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FA21-BEE-017

Q1

We have $A(4,-4,1)$

$B(-4,3,-4)$

$C(4,-1,-2)$

$AB = OB - OA$

$$= \begin{bmatrix} -4 \\ 3 \\ -4 \end{bmatrix} - \begin{bmatrix} 4 \\ -4 \\ 1 \end{bmatrix} = \begin{bmatrix} -8 \\ 7 \\ -5 \end{bmatrix} = -8i + 7j - 5k$$

$AC = OC - OA$

$$= \begin{bmatrix} 4 \\ -1 \\ -2 \end{bmatrix} - \begin{bmatrix} 4 \\ -4 \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ 3 \\ -3 \end{bmatrix} = 3j - 3k$$

$$AB \times AC = \begin{vmatrix} i & j & k \\ -8 & 7 & -5 \\ 0 & 3 & -3 \end{vmatrix}$$

$$= (-21+15)i - (24-0)j + (-24)k$$

$$= -6i - 24j - 24k$$

$n = i + 4j + 4k$

$d = a.n = 4i - 4j + k \cdot (i + 4j + 4k)$

$= 4 - 16 + 4 = -8$

Eq of plane

$r \cdot n = d$

$r \cdot (i + 4j + 4k) = -8$

$(xi + yj + zk) \cdot (i + 4j + 4k) = -8$

$x + 4y + 4z = -8$

$x + 4y + 4z + 8 = 0$

(b)

perp $= \frac{d}{|n|}$ $\frac{+8}{\sqrt{1^2 + 4^2 + 4^2}} = \frac{8}{33}$

(c)

Line : $r = a + \lambda b$

$$r = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} + \lambda \begin{pmatrix} 2 \\ +3 \\ -3 \end{pmatrix}$$

value of λ

$$\begin{pmatrix} 2\lambda \\ +3\lambda \\ -3\lambda \end{pmatrix} \cdot \begin{pmatrix} 1 \\ 4 \\ 4 \end{pmatrix} = -8$$

$2\lambda + 12\lambda - 12\lambda = -8$

$2\lambda = -8$

$\boxed{\lambda = -4}$

$$r = \begin{pmatrix} 2(-4) \\ 3(-4) \\ -3(-4) \end{pmatrix} = (-8, -12, 12)$$

Q3

ⓐ find value of t.

$l_1 = ti + j \qquad -2i - j$

$l_2 = j + tk \qquad -2j + k$

shortest distance b/w $l_1, l_2 = \sqrt{2}1$

$r_1 = OA + \lambda AB$

$r_2 = OA + \lambda AB$

$r_1 = ti + \hat{j} + \lambda\, 2\hat{i} - \hat{j}$

$$L_1 = r_1 \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix} + \lambda \begin{bmatrix} -2 \\ 10 \\ 0 \end{bmatrix}$$

$$L_2 = r_2 - \begin{bmatrix} 0 \\ 1 \\ t \end{bmatrix} + u \begin{bmatrix} 0 \\ -2 \\ 1 \end{bmatrix}$$

$$D = \frac{(b_2 \times b_1) \cdot (a_2 - a_1)}{|b_1 \times b_2|}$$

$b_1 \times b_2$ $\begin{vmatrix} i & j & k \\ 2 & -1 & 0 \\ 0 & -2 & 1 \end{vmatrix}$

$= i \begin{vmatrix} -1 & 0 \\ -2 & 1 \end{vmatrix} - j \begin{vmatrix} -2 & 0 \\ 0 & 1 \end{vmatrix} + k \begin{vmatrix} -2 & -1 \\ 0 & -2 \end{vmatrix}$

$= i(-1) - j(-2) + \hat{k}(4)$

$|b_1 \times b_2| = \sqrt{(-1)^2 + (2)^2 + (4)^2}$

$= \sqrt{21}$

$a_2 - a_1$

$= \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix} - \begin{bmatrix} t \\ t \\ 0 \end{bmatrix}$

$- ki + tk$

$D = \dfrac{(-i + 2j + 4k) \cdot (-ti + tk)}{\sqrt{21}}$

$\sqrt{21} = \dfrac{t + 4t}{\sqrt{21}}$

$21 = t + 4t$

$21 = 5t$

$\boxed{t = \dfrac{21}{5}}$

part(i)

$r_1 = \frac{21}{5}\hat{i} + j + \lambda(2i - j)$

$r_2 = j - \frac{21}{5}\hat{k} + \mu(-2j + k)$

$\lambda_1 = r = a + \lambda AB + \mu AC$

$r = -\frac{21}{5}\hat{i} + j + \lambda(2i - \hat{j}) + \mu(-2j +$

part(ii)

$\lambda_2 = 5x - 6y + 5z = 0$

$\lambda_2 = \frac{x-0}{0}$, $\lambda_2 = \frac{y-1}{-2}$

$\lambda_2 = \frac{2 - 4 \cdot 2}{1}$

from l_2 direction vector is

$= \{0, -2, 1\}$

from π_2 normal vector is

$= (5, -6, 7)$

$$\cos\theta = \frac{a_1 b_1 + a_2 b_2 + a_3 b_3}{|a| \, |b|}$$

$$a \cdot b \quad \begin{bmatrix} 0 \\ 1 \\ -21/5 \end{bmatrix} \cdot \begin{bmatrix} 5 \\ -6 \\ 7 \end{bmatrix}$$

$= \cancel{6 + 49}$

$= -6 + 29.4 = 23.4$

$|a| = \sqrt{1^2 + \left(\frac{21}{5}\right)^2} = 1 + 17.64$

$\boxed{|a| = 4.3}$

$|b| = \sqrt{(5^2 + 6^2 + 7^2)}$

$|b| = 10.49$

$\theta = \cos^{-1}\left(\frac{23.4}{4.3 \times 10.49}\right)$

$\boxed{\theta = 59.34}$

Question 5

(part e)

~~if p = (-2, -1) and Ø = -6 -3~~

Mid point

p = (-2, -1) Ø = (-6,

$\left(\frac{-2-6}{2}\right) \quad \left(\frac{-1-3}{2}\right)$

$\frac{-8}{2}, \frac{-4}{2}$

$(-4, -2)$

equation of circle

$(x+4)^2 + (y+2)^2 = r^2$

$(x, y) = (-2, -1)$

$(-2+4)^2 + (-1+2)^2 = r^2$

$(2)^2 + (1)^2 = r^2$

$\boxed{r = 5}$

values

$\boxed{(x+4)^2 + (y+2)^2 = 5}$

equation of circle

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(b)

equation of circle

$(x-b)^2 + (y-b)^2 = r^2$

$x=0$ $y=b$

at point $(4,0)$

$4^2 + (-b)^2 = r$

at point $(0,2)$

$(0)^2 + (2-b)^2 = r^2$

$(2-b)^2 = r^2$

$16+b^2 = (2-b)^2$

$16+b^2 = 4-4b+b^2$

$12 + 4b = 0$

$\boxed{b = -3}$

values

$r^2 = 4^2 + 3^2$

$\boxed{= \pm 5}$ $\boxed{r = 5}$

(c)

$y^2 = 100x$

compare

$y^2 = 4ax$

$4a = 100$

$\boxed{a = \cancel{100}\ 25}$

equation of diretix $= x = -a$

$x = -25$

(d)

$x^2 = 24y$

Compare $x^2 = 4ay$

$4a = 24$

$\boxed{a = 8}$

so focus is $f(a, 0)$

and equation of direchrix

$x = -a$

$\boxed{x = -8}$

(e)

$$\left(\frac{x^2}{25^2}\right) + \left(\frac{y}{16}\right)^2 = 1$$

Compare

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = \frac{x^2}{5^2} + \frac{y^2}{4^2}$$

$a = 5$

$b = 4$

$c = \sqrt{a^2 - b^2} = 25 - 16$

$= \pm 3$

$F_1 = (3, 0)$

$F_2 = (-3, 0)$

major axis $= 2a$

$2(5) = 10$

(f)

major axis $= 10$

meqor axis $= 8$

$2a = 10$

$\boxed{a = 5}$

$2b = 8$

$b = 4$

Equation of ellipse

$$\left(\frac{x}{a}\right)^2 + \left(\frac{y}{b}\right)^2 = 1$$

$$\left(\frac{x}{5}\right)^2 + \left(\frac{y}{4}\right)^2 = 1$$

Question no. 02

(a)

$r_1 = OA - OB$

$AB = OB - OA$

$$\begin{bmatrix} 11 \\ 3 \\ 0 \end{bmatrix} - \begin{bmatrix} 7 \\ 4 \\ -1 \end{bmatrix} = \begin{bmatrix} 4 \\ -1 \\ 1 \end{bmatrix}$$

$$r_1 = \begin{bmatrix} 7 \\ 4 \\ -1 \end{bmatrix} + \lambda \begin{bmatrix} 4 \\ -1 \\ 1 \end{bmatrix}$$

$r_2 = OC - CD$

$CD = OD - OC$

$$\begin{bmatrix} 2 \\ 7 \\ \lambda \end{bmatrix} - \begin{bmatrix} 2 \\ 6 \\ 3 \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \\ \lambda - 3 \end{bmatrix}$$

$$|b_1 \times b_2| = \begin{vmatrix} i & j & k \\ 4 & -1 & 1 \\ 0 & 1 & \lambda-3 \end{vmatrix}$$

$$= i \begin{vmatrix} -1 & 1 \\ 1 & \lambda-3 \end{vmatrix} - j \begin{vmatrix} 4 & 1 \\ 0 & \lambda-3 \end{vmatrix} + k \begin{vmatrix} 4 & -1 \\ 0 & 1 \end{vmatrix}$$

$$= i\,[-(\lambda-3)-1] - j\,[4(\lambda-3)] + k(4)$$

$$a_2 - a_1 = \begin{bmatrix} 2 \\ 6 \\ 3 \end{bmatrix} - \begin{bmatrix} 7 \\ 4 \\ -1 \end{bmatrix} = \begin{bmatrix} -5 \\ 2 \\ 4 \end{bmatrix}$$

$$|b_1 \times b_2| = \sqrt{(2-\lambda)^2 + (4\lambda-12)^2 + (4)^2}$$

$$= \sqrt{4 + \lambda^2 - 4\lambda + 16\lambda^2 + 144 - 96\lambda + 16}$$

$$= \sqrt{17\lambda^2 - 100\lambda + 164}$$

$$(b_1 \times b_2) \cdot a_1 - a_2 \; [(2-\lambda)i - j(4\lambda-12) + 4k]$$

$$\cdot (-5i + 2j + 4k)$$

$$= -5(2-\lambda) - 2(4\lambda-12) + 16$$

$$= 30 - 3\lambda$$

$$d = \frac{(b_1 \times b_2)\cdot(a_1 - a_2)}{|b_1 \times b_2|}$$

$$3 = \frac{30 - 3\lambda}{\sqrt{17\lambda^2 - 100\lambda + 164}}$$

square

$$9 = \frac{900 + 9\lambda^2 - 180\lambda}{17\lambda^2 - 100\lambda + 164}$$

$$9(17\lambda^2 - 100\lambda + 164) = 900 + 9\lambda^2 - 180\lambda$$

$$144\lambda^2 - 720\lambda + 576 = 0$$

$$16(9\lambda^2 - 45\lambda + 36) = 0$$

$$9\lambda^2 - 45\lambda + 36$$

$$\lambda^2 - 5\lambda + 4 = 0$$

$$\lambda(\lambda - 4) - 1(\lambda - 4) = 0$$

$$\boxed{\lambda = 4} \qquad \boxed{\lambda = 1}$$

$\boxed{\lambda = 4}$ in eq 1

$$(4)^2 - 5(4) + 4 = 0$$

$$\boxed{0 = 0}$$

$\boxed{\lambda = 1}$ in eq ①

$1 - 5 + 4 = 0$

$0 = 0$

part b

① $\lambda_1 = r_1 = OA + s AB + t AD$

$\boxed{\lambda = 4}$

AD = OD - OA

$$\begin{bmatrix} 2 \\ 7 \\ 1 \end{bmatrix} - \begin{bmatrix} 7 \\ 4 \\ 1 \end{bmatrix} = \begin{bmatrix} -5 \\ 3 \\ 2 \end{bmatrix}$$

$\pi_1 = 7i + 4j - k + s(4i - j + k) + t(-5i + 3j + 2k)$

② $\pi_2 = r_2 = OA + \lambda AB + \mu AD$

$\lambda = 4$

AD = OD - OA

$\bar{r}_2 = r_2 = OA + \lambda AB + \mu AD$

$$= \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 7 \\ 4 \\ -1 \end{bmatrix} + \lambda \begin{bmatrix} 4 \\ -1 \\ 1 \end{bmatrix} + \mu \begin{bmatrix} -5 \\ 3 \\ 5 \end{bmatrix}$$

$x = 7 + 4\lambda - 5\mu$ (i)

$y = 4 - \lambda + 3\mu$ (ii)

$z = -1 + \lambda + 5\mu$ (iii)

eq (ii) to (iii)

$y = 4 - \lambda + 3\mu$

$z = -1 + \lambda + 5\mu$

$y + z = 3 + 8\mu$ — (iv)

eq (ii) by 2

$y = 4 - \lambda + 3\mu$

$\times 4$

$4y = 16 - 4\lambda + 12\mu$ — (v)

eq (i) to eq (v)

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$x = \cancel{10}\ 7 + 4\lambda - 5\mu$

$4y = 16 - 4\lambda + 12\mu$

$x + 4y = 23 + 7\mu$

$\mu = \frac{x + 4y - 23}{7}$

values in eq (ii)

$y + z = 3 + 8\left(\frac{x + 4y - 23}{7}\right)$

$y + z = 3 + \frac{8x + 32y - 184}{7}$

$7y + 7z = 21 + 8x + 32y - 184$

$-8x - 25y + 7z + 163 = 0$

$-(8x + 25y - 7z - 163) = 0$

$8x + 25y - 7z = 163$

$-(5x + 13y - 7z - 94)$

$\theta = \cos^{-1} \frac{\bar{\lambda}_1 \cdot \bar{\lambda}_2}{|\bar{\lambda}_1| \cdot |\bar{\lambda}_2|}$

$\bar{\lambda}_1 \cdot \bar{\lambda}_2 = \begin{bmatrix} 5 \\ 13 \\ -7 \end{bmatrix} \begin{bmatrix} 8 \\ -25 \\ -7 \end{bmatrix}$

$40 + 325 + 49 = 414$

$|n_1| = \sqrt{5^2 + 13^2 + (-7)^2} = 9\sqrt{3}$

$|n_2| = \sqrt{8^2 + 25^2 + (-7)^2} = 3\sqrt{82}$

$\theta = \cos^{-1}\left(\frac{414}{27\sqrt{246}}\right)$

$\boxed{\theta = 12.15^0}$

(c)

$\bar{\lambda}_1 = r_1 = OA + \lambda AB + \mu AD$

$x = 7 + 4\lambda - 5\mu$ (1)

$y = 4 - \lambda + 3\mu$ (2)

$z = -1 + \lambda + 2\mu$ (3)

eq (2) ~~to~~ eq (3)

$y = 4 - \lambda + 3\mu$

$z = -1 + \lambda + 2\mu$

$y + z = 3 + 5\mu$ — (4)

eq ② by 4

$4y = 16 - 4\lambda + 12$

add eq ① and eq ②

$x = 7 + 4\lambda - 54$

$16 - 4\lambda + 12$

$x + 4y \quad 23 + 74$

$\frac{x + 4y - 23}{7} = 4$

$y + z \quad 3 + \left(\frac{5x + 20y - 115}{7} \right)$

$7y + 7z - 21 \quad 5x + 20y - 115$

$-5x - 13y + 7z + 94 = 0$