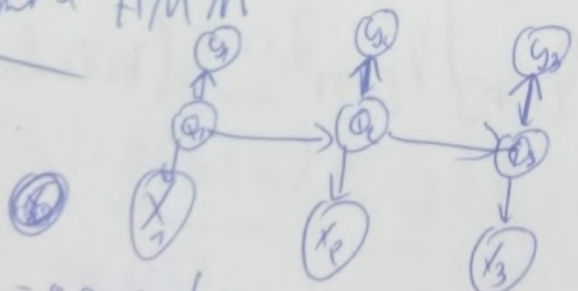
X_R : $\emptyset$ // des // à maximiser

X_I : $X_V \setminus (X_R \cup X_O)$ variables à éliminer

$\underset{X_R}{\operatorname{argmax}}\, p(X_R, X_I, X_O) = \underset{X_{X_{R_1}, X_{R_2}}}{\operatorname{argmax}}\, p(X_R, X_I, X_O)$

$\underset{X_{R_i} \notin X_R}{\operatorname{argmax}} \sum p(X_R, X_I, X_O) \prod_{i} F_i(\,) \quad F_i(\,) \notin X_{R_i}$

2/ Double HMM



$q^*_{[1:T]} = \underset{q_{[1:T]}}{\operatorname{argmax}}\, (x_{q_{[1:T]}} \mid x_{[1:T]})$

$$p(q_{[1:T]} \mid x_{[1:T]}) = \sum_{y_{[1:T]}} \left(p(q_1)\, p(x_1 \mid q_1)\, p(y_1 \mid q_1) \prod_{t=2}^{T} p(q_t \mid q_{t-1})\, p(x_t \mid y_t) \right)$$

$p(y_t \mid q_t)$ (1)

$$p(q_{1:T} | x_{[1:T]}) = \sum_{q_{[1:T-1]}} p(q_1) p(x_1|q_1) p(y_1|q_1) \prod_{t=1}^{T-1} p(q_t|q_{t-1})$$

$$p(y_t | q_t)\ p(q_t|q_{t-1}) \sum_{q_T} p(q_t|q_{t-1})\, p(q_T|q_T)$$

$$= \sum_{q_{[1:T-1]}} \Big(p(q_1) \cdot p(y_1|q_1)\, p(q_1|q_1) \prod_{t=1}^{T-1} p(q_t|q_{t-1}) p(x_t|q_t) p(y_t|q_t) \Big)$$

$$p(q_t|q_{t-1})\, p(y_t|q_t) \Big)$$

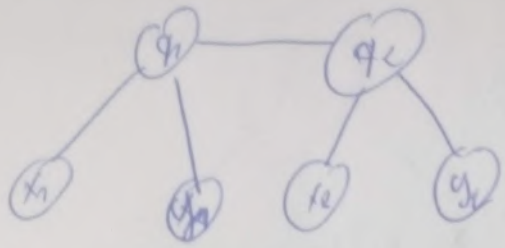
$$p(q_{[1:T]} | x_{[1:T]}) = p(q_1)\, p(x_1|q_1) \prod_{t=1}^{T} p(q_t|q_{t-1}) p(y_t|q_t)$$

$$q^*_{[1:T]} = \underset{q_{[1:T]}}{\operatorname{argmax}}\ p(q_{[1:T]} | x_{[1:T]}) \quad \text{(Viterbi)}$$

2. Calculer $\underset{q_{[1:T]}}{\operatorname{argmax}}\ p(q_{[1:T]} | x_{[1:T]})$
on applique la moralisation et triangularisation
(algorithme de passage de message)

(2)

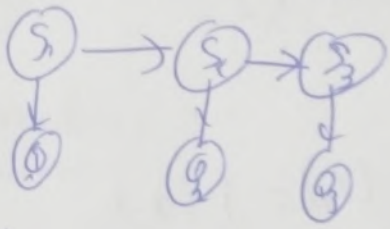
On obtient cet arbre



3/ refaire les calculs

-1- Forward backward comme passage de message

On suppose



S_1 : etat

Q_1 : observation

$m_{i \to j}(j)$: message de i à j

$$p(S_2 | Q_{[1:3]}) \propto p(Q_{[1:2]}, S_2)\, p(Q_3 | S_2)$$

$$p(Q_{[1:2]}, S_2) = \underbrace{p(S_1)\, p(Q_1 | S_1)}_{m(S_1)} \; p(S_2 | S_1) \to m(S_2)$$

$Q_1 \to S_1$; $S_2 \to S_2$

$$p(Q_3 | S_2) = \underbrace{p(S_3 | S_2)}_{m_{S_3, S_2}(S_2)} \; \underbrace{p(Q_3 | S_3)}_{m(S_3)}$$

$Q_3 \to S_3$

$$p(S_2 | Q_{[1:3]}) \propto \left(m(S_1), m(S_2) \right) \left(m(S_3), m(S_2) \right)$$

$Q_2 \to S_2$; $S_1 \to S_2$

Forward-backward $\Longleftrightarrow$ passage de message

4/ application de JTA(1)

1- arbre de jonction

$$p(x_1,x_2,x_3,x_4) = \varphi_1(x_1,x_2)\,\varphi_2(x_2,x_3)\,\varphi_3(x_3,x_4)\,\varphi_4(x_4,x_2)$$

$$p(x_1) = \sum_{x_2 x_3 x_4} \varphi_1\varphi_2\varphi_3\varphi_4 = \left(\sum_{x_4}\varphi_4\right)\left(\sum_{x_2 x_3}\varphi_2\varphi_3\varphi_1\right)$$

$$\sum_{x_4}\varphi_4 = m_{4\to 1}(x_2) = f_2(x_1)$$

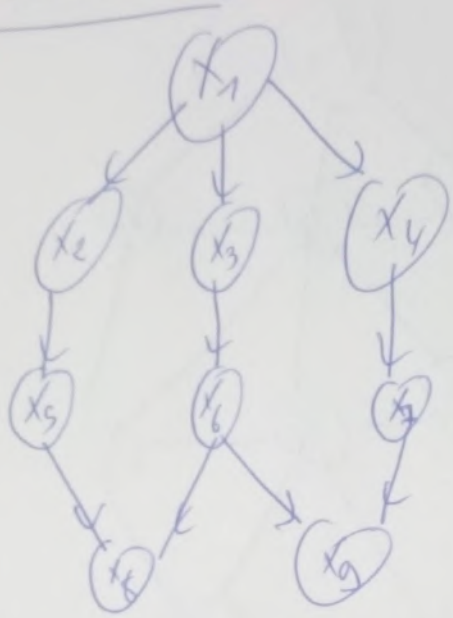
$$\sum_{x_2 x_3}\varphi_2\varphi_3\varphi_1 = m_{2\to 1}(x_1) = f_2(x_1)$$

d'où $p(x_1) = \left(f_1(x_1)\,f_2(x_1) + f_2(x_1)\right)/Z$

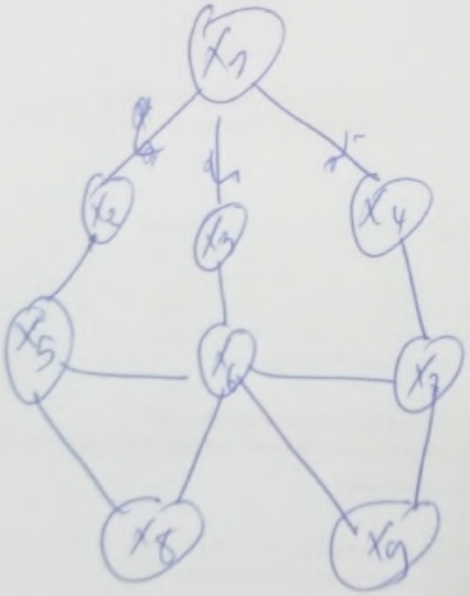
dont $Z = \sum_{x_1} f_1(x_1) f_2(x_1)$

5/ application de JTA(2)

1. le réseau bayesien



2. graphe moralisé



3. grafo triangolare